Package 'FRK'

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Type Package

Title Fixed Rank Kriging

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VignetteBuilder knitr

Description A tool for spatial/spatio-temporal modelling and prediction with large datasets. The approach models the field, and hence the covariance function, using a set of basis functions. This fixed-rank basis-function representation facilitates the modelling of big data, and the method naturally allows for non-stationary, anisotropic covariance functions. Discretisation of the spatial domain into so-called basic areal units (BAUs) facilitates the use of observations with varying support (i.e., both point-referenced and areal supports, potentially simultaneously), and prediction over arbitrary user-specified regions. `FRK` also supports inference over various manifolds, including the 2D plane and 3D sphere, and it provides helper functions to model, fit, predict, and plot with relative ease. Version 2.0.0 and above also supports the modelling of non-Gaussian data (e.g., Poisson, binomial, negative-binomial, gamma, and inverse-Gaussian) by employing a generalised linear mixed model (GLMM) framework. Zammit-Mangion and Cressie <doi:10.18637/jss.v098.i04> describe `FRK` in a Gaussian setting, and detail its use of basis functions and BAUs, while Sainsbury-Dale et al. <arXiv:2110.02507> describe `FRK` in a non-Gaussian setting; two vignettes are available that summarise these papers and provide additional examples.

BugReports https://github.com/andrewzm/FRK/issues/

Depends R (>= 3.5.0)

Suggests covr, dggrids, gstat, INLA, knitr, mapproj, parallel, rgdal, sf, spdep, splancs, testthat, verification

Imports digest, dplyr, ggplot2, grDevices, Hmisc (>= 4.1), Matrix, methods, plyr, Rcpp (>= 0.12.12), sp, spacetime, sparseinv, statmod, stats, TMB, utils, ggpubr, reshape2, scales

Additional_repositories https://andrewzm.github.io/dggrids-repo/, https://inla.r-inla-download.org/R/stable/

License GPL (>= 2)

NeedsCompilation yes

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R **topics documented:**

AIRS_05_2003   *AIRS data for May 2003*

Description

Mid-tropospheric CO_2 measurements from the Atmospheric InfraRed Sounder (AIRS). The data are measurements between 60 degrees S and 90 degrees N at roughly 1:30 pm local time on 1 May through to 15 May 2003. (AIRS does not release data below 60 degrees S.)

Usage

```
AIRS_05_2003
```

Format

A data frame with 209631 rows and 7 variables:

year year of retrieval

month month of retrieval

day day of retrieval

lon longitude coordinate of retrieval

lat latitude coordinate of retrieval

co2avgret CO2 mole fraction retrieval in ppm

co2std standard error of CO2 retrieval in ppm

References

Chahine, M. et al. (2006). AIRS: Improving weather forecasting and providing new data on greenhouse gases. Bulletin of the American Meteorological Society 87, 911–26.

Am_data   *Americium soil data*

Description

Americium (Am) concentrations in a spatial domain immediately surrounding the location at which nuclear devices were detonated at Area 13 of the Nevada Test Site, between 1954 and 1963.

Usage

```
Am_data
```

Format

A data frame with 212 rows and 3 variables:

Easting Easting in metres

Northing Northing in metres

Am Americium concentration in 1000 counts per minute

References

Paul R, Cressie N (2011). "Lognormal block kriging for contaminated soil." European Journal of Soil Science, 62, 337–345.

---

auto_basis   *Automatic basis-function placement*

Description

Automatically generate a set of local basis functions in the domain, and automatically prune in regions of sparse data.

Usage

```
auto_basis(
  manifold = plane(),
  data,
  regular = 1,
  nres = 3,
  prune = 0,
  max_basis = NULL,
  subsamp = 10000,
  type = c("bisquare", "Gaussian", "exp", "Matern32"),
  isea3h_lo = 2,
```

```
        bndary = NULL,
        scale_aperture = ifelse(is(manifold, "sphere"), 1, 1.25),
        verbose = 0L,
        buffer = 0,
        tunit = NULL,
        ...
)
```

Arguments

manifold	object of class `manifold`, for example, `sphere` or `plane`
data	object of class `SpatialPointsDataFrame` or `SpatialPolygonsDataFrame` containing the data on which basis-function placement is based, or a list of these; see details
regular	an integer indicating the number of regularly-placed basis functions at the first resolution. In two dimensions, this dictates the smallest number of basis functions in a row or column at the coarsest resolution. If `regular=0`, an irregular grid is used, one that is based on the triangulation of the domain with increased mesh density in areas of high data density; see details
nres	the number of basis-function resolutions to use
prune	a threshold parameter that dictates when a basis function is considered irrelevent or unidentifiable, and thus removed; see details
max_basis	maximum number of basis functions. This overrides the parameter `nres`
subsamp	the maximum amount of data points to consider when carrying out basis-function placement: these data objects are randomly sampled from the full dataset. Keep this number fairly high (on the order of 10^5), otherwise fine-resolution basis functions may be spuriously removed
type	the type of basis functions to use; see details
isea3h_lo	if `manifold = sphere()`, this argument dictates which ISEA3H resolution is the coarsest one that should be used for the first resolution
bndary	a `matrix` containing points containing the boundary. If `regular == 0` this can be used to define a boundary in which irregularly-spaced basis functions are placed
scale_aperture	the aperture (in the case of the bisquare, but similar interpretation for other basis) width of the basis function is the minimum distance between all the basis function centroids multiplied by `scale_aperture`. Typically this ranges between 1 and 1.5 and is defaulted to 1 on the sphere and 1.25 on the other manifolds.
verbose	a logical variable indicating whether to output a summary of the basis functions created or not
buffer	a numeric between 0 and 0.5 indicating the size of the buffer of basis functions along the boundary. The buffer is added by computing the number of basis functions in each dimension, and increasing this number by a factor of `buffer`. A buffer may be needed when the prior distribution of the basis-function coefficients is formulated in terms of a precision matrix
tunit	temporal unit, required when constructing a spatio-temporal basis. Should be the same as used for the BAUs. Can be "secs", "mins", "hours", "days", "years", etc.

... unused

Details

This function automatically places basis functions within the domain of interest. If the domain is a plane or the real line, then the object `data` is used to establish the domain boundary.

The argument `type` can be either "Gaussian", in which case

$$\phi(u) = \exp\left(-\frac{\|u\|^2}{2\sigma^2}\right),$$

"bisquare", in which case

$$\phi(u) = \left(1 - \left(\frac{\|u\|}{R}\right)^2\right)^2 I(\|u\| < R),$$

"exp", in which case

$$\phi(u) = \exp\left(-\frac{\|u\|}{\tau}\right),$$

or "Matern32", in which case

$$\phi(u) = \left(1 + \frac{\sqrt{3}\|u\|}{\kappa}\right)\exp\left(-\frac{\sqrt{3}\|u\|}{\kappa}\right),$$

where the parameters σ, R, τ and κ are `scale` arguments.

If the manifold is the real line, the basis functions are placed regularly inside the domain, and the number of basis functions at the coarsest resolution is dictated by the integer parameter `regular` which has to be greater than zero. On the real line, each subsequent resolution has twice as many basis functions. The scale of the basis function is set based on the minimum distance between the centre locations following placement. The scale is equal to the minimum distance if the type of basis function is Gaussian, exponential, or Matern32, and is equal to 1.5 times this value if the function is bisquare.

If the manifold is a plane, and `regular > 0`, then basis functions are placed regularly within the bounding box of `data`, with the smallest number of basis functions in each row or column equal to the value of `regular` in the coarsest resolution (note, this is just the smallest number of basis functions). Subsequent resolutions have twice the number of basis functions in each row or column. If `regular = 0`, then the function `INLA::inla.nonconvex.hull` is used to construct a (non-convex) hull around the data. The buffer and smoothness of the hull is determined by the parameter `convex`. Once the domain boundary is found, `INLA::inla.mesh.2d` is used to construct a triangular mesh such that the node vertices coincide with data locations, subject to some minimum and maximum triangular-side-length constraints. The result is a mesh that is dense in regions of high data density and not dense in regions of sparse data. Even basis functions are irregularly placed, the scale is taken to be a function of the minimum distance between basis function centres, as detailed above. This may be changed in a future revision of the package.

If the manifold is the surface of a sphere, then basis functions are placed on the centroids of the discrete global grid (DGG), with the first basis resolution corresponding to the third resolution of the DGG (ISEA3H resolution 2, which yields 92 basis functions globally). It is not recommended

to go above `nres == 3` (ISEA3H resolutions 2–4) for the whole sphere; `nres=3` yields a total of 1176 basis functions. Up to ISEA3H resolution 6 is available with FRK; for finer resolutions; please install dggrids from `https://github.com/andrewzm/dggrids` using `devtools`.

Basis functions that are not influenced by data points may hinder convergence of the EM algorithm when `K_type = "unstructured"`, since the associated hidden states are, by and large, unidentifiable. We hence provide a means to automatically remove such basis functions through the parameter `prune`. The final set only contains basis functions for which the column sums in the associated matrix S (which, recall, is the value/average of the basis functions at/over the data points/polygons) is greater than prune. If `prune == 0`, no basis functions are removed from the original design.

See Also

remove_basis for removing basis functions and show_basis for visualising basis functions

Examples

```
## Not run:
library(sp)
library(ggplot2)

## Create a synthetic dataset
set.seed(1)
d <- data.frame(lon = runif(n=1000,min = -179, max = 179),
                lat = runif(n=1000,min = -90, max = 90),
                z = rnorm(5000))
coordinates(d) <- ~lon + lat
proj4string(d)=CRS("+proj=longlat +ellps=sphere")

## Now create basis functions over sphere
G <- auto_basis(manifold = sphere(),data=d,
                nres = 2,prune=15,
                type = "bisquare",
                subsamp = 20000)

## Plot
show_basis(G,draw_world())

## End(Not run)
```

---

auto_BAUs   *Automatic BAU generation*

Description

This function calls the generic function `auto_BAU` (not exported) after a series of checks and is the easiest way to generate a set of Basic Areal Units (BAUs) on the manifold being used; see details.

Usage

```
auto_BAUs(
  manifold,
  type = NULL,
  cellsize = NULL,
  isea3h_res = NULL,
  data = NULL,
  nonconvex_hull = TRUE,
  convex = -0.05,
  tunit = NULL,
  xlims = NULL,
  ylims = NULL,
  spatial_BAUs = NULL,
  ...
)
```

Arguments

manifold	object of class `manifold`
type	either "grid" or "hex", indicating whether gridded or hexagonal BAUs should be used. If `type` is unspecified, "hex" will be used if we are on the sphere, and "grid" will used otherwise
cellsize	denotes size of gridcell when `type` = "grid". Needs to be of length 1 (square-grid case) or a vector of length `dimensions(manifold)` (rectangular-grid case)
isea3h_res	resolution number of the isea3h DGGRID cells for when type is "hex" and manifold is the surface of a `sphere`
data	object of class `SpatialPointsDataFrame`, `SpatialPolygonsDataFrame`, `STIDF`, or `STFDF`. Provision of `data` implies that the domain is bounded, and is thus necessary when the manifold is a `real_line`, `plane`, or `STplane`, but is not necessary when the manifold is the surface of a `sphere`
nonconvex_hull	flag indicating whether to use `INLA` to generate a non-convex hull. Otherwise a convex hull is used
convex	convex parameter used for smoothing an extended boundary when working on a bounded domain (that is, when the object `data` is supplied); see details
tunit	temporal unit when requiring space-time BAUs. Can be "secs", "mins", "hours", etc.
xlims	limits of the horizontal axis (overrides automatic selection)
ylims	limits of the vertical axis (overrides automatic selection)
spatial_BAUs	object of class `SpatialPolygonsDataFrame` or `SpatialPixelsDataFrame` representing the spatial BAUs to be used in a spatio-temporal setting (if left NULL, the spatial BAUs are constructed automatically using the data)
...	currently unused

Details

> `auto_BAUs` constructs a set of Basic Areal Units (BAUs) used both for data pre-processing and for prediction. As such, the BAUs need to be of sufficienly fine resolution so that inferences are not affected due to binning.
>
> Two types of BAUs are supported by FRK: "hex" (hexagonal) and "grid" (rectangular). In order to have a "grid" set of BAUs, the user should specify a cellsize of length one, or of length equal to the dimensions of the manifold, that is, of length 1 for `real_line` and of length 2 for the surface of a `sphere` and `plane`. When a "hex" set of BAUs is desired, the first element of `cellsize` is used to determine the side length by dividing this value by approximately 2. The argument `type` is ignored with `real_line` and "hex" is not available for this manifold.
>
> If the object `data` is provided, then automatic domain selection may be carried out by employing the INLA function `inla.nonconvex.hull`, which finds a (non-convex) hull surrounding the data points (or centroids of the data polygons). This domain is extended and smoothed using the parameter `convex`. The parameter `convex` should be negative, and a larger absolute value for `convex` results in a larger domain with smoother boundaries (note that INLA was not available on CRAN at the time of writing).

See Also

> [auto_basis](#) for automatically constructing basis functions.

Examples

```
## First a 1D example
library(sp)
set.seed(1)
data <- data.frame(x = runif(10)*10, y = 0, z= runif(10)*10)
coordinates(data) <- ~x+y
Grid1D_df <- auto_BAUs(manifold = real_line(),
                       cellsize = 1,
                       data=data)
## Not run: spplot(Grid1D_df)

## Now a 2D example
data(meuse)
coordinates(meuse) = ~x+y # change into an sp object

## Grid BAUs
GridPols_df <- auto_BAUs(manifold = plane(),
                         cellsize = 200,
                         type = "grid",
                         data = meuse,
                         nonconvex_hull = 0)
## Not run: plot(GridPols_df)

## Hex BAUs
HexPols_df <- auto_BAUs(manifold = plane(),
                        cellsize = 200,
                        type = "hex",
                        data = meuse,
```

```
                      nonconvex_hull = 0)
## Not run: plot(HexPols_df)
```

Basis   *Generic basis-function constructor*

Description

This function is meant to be used for manual construction of arbitrary basis functions. For 'local' basis functions, please use the function [local_basis](#) instead.

Usage

```
Basis(manifold, n, fn, pars, df, regular = FALSE)
```

Arguments

manifold	object of class `manifold`, for example, `sphere`
n	number of basis functions (should be an integer)
fn	a list of functions, one for each basis function. Each function should be encapsulated within an environment in which the manifold and any other parameters required to evaluate the function are defined. The function itself takes a single input s which can be of class `numeric`, `matrix`, or `Matrix`, and returns a vector which contains the basis function evaluations at s.
pars	A list containing a list of parameters for each function. For local basis functions these would correspond to location and scale parameters.
df	A data frame containing one row per basis function, typically for providing informative summaries.
regular	logical indicating if the basis functions (of each resolution) are in a regular grid

Details

This constructor checks that all parameters are valid before constructing the basis functions. The requirement that every function is encapsulated is tedious, but necessary for FRK to work with a large range of basis functions in the future. Please see the example below which exemplifies the process of constructing linear basis functions from scratch using this function.

See Also

[auto_basis](#) for constructing basis functions automatically, [local_basis](#) for constructing 'local' basis functions, and [show_basis](#) for visualising basis functions.

Examples

```
## Construct two linear basis functions on [0, 1]
manifold <- real_line()
n <- 2
lin_basis_fn <- function(manifold, grad, intercept) {
    function(s) grad*s + intercept
}
pars <- list(list(grad = 1, intercept = 0),
             list(grad = -1, intercept = 1))
fn <- list(lin_basis_fn(manifold, 1, 0),
           lin_basis_fn(manifold, -1, 1))
df <- data.frame(n = 1:2, grad = c(1, -1), m = c(1, -1))
G <- Basis(manifold = manifold, n = n, fn = fn, pars = pars, df = df)
## Not run:
eval_basis(G, s = matrix(seq(0,1, by = 0.1), 11, 1))
## End(Not run)
```

Basis_obj-class   *Basis functions*

Description

An object of class Basis contains the basis functions used to construct the matrix S in FRK.

Details

Basis functions are a central component of FRK, and the package is designed to work with user-defined specifications of these. For convenience, however, several functions are available to aid the user to construct a basis set for a given set of data points. Please see auto_basis for more details. The function local_basis helps the user construct a set of local basis functions (e.g., bisquare functions) from a collection of location and scale parameters.

Slots

manifold an object of class manifold that contains information on the manifold and the distance measure used on the manifold. See manifold-class for more details

n the number of basis functions in this set

fn a list of length n, with each item the function of a specific basis function

pars a list of parameters where the i-th item in the list contains the parameters of the i-th basis function, fn[[i]]

df a data frame containing other attributes specific to each basis function (for example the geometric centre of the local basis function)

regular logical indicating if the basis functions (of each resolution) are in a regular grid

See Also

auto_basis for automatically constructing basis functions and show_basis for visualising basis functions.

BAUs_from_points   *Creates pixels around points*

Description

Takes a SpatialPointsDataFrame and converts it into SpatialPolygonsDataFrame by constructing a tiny (within machine tolerance) BAU around each SpatialPoint.

Usage

```
BAUs_from_points(obj, offset = 1e-10)

## S4 method for signature 'SpatialPoints'
BAUs_from_points(obj, offset = 1e-10)

## S4 method for signature 'ST'
BAUs_from_points(obj, offset = 1e-10)
```

Arguments

obj	object of class SpatialPointsDataFrame
offset	edge size of the mini-BAU (default 1e-10)

Details

This function allows users to mimic standard geospatial analysis where BAUs are not used. Since FRK is built on the concept of a BAU, this function constructs tiny BAUs around the observation and prediction locations that can be subsequently passed on to the functions SRE and FRK. With BAUs_from_points, the user supplies both the data and prediction locations accompanied with covariates.

See Also

[auto_BAUs](#) for automatically constructing generic BAUs.

Examples

```
library(sp)
opts_FRK$set("parallel",0L)
df <- data.frame(x = rnorm(10),
                 y = rnorm(10))
coordinates(df) <- ~x+y
BAUs <- BAUs_from_points(df)
```

---

combine_basis *Combine basis functions*

Description

Takes a list of objects of class Basis and returns a single object of class Basis.

Usage

```
combine_basis(Basis_list)

## S4 method for signature 'list'
combine_basis(Basis_list)
```

Arguments

Basis_list   a list of objects of class Basis. Each element of the list is assumed to represent a single resolution of basis functions

See Also

[auto_basis](#) for automatically constructing basis functions and [show_basis](#) for visualising basis functions

Examples

```
## Construct two resolutions of basis functions using local_basis()
Basis1 <- local_basis(manifold = real_line(),
                      loc = matrix(seq(0, 1, length.out = 3), ncol = 1),
                      scale = rep(0.4, 3))

Basis2 <- local_basis(manifold = real_line(),
                      loc = matrix(seq(0, 1, length.out = 6), ncol = 1),
                      scale = rep(0.2, 6))

## Combine basis-function resolutions into a single Basis object
combine_basis(list(Basis1, Basis2))
```

---

data.frame<- *Basis-function data frame object*

Description

Tools for retrieving and manipulating the data frame within Basis objects. Use the assignment data.frame()<- with care; no checks are made to ensure the data frame conforms with the object.

Usage

```
data.frame(x) <- value

## S4 method for signature 'Basis'
x$name

## S4 replacement method for signature 'Basis'
x$name <- value

## S4 replacement method for signature 'Basis'
data.frame(x) <- value

## S4 replacement method for signature 'TensorP_Basis'
data.frame(x) <- value

## S3 method for class 'Basis'
as.data.frame(x, ...)

## S3 method for class 'TensorP_Basis'
as.data.frame(x, ...)
```

Arguments

x	the obect of class Basis we are assigning the new data to or retrieving data from
value	the new data being assigned to the Basis object
name	the field name to which values will be retrieved or assigned inside the Basis object's data frame
...	unused

Examples

```
G <- local_basis()
df <- data.frame(G)
print(df$res)
df$res <- 2
data.frame(G) <- df
```

---

df_to_SpatialPolygons   *Convert data frame to SpatialPolygons*

Description

Convert data frame to SpatialPolygons object.

Usage

```
df_to_SpatialPolygons(df, keys, coords, proj)
```

Arguments

df	data frame containing polygon information, see details
keys	vector of variable names used to group rows belonging to the same polygon
coords	vector of variable names identifying the coordinate columns
proj	the projection of the `SpatialPolygons` object. Needs to be of class CRS

Details

Each row in the data frame `df` contains both coordinates and labels (or keys) that identify to which polygon the coordinates belong. This function groups the data frame according to `keys` and forms a `SpatialPolygons` object from the coordinates in each group. It is important that all rings are closed, that is, that the last row of each group is identical to the first row. Since `keys` can be of length greater than one, we identify each polygon with a new key by forming an MD5 hash made out of the respective `keys` variables that in themselves are unique (and therefore the hashed key is also unique). For lon-lat coordinates use `proj = CRS("+proj=longlat +ellps=sphere")`.

Examples

```
library(sp)
df <- data.frame(id = c(rep(1,4),rep(2,4)),
                 x = c(0,1,0,0,2,3,2,2),
                 y=c(0,0,1,0,0,1,1,0))
pols <- df_to_SpatialPolygons(df,"id",c("x","y"),CRS())
## Not run: plot(pols)
```

---

dist-matrix	*Distance Matrix Computation from Two Matrices*

Description

This function extends `dist` to accept two arguments.

Usage

```
distR(x1, x2 = NULL)
```

Arguments

x1	matrix of size N1 x n
x2	matrix of size N2 x n

Details

Computes the distances between the coordinates in `x1` and the coordinates in `x2`. The matrices `x1` and `x2` do not need to have the same number of rows, but need to have the same number of columns (e.g., manifold dimensions).

Value

Matrix of size N1 x N2

Examples

```
A <- matrix(rnorm(50),5,10)
D <- distR(A,A[-3,])
```

distance   *Compute distance*

Description

Compute distance using object of class measure or manifold.

Usage

```
distance(d, x1, x2 = NULL)

## S4 method for signature 'measure'
distance(d, x1, x2 = NULL)

## S4 method for signature 'manifold'
distance(d, x1, x2 = NULL)
```

Arguments

d	object of class measure or manifold
x1	first coordinate
x2	second coordinate

See Also

[real_line](#), [plane](#), [sphere](#), [STplane](#) and [STsphere](#) for constructing manifolds, and [distances](#) for the type of distances available.

Examples

```
distance(sphere(),matrix(0,1,2),matrix(10,1,2))
distance(plane(),matrix(0,1,2),matrix(10,1,2))
```

---

distances   *Pre-configured distances*

Description

Useful objects of class distance included in package.

Usage

```
measure(dist, dim)

Euclid_dist(dim = 2L)

gc_dist(R = NULL)

gc_dist_time(R = NULL)
```

Arguments

dist	a function taking two arguments x1,x2
dim	the dimension of the manifold (e.g., 2 for a plane)
R	great-circle radius

Details

Initialises an object of class measure which contains a function dist used for computing the distance between two points. Currently the Euclidean distance and the great-circle distance are included with FRK.

Examples

```
M1 <- measure(distR,2)
D <- distance(M1,matrix(rnorm(10),5,2))
```

---

draw_world   *Draw a map of the world with country boundaries.*

Description

Layers a ggplot2 map of the world over the current ggplot2 object.

Usage

```
draw_world(g = ggplot() + theme_bw() + xlab("") + ylab(""), inc_border = TRUE)
```

Arguments

g	initial ggplot object
inc_border	flag indicating whether a map border should be drawn or not; see details.

Details

This function uses `ggplot2::map_data()` in order to create a world map. Since, by default, this creates lines crossing the world at the (-180,180) longitude boundary, the function `.homogenise_maps()` is used to split the polygons at this boundary into two. If `inc_border` is TRUE, then a border is drawn around the lon-lat space; this option is most useful for projections that do not yield rectangular plots (e.g., the sinusoidal global projection).

See Also

the help file for the dataset `worldmap`

Examples

```
## Not run:
library(ggplot2)
draw_world(g = ggplot())
## End(Not run)
```

---

eval_basis	*Evaluate basis functions*

Description

Evaluate basis functions at points or average functions over polygons.

Usage

```
eval_basis(basis, s)

## S4 method for signature 'Basis,matrix'
eval_basis(basis, s)

## S4 method for signature 'Basis,SpatialPointsDataFrame'
eval_basis(basis, s)

## S4 method for signature 'Basis,SpatialPolygonsDataFrame'
eval_basis(basis, s)

## S4 method for signature 'Basis,STIDF'
eval_basis(basis, s)

## S4 method for signature 'TensorP_Basis,matrix'
```

```
eval_basis(basis, s)

## S4 method for signature 'TensorP_Basis,STIDF'
eval_basis(basis, s)

## S4 method for signature 'TensorP_Basis,STFDF'
eval_basis(basis, s)
```

Arguments

basis	object of class Basis
s	object of class matrix, SpatialPointsDataFrame or SpatialPolygonsDataFrame containing the spatial locations/footprints

Details

This function evaluates the basis functions at isolated points, or averages the basis functions over polygons, for computing the matrix S. The latter operation is carried out using Monte Carlo integration with 1000 samples per polygon. When using space-time basis functions, the object must contain a field t containing a numeric representation of the time, for example, containing the number of seconds, hours, or days since the first data point.

See Also

[auto_basis](#) for automatically constructing basis functions.

Examples

```
library(sp)

### Create a synthetic dataset
set.seed(1)
d <- data.frame(lon = runif(n=500,min = -179, max = 179),
                lat = runif(n=500,min = -90, max = 90),
                z = rnorm(500))
coordinates(d) <- ~lon + lat
proj4string(d)=CRS("+proj=longlat")

### Now create basis functions on sphere
G <- auto_basis(manifold = sphere(),data=d,
                nres = 2,prune=15,
                type = "bisquare",
                subsamp = 20000)

### Now evaluate basis functions at origin
S <- eval_basis(G,matrix(c(0,0),1,2))
```

FRK   *Construct SRE object, fit and predict*

Description

The Spatial Random Effects (SRE) model is the central object in **FRK**. The function FRK() provides a wrapper for the construction and estimation of the SRE object from data, using the functions SRE() (the object constructor) and SRE.fit() (for fitting it to the data). Please see SRE-class for more details on the SRE object's properties and methods.

Usage

```
FRK(
  f,
  data,
  basis = NULL,
  BAUs = NULL,
  est_error = TRUE,
  average_in_BAU = TRUE,
  sum_variables = NULL,
  normalise_wts = TRUE,
  fs_model = "ind",
  vgm_model = NULL,
  K_type = c("block-exponential", "precision", "unstructured"),
  n_EM = 100,
  tol = 0.01,
  method = c("EM", "TMB"),
  lambda = 0,
  print_lik = FALSE,
 response = c("gaussian", "poisson", "gamma", "inverse-gaussian", "negative-binomial",
    "binomial"),
  link = c("identity", "log", "sqrt", "logit", "probit", "cloglog", "inverse",
    "inverse-squared"),
  optimiser = nlminb,
  fs_by_spatial_BAU = FALSE,
  known_sigma2fs = NULL,
  taper = NULL,
  simple_kriging_fixed = TRUE,
  ...
)

SRE(
  f,
  data,
  basis,
  BAUs,
  est_error = TRUE,
```

```
    average_in_BAU = TRUE,
    sum_variables = NULL,
    normalise_wts = TRUE,
    fs_model = "ind",
    vgm_model = NULL,
    K_type = c("block-exponential", "precision", "unstructured"),
    normalise_basis = TRUE,
   response = c("gaussian", "poisson", "gamma", "inverse-gaussian", "negative-binomial",
      "binomial"),
    link = c("identity", "log", "sqrt", "logit", "probit", "cloglog", "inverse",
      "inverse-squared"),
    include_fs = TRUE,
    fs_by_spatial_BAU = FALSE,
    ...
  )

  SRE.fit(
    object,
    n_EM = 100L,
    tol = 0.01,
    method = c("EM", "TMB"),
    lambda = 0,
    print_lik = FALSE,
    optimiser = nlminb,
    known_sigma2fs = NULL,
    taper = NULL,
    simple_kriging_fixed = TRUE,
    ...
  )

  ## S4 method for signature 'SRE'
  predict(
    object,
    newdata = NULL,
    obs_fs = FALSE,
    pred_time = NULL,
    covariances = FALSE,
    nsim = 400,
    type = "mean",
    k = NULL,
    percentiles = c(5, 95),
    kriging = "simple"
  )

  ## S4 method for signature 'SRE'
  logLik(object)

  ## S4 method for signature 'SRE'
```

```
nobs(object, ...)

## S4 method for signature 'SRE'
coef(object, ...)

simulate(object, newdata = NULL, nsim = 400, conditional_fs = FALSE, ...)

## S4 method for signature 'SRE'
fitted(object, ...)

## S4 method for signature 'SRE'
residuals(object, type = "pearson")

## S4 method for signature 'SRE'
AIC(object, k = 2)

## S4 method for signature 'SRE'
BIC(object)
```

Arguments

f	R formula relating the dependent variable (or transformations thereof) to covariates
data	list of objects of class SpatialPointsDataFrame, SpatialPolygonsDataFrame, STIDF, or STFDF. If using space-time objects, the data frame must have another field, t, containing the time index of the data point
basis	object of class Basis (or TensorP_Basis)
BAUs	object of class SpatialPolygonsDataFrame, SpatialPixelsDataFrame, STIDF, or STFDF. The object's data frame must contain covariate information as well as a field fs describing the fine-scale variation up to a constant of proportionality. If the function FRK() is used directly, then BAUs are created automatically, but only coordinates can then be used as covariates
est_error	(applicable only if response = "gaussian") flag indicating whether the measurement-error variance should be estimated from variogram techniques. If this is set to 0, then data must contain a field std. Measurement-error estimation is currently not implemented for spatio-temporal datasets
average_in_BAU	if TRUE, then multiple data points falling in the same BAU are averaged; the measurement error of the averaged data point is taken as the average of the individual measurement errors
sum_variables	if average_in_BAU == TRUE, the string sum_variables indicates which data variables (can be observations or covariates) are to be summed rather than averaged
normalise_wts	if TRUE, the rows of the incidence matrices C_Z and C_P are normalised to sum to 1, so that the mapping represents a weighted average; if false, no normalisation of the weights occurs (i.e., the mapping corresponds to a weighted sum)

fs_model	if "ind" then the fine-scale variation is independent at the BAU level. Only the independent model is allowed for now, future implementation will include CAR/ICAR (in development)
vgm_model	(applicable only if response = "gaussian") an object of class variogramModel from the package gstat constructed using the function vgm. This object contains the variogram model that will be fit to the data. The nugget is taken as the measurement error when est_error = TRUE. If unspecified, the variogram used is gstat::vgm(1, ″Lin″, d, 1), where d is approximately one third of the maximum distance between any two data points
K_type	the parameterisation used for the basis-function covariance matrix, K. If method = "EM", K_type can be "unstructured" or "block-exponential". If method = "TMB", K_type can be "precision" or "block-exponential". The default is "block-exponential", however if FRK() is used and method = "TMB", for computational reasons K_type is set to "precision"
n_EM	(applicable only if method = "EM") maximum number of iterations for the EM algorithm
tol	(applicable only if method = "EM") convergence tolerance for the EM algorithm
method	parameter estimation method to employ. Currently "EM" and "TMB" are supported
lambda	(applicable only if K_type = "unstructured") ridge-regression regularisation parameter (0 by default). Can be a single number, or a vector (one parameter for each resolution)
print_lik	(applicable only if method = "EM") flag indicating whether to plot log-likelihood vs. iteration after convergence of the EM estimation algorithm
response	string indicating the assumed distribution of the response variable. It can be "gaussian", "poisson", "negative-binomial", "binomial", "gamma", or "inverse-gaussian". If method = "EM", only "gaussian" can be used. Two distributions considered in this framework, namely the binomial distribution and the negative-binomial distribution, have an assumed-known 'size' parameter and a 'probability of success' parameter; see the details below for the exact parameterisations used, and how to provide these 'size' parameters
link	string indicating the desired link function. Can be "log", "identity", "logit", "probit", "cloglog", "reciprocal", or "reciprocal-squared". Note that only sensible link-function and response-distribution combinations are permitted. If method = "EM", only "identity" can be used
optimiser	(applicable only if method = "TMB") the optimising function used for model fitting when method = "TMB" (default is nlminb). Users may pass in a function object or a string corresponding to a named function. Optional parameters may be passed to optimiser via .... The only requirement of optimiser is that the first three arguments correspond to the initial parameters, the objective function, and the gradient, respectively (this may be achieved by simply constructing a wrapper function)
fs_by_spatial_BAU	
	(applicable only in a spatio-temporal setting and if method = "TMB") if TRUE, then each spatial BAU is associated with its own fine-scale variance parameter; otherwise, a single fine-scale variance parameter is used

known_sigma2fs   known value of the fine-scale variance parameter. If NULL (the default), the fine-scale variance parameter is estimated as usual. If known_sigma2fs is not NULL, the fine-scale variance is fixed to the supplied value; this may be a scalar, or vector of length equal to the number of spatial BAUs (if fs_by_spatial_BAU = TRUE)

taper   positive numeric indicating the strength of the covariance/partial-correlation tapering. Only applicable if K_type = "block-exponential", or if K_type = "precision" and the the basis-functions are irregular or the manifold is not the plane. If taper is NULL (default) and method = "EM", no tapering is applied; if method = "TMB", tapering must be applied (for computational reasons), and we set it to 3 if it is unspecified

simple_kriging_fixed
                  commit to simple kriging at the fitting stage? If TRUE, model fitting is faster, but the option to conduct universal kriging at the prediction stage is removed

...   other parameters passed on to auto_basis() and auto_BAUs() when calling FRK(), or the user specified function optimiser() when calling FRK() or SRE.fit()

normalise_basis
                  flag indicating whether to normalise the basis functions so that they reproduce a stochastic process with approximately constant variance spatially

include_fs   (applicable only if method = "TMB") flag indicating whether the fine-scale variation should be included in the model

object   object of class SRE returned from the constructor SRE() containing all the parameters and information on the SRE model

newdata   object of class SpatialPoylgons, SpatialPoints, or STI, indicating the regions or points over which prediction will be carried out. The BAUs are used if this option is not specified.

obs_fs   flag indicating whether the fine-scale variation sits in the observation model (systematic error; indicated by obs_fs = TRUE) or in the process model (process fine-scale variation; indicated by obs_fs = FALSE, default). For non-Gaussian data models, and/or non-identity link functions, if obs_fs = TRUE, then the fine-scale variation is removed from the latent process Y; however, they are re-introduced for prediction of the conditonal mean $\boldsymbol{\mu}$ and simulated data $\boldsymbol{Z}^*$

pred_time   vector of time indices at which prediction will be carried out. All time points are used if this option is not specified

covariances   (applicable only for method = "EM") logical variable indicating whether prediction covariances should be returned or not. If set to TRUE, a maximum of 4000 prediction locations or polygons are allowed

nsim   number of i) MC samples at each location when using predict or ii) response vectors when using simulate

type   (applicable only if method = "TMB") vector of strings indicating the quantities for which inference is desired. If "link" is in type, inference on the latent Gaussian process $Y(\cdot)$ is included; if "mean" is in type, inference on the mean process $\mu(\cdot)$ is included (and the probability process, $\pi(\cdot)$, if applicable); if "response" is in type, inference on the noisy data $\boldsymbol{Z}^*$ is included

k	(applicable only if response is "binomial" or "negative-binomial") vector of size parameters at each BAU
percentiles	(applicable only if method = "TMB") a vector of scalars in (0, 100) specifying the desired percentiles of the posterior predictive distribution; if NULL, no percentiles are computed
kriging	(applicable only if method = "TMB") string indicating the kind of kriging: "simple" ignores uncertainty due to estimation of the fixed effects, while "universal" accounts for this source of uncertainty
conditional_fs	condition on the fitted fine-scale random effects?

Details

The following details provide a summary of the model and basic workflow used in **FRK**. See Zammit-Mangion and Cressie (2021) and Sainsbury-Dale, Zammit-Mangion and Cressie (2021) for further details.

Model description

The hierarchical model implemented in **FRK** is a spatial generalised linear mixed model (GLMM), which may be summarised as

$$Z_j \mid \boldsymbol{\mu}_Z, \psi \sim EF(\mu_{Z_j}, \psi); \quad j = 1, \dots, m,$$

$$\boldsymbol{\mu}_Z = \boldsymbol{C}_Z \boldsymbol{\mu}$$

$$g(\boldsymbol{\mu}) = \boldsymbol{Y}$$

$$\boldsymbol{Y} = \boldsymbol{T}\boldsymbol{\alpha} + \boldsymbol{S}\boldsymbol{\eta} + \boldsymbol{\xi}$$

$$\boldsymbol{\eta} \mid \boldsymbol{\vartheta} \sim N(\boldsymbol{0}, \boldsymbol{K})$$

$$\boldsymbol{\xi} \mid \sigma_\xi^2 \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_\xi),$$

where Z_j denotes a datum, EF corresponds to a probability distribution in the exponential family with dispersion parameter ψ, $\boldsymbol{\mu}_Z$ is the vector containing the conditional expectations of each datum, $\boldsymbol{C}_Z$ is a matrix which aggregates the BAU-level mean process over the observation supports, $\boldsymbol{\mu}$ is the mean process evaluated over the BAUs, g is a link function, $\boldsymbol{Y}$ is a latent Gaussian process evaluated over the BAUs, the matrix $\boldsymbol{T}$ contains regression covariates at the BAU level associated with the fixed effects $\boldsymbol{\alpha}$, the matrix $\boldsymbol{S}$ contains basis function evaluations over the BAUs, $\boldsymbol{\eta}$ are the random coefficients associated with the basis functions, and $\boldsymbol{\xi}$ is a vector containing fine-scale variation at the BAU level.

The prior distribution of the basis-function coefficients, $\boldsymbol{\eta}$, are formulated using either a covariance matrix $\boldsymbol{K}$ or precision matrix $\boldsymbol{Q}$, depending on the argument K_type; the parameters of these matrices, $\boldsymbol{\vartheta}$, are estimated during model fitting. The covariance matrix of $\boldsymbol{\xi}$, $\boldsymbol{\Sigma}_\xi$, is diagonal. By default, $\boldsymbol{\Sigma}_\xi = \sigma_\xi^2 \boldsymbol{V}$, where $\boldsymbol{V}$ is a known, positive-definite diagonal matrix whose elements are provided in the field 'fs' in the BAUs; in the absence of problem specific fine-scale information, 'fs' can simply be set to 1, so that $\boldsymbol{V} = \boldsymbol{I}$. In a spatio-temporal setting, another model for $\boldsymbol{\Sigma}_\xi$ can be used by setting fs_by_spatial_BAU = TRUE, in which case each spatial BAU is associated with its own fine-scale variance parameter (see Section 2.6 of Sainsbury-Dale, Zammit-Mangion and Cressie (2021) for details). In either case, the fine-scale variance parameter(s) are either estimated during model fitting, or provided by the user via the argument known_sigma2fs.

Gaussian data model with an identity link function

When the data is Gaussian, and an identity link function is used, the preceding model simplifies considerably: Specifically,

$$\boldsymbol{Z} = \boldsymbol{C}_Z \boldsymbol{Y} + \boldsymbol{C}_Z \boldsymbol{\delta} + \boldsymbol{e},$$

where $\boldsymbol{Z}$ is the data vector, $\boldsymbol{\delta}$ is systematic error at the BAU level, and $\boldsymbol{e}$ represents independent measurement error.

Distributions with size parameters

Two distributions considered in this framework, namely the binomial distribution and the negative-binomial distribution, have an assumed-known 'size' parameter and a 'probability of success' parameter. Given the vector of size parameters associated with the data, $\boldsymbol{k}_Z$, the parameterisation used in **FRK** assumes that Z_j represents either the number of 'successes' from k_{Z_j} trials (binomial data model) or that it represents the number of failures before k_{Z_j} successes (negative-binomial data model).

When model fitting, the BAU-level size parameters $\boldsymbol{k}$ are needed. The user must supply these size parameters either through the data or though the BAUs. How this is done depends on whether the data are areal or point-referenced, and whether they overlap common BAUs or not. The simplest case is when each observation is associated with a single BAU only and each BAU is associated with at most one observation support; then, it is straightforward to assign elements from $\boldsymbol{k}_Z$ to elements of $\boldsymbol{k}$ and vice-versa, and so the user may provide either $\boldsymbol{k}$ or $\boldsymbol{k}_Z$. If each observation is associated with exactly one BAU, but some BAUs are associated with multiple observations, the user must provide $\boldsymbol{k}_Z$, which is used to infer $\boldsymbol{k}$; in particular, $k_i = \sum_{j \in a_i} k_{Z_j}, i = 1, \dots, N$, where a_i denotes the indices of the observations associated with BAU A_i. If one or more observations encompass multiple BAUs, $\boldsymbol{k}$ must be provided with the BAUs, as we cannot meaningfully distribute k_{Z_j} over multiple BAUs associated with datum Z_j. In this case, we infer $\boldsymbol{k}_Z$ using $k_{Z_j} = \sum_{i \in c_j} k_i$, $j = 1, \dots, m$, where c_j denotes the indices of the BAUs associated with observation Z_j.

Set-up

SRE() constructs a spatial random effects model from the user-defined formula, data object (a list of spatially-referenced data), basis functions and a set of Basic Areal Units (BAUs). It first takes each object in the list data and maps it to the BAUs – this entails binning point-referenced data into the BAUs (and averaging within the BAU if average_in_BAU = TRUE), and finding which BAUs are associated with observations. Following this, the incidence matrix, $\boldsymbol{C}_Z$, is constructed. All required matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{C}_Z$, etc.) are constructed within SRE() and returned as part of the SRE object. SRE() also intitialises the parameters and random effects using sensible defaults. Please see SRE-class for more details. The functions observed_BAUs() and unobserved_BAUs() return the indices of the observed and unobserved BAUs, respectively.

Model fitting

SRE.fit() takes an object of class SRE and estimates all unknown parameters, namely the covariance matrix $\boldsymbol{K}$, the fine scale variance (σ_ξ^2 or σ_δ^2, depending on whether Case 1 or Case 2 is chosen; see the vignette "FRK_intro") and the regression parameters $\boldsymbol{\alpha}$. There are two methods of model fitting currently implemented, both of which implement maximum likelihood estimation (MLE).

- MLE via the expectation maximisation (EM) algorithm. This method is implemented only for Gaussian data and an identity link function. The log-likelihood (given in Section 2.2 of the vignette) is evaluated at each iteration at the current parameter estimate. Optimation continues

until convergence is reached (when the log-likelihood stops changing by more than `tol`), or when the number of EM iterations reaches `n_EM`. The actual computations for the E-step and M-step are relatively straightforward. The E-step contains an inverse of an $r \times r$ matrix, where r is the number of basis functions which should not exceed 2000. The M-step first updates the matrix K, which only depends on the sufficient statistics of the basis-function coefficients η. Then, the regression parameters α are updated and a simple optimisation routine (a line search) is used to update the fine-scale variance σ_δ^2 or σ_ξ^2. If the fine-scale errors and measurement random errors are homoscedastic, then a closed-form solution is available for the update of σ_ξ^2 or σ_δ^2. Irrespectively, since the updates of α, and σ_δ^2 or σ_ξ^2, are dependent, these two updates are iterated until the change in $\sigma_\cdot^2$ is no more than 0.1%.

- MLE via `TMB`. This method is implemented for all available data models and link functions offered by **FRK**. Furthermore, this method faciliates the inclusion of many more basis function than possible with the EM algorithm (in excess of 10,000). `TMB` applies the Laplace approximation to integrate out the latent random effects from the complete-data likelihood. The resulting approximation of the marginal log-likelihood, and its derivatives with respect to the parameters, are then called from within R using the optimising function `optimiser` (default `nlminb()`).

Wrapper for set-up and model fitting

The function `FRK()` acts as a wrapper for the functions `SRE()` and `SRE.fit()`. An added advantage of using `FRK()` directly is that it automatically generates BAUs and basis functions based on the data. Hence `FRK()` can be called using only a list of data objects and an R formula, although the R formula can only contain space or time as covariates when BAUs are not explicitly supplied with the covariate data.

Prediction

Once the parameters are estimated, the SRE object is passed onto the function `predict()` in order to carry out optimal predictions over the same BAUs used to construct the SRE model with `SRE()`. The first part of the prediction process is to construct the matrix S over the prediction polygons. This is made computationally efficient by treating the prediction over polygons as that of the prediction over a combination of BAUs. This will yield valid results only if the BAUs are relatively small. Once the matrix S is found, a standard Gaussian inversion (through conditioning) using the estimated parameters is used for prediction.

`predict()` returns the BAUs (or an object specified in `newdata`), which are of class `SpatialPixelsDataFrame`, `SpatialPolygonsDataFrame`, or `STFDF`, with predictions and uncertainty quantification added. If `method = "TMB"`, the returned object is a list, containing the previously described predictions, and a list of Monte Carlo samples. The predictions and uncertainties can be easily plotted using [plot] or spplot from the package sp.

References

Zammit-Mangion, A. and Cressie, N. (2021). FRK: An R package for spatial and spatio-temporal prediction with large datasets. Journal of Statistical Software, 98(4), 1-48. doi:10.18637/jss.v098.i04.

Sainsbury-Dale, M. and Zammit-Mangion, A. and Cressie, N. (2021) Modelling with Non-Gaussian Spatial and Spatio-Temporal Data using FRK, arXiv:2110.02507

See Also

SRE-class for details on the SRE object internals, auto_basis for automatically constructing basis functions, and auto_BAUs for automatically constructing BAUs.

Examples

```
library("FRK")
library("sp")
## Generate process and data
m <- 250                                                # Sample size
zdf <- data.frame(x = runif(m), y= runif(m))            # Generate random locs
zdf$Y <- 3 + sin(7 * zdf$x) + cos(9 * zdf$y)            # Latent process
zdf$z <- rnorm(m, mean = zdf$Y)                         # Simulate data
coordinates(zdf) = ~x+y                                 # Turn into sp object

## Construct BAUs and basis functions
BAUs <- auto_BAUs(manifold = plane(), data = zdf,
                  nonconvex_hull = FALSE, cellsize = c(0.03, 0.03), type="grid")
BAUs$fs <- 1 # scalar fine-scale covariance matrix
basis <- auto_basis(manifold =  plane(), data = zdf, nres = 2)

## Construct the SRE model
S <- SRE(f = z ~ 1, list(zdf), basis = basis, BAUs = BAUs)

## Fit with 2 EM iterations so to take as little time as possible
S <- SRE.fit(S, n_EM = 2, tol = 0.01, print_lik = TRUE)

## Check fit info, final log-likelihood, and estimated regression coefficients
info_fit(S)
logLik(S)
coef(S)

## Predict over BAUs
pred <- predict(S)

## Plot
## Not run:
plotlist <- plot(S, pred)
ggpubr::ggarrange(plotlist = plotlist, nrow = 1, align = "hv", legend = "top")
## End(Not run)
```

---

info_fit   *Retrieve fit information for SRE model*

Description

Takes an object of class SRE and returns a list containing all the relevant information on parameter estimation

Usage

```
info_fit(object)

## S4 method for signature 'SRE'
info_fit(object)
```

Arguments

object   object of class SRE

See Also

See FRK for more information on the SRE model and available fitting methods.

Examples

```
# See example in the help file for FRK
```

---

initialize,manifold-method
manifold

Description

Manifold initialisation. This function should not be called directly as manifold is a virtual class.

Usage

```
## S4 method for signature 'manifold'
initialize(.Object)
```

Arguments

.Object   manifold object passed up from lower-level constructor

---

isea3h   *ISEA Aperture 3 Hexagon (ISEA3H) Discrete Global Grid*

Description

The data used here were obtained from https://webpages.sou.edu/~sahrk/dgg/isea.old/gen/isea3h.html and represent ISEA discrete global grids (DGGRIDs) generated using the DGGRID software. The original .gen files were converted to a data frame using the function dggrid_gen_to_df, available with the dggrids package. Only resolutions 0–6 are supplied with FRK and note that resolution 0 of ISEA3H is equal to resolution 1 in FRK. For higher resolutions dggrids can be installed from https://github.com/andrewzm/dggrids/ using devtools.

Usage

```
isea3h
```

Format

A data frame with 284,208 rows and 5 variables:

id grid identification number within the given resolution

lon longitude coordinate

lat latitude coordinate

res DGGRID resolution (0 – 6)

centroid A 0-1 variable, indicating whether the point describes the centroid of the polygon, or whether it is a boundary point of the polygon

References

Sahr, K. (2008). Location coding on icosahedral aperture 3 hexagon discrete global grids. Computers, Environment and Urban Systems, 32, 174–187.

---

local_basis   *Construct a set of local basis functions*

Description

Construct a set of local basis functions based on pre-specified location and scale parameters.

Usage

```
local_basis(
  manifold = sphere(),
  loc = matrix(c(1, 0), nrow = 1),
  scale = 1,
  type = c("bisquare", "Gaussian", "exp", "Matern32"),
  res = 1,
  regular = FALSE
)

radial_basis(
  manifold = sphere(),
  loc = matrix(c(1, 0), nrow = 1),
  scale = 1,
  type = c("bisquare", "Gaussian", "exp", "Matern32")
)
```

Arguments

manifold	object of class `manifold`, for example, `sphere`
loc	a matrix of size n by `dimensions(manifold)` indicating centres of basis functions
scale	vector of length n containing the scale parameters of the basis functions; see details
type	either "bisquare", "Gaussian", "exp", or "Matern32"
res	vector of length n containing the resolutions of the basis functions
regular	logical indicating if the basis functions (of each resolution) are in a regular grid

Details

This functions lays out local basis functions in a domain of interest based on pre-specified location and scale parameters. If `type` is "bisquare", then

$$\phi(u) = \left(1 - \left(\frac{\|u\|}{R} \right)^2 \right)^2 I(\|u\| < R),$$

and `scale` is given by R, the range of support of the bisquare function. If `type` is "Gaussian", then

$$\phi(u) = \exp \left(-\frac{\|u\|^2}{2\sigma^2} \right),$$

and `scale` is given by σ, the standard deviation. If `type` is "exp", then

$$\phi(u) = \exp \left(-\frac{\|u\|}{\tau} \right),$$

and `scale` is given by τ, the e-folding length. If `type` is "Matern32", then

$$\phi(u) = \left(1 + \frac{\sqrt{3}\|u\|}{\kappa} \right) \exp \left(-\frac{\sqrt{3}\|u\|}{\kappa} \right),$$

and `scale` is given by κ, the function's scale.

See Also

[`auto_basis`](#) for constructing basis functions automatically, and [`show_basis`](#) for visualising basis functions.

Examples

```
library(ggplot2)
G <- local_basis(manifold = real_line(),
                 loc=matrix(1:10,10,1),
                 scale=rep(2,10),
                 type="bisquare")
## Not run: show_basis(G)
```

---

loglik   *(Deprecated) Retrieve log-likelihood*

Description

This function is deprecated; please use `logLik`

Usage

```
loglik(object)

## S4 method for signature 'SRE'
loglik(object)
```

Arguments

object   object of class SRE

---

manifold   *Retrieve manifold*

Description

Retrieve manifold from FRK object.

Usage

```
manifold(.Object)

## S4 method for signature 'Basis'
manifold(.Object)

## S4 method for signature 'TensorP_Basis'
manifold(.Object)
```

Arguments

.Object   FRK object

See Also

[real_line](), [plane](), [sphere](), [STplane]() and [STsphere]() for constructing manifolds.

Examples

```
G <-  local_basis(manifold = plane(),
                  loc=matrix(0,1,2),
                  scale=0.2,
                  type="bisquare")
manifold(G)
```

---

manifold-class   *manifold*

Description

The class manifold is virtual; other manifold classes inherit from this class.

Details

A manifold object is characterised by a character variable type, which contains a description of the manifold, and a variable measure of type measure. A typical measure is the Euclidean distance.

FRK supports five manifolds; the real line (in one dimension), instantiated by using real_line(); the 2D plane, instantiated by using plane(); the 2D-sphere surface S2, instantiated by using sphere(); the R2 space-time manifold, instantiated by using STplane(), and the S2 space-time manifold, instantiated by using STsphere(). User-specific manifolds can also be specified, however helper functions that are manifold specific, such as auto_BAUs and auto_basis, only work with the pre-configured manifolds. Importantly, one can change the distance function used on the manifold to synthesise anisotropy or heterogeneity. See the vignette for one such example.

See Also

[real_line](), [plane](), [sphere](), [STplane]() and [STsphere]() for constructing manifolds.

---

measure-class   *measure*

Description

Measure class used for defining measures used to compute distances between points in objects constructed with the FRK package.

Details

An object of class measure contains a distance function and a variable dim with the dimensions of the Riemannian manifold over which the distance is computed.

See Also

distance for computing a distance and distances for a list of implemented distance functions.

MODIS_cloud_df   *MODIS cloud data*

Description

An image of a cloud taken by the Moderate Resolution Imaging Spectroradiometer (MODIS) instrument aboard the Aqua satellite (MODIS Characterization Support Team, 2015).

Usage

```
MODIS_cloud_df
```

Format

A data frame with 33,750 rows and 3 variables:

x x-coordinate

y y-coordinate

z binary dependent variable: 1 if cloud is present, 0 if no cloud. This variable has been thresholded from the original continuous measurement of radiance supplied by the MODIS instrument

z_unthresholded The original continuous measurement of radiance supplied by the MODIS instrument

References

MODIS Characterization Support Team (2015). MODIS 500m Calibrated Radiance Product.NASA MODIS Adaptive Processing System, Goddard Space Flight Center, USA.

---

nbasis	*Number of basis functions*

Description

Retrieve the number of basis functions from `Basis` or `SRE` object.

Usage

```
nbasis(.Object)

## S4 method for signature 'Basis_obj'
nbasis(.Object)

## S4 method for signature 'SRE'
nbasis(.Object)
```

Arguments

.Object   object of class `Basis` or `SRE`

See Also

[auto_basis](#) for automatically constructing basis functions.

Examples

```
library(sp)
data(meuse)
coordinates(meuse) = ~x+y # change into an sp object
G <- auto_basis(manifold = plane(),
                data=meuse,
                nres = 2,
                regular=1,
                type = "Gaussian")
print(nbasis(G))
```

NOAA_df_1990	*NOAA maximum temperature data for 1990–1993*

Description

Maximum temperature data obtained from the National Oceanic and Atmospheric Administration
(NOAA) for a part of the USA between 1990 and 1993 (inclusive). See https://iridl.ldeo.columbia.edu/
SOURCES/.NOAA/.NCDC/.DAILY/.FSOD/.

Usage

```
NOAA_df_1990
```

Format

A data frame with 196,253 rows and 8 variables:

year year of retrieval

month month of retrieval

day day of retrieval

z dependent variable

proc variable name (Tmax)

id station id

lon longitude coordinate of measurement station

lat latitude coordinate of measurement station

References

National Climatic Data Center, March 1993: Local Climatological Data. Environmental Information summary (C-2), NOAA-NCDC, Asheville, NC.

---

nres   *Return the number of resolutions*

Description

Return the number of resolutions from a basis function object.

Usage

```
nres(b)

## S4 method for signature 'Basis'
nres(b)

## S4 method for signature 'TensorP_Basis'
nres(b)

## S4 method for signature 'SRE'
nres(b)
```

Arguments

b   object of class `Basis` or `SRE`

See Also

auto_basis for automatically constructing basis functions and show_basis for visualising basis functions.

Examples

```
library(sp)
set.seed(1)
d <- data.frame(lon = runif(n=500,min = -179, max = 179),
                lat = runif(n=500,min = -90, max = 90),
                z = rnorm(500))
coordinates(d) <- ~lon + lat
proj4string(d)=CRS("+proj=longlat")

### Now create basis functions on sphere
G <- auto_basis(manifold = sphere(),data=d,
                nres = 2,prune=15,
                type = "bisquare",
                subsamp = 20000)
nres(G)
```

---

observed_BAUs   *Observed (or unobserved) BAUs*

Description

Computes the indices (a numeric vector) of the observed (or unobserved) BAUs

Usage

```
observed_BAUs(object)

unobserved_BAUs(object)

## S4 method for signature 'SRE'
observed_BAUs(object)

## S4 method for signature 'SRE'
unobserved_BAUs(object)
```

Arguments

object   object of class SRE

See Also

See FRK for more information on the SRE model and available fitting methods.

Examples

```
# See example in the help file for FRK
```

---

opts_FRK   *FRK options*

Description

The main options list for the FRK package.

Usage

```
opts_FRK
```

Format

List of 2

- `$ set:function(opt,value)`
- `$ get:function(opt)`

Details

`opts_FRK` is a list containing two functions, `set` and `get`, which can be used to set options and retrieve options, respectively. Currently FRK uses three options:

- "progress": a flag indicating whether progress bars should be displayed or not
- "verbose": a flag indicating whether certain progress messages should be shown or not. Currently this is the only option applicable to `method = "TMB"`
- "parallel": an integer indicating the number of cores to use. A number 0 or 1 indicates no parallelism

Examples

```
opts_FRK$set("progress",1L)
opts_FRK$get("parallel")
```

---

plane   *plane*

Description

Initialisation of a 2D plane.

Usage

```
plane(measure = Euclid_dist(dim = 2L))
```

Arguments

measure   an object of class measure

Details

A 2D plane is initialised using a measure object. By default, the measure object (measure) is the
Euclidean distance in 2 dimensions, Euclid_dist.

Examples

```
P <- plane()
print(type(P))
print(sp::dimensions(P))
```

---

plot   *Plot predictions from FRK analysis*

Description

This function acts as a wrapper around `plot_spatial_or_ST`. It plots the fields of the Spatial*DataFrame
or STFDF object corresponding to prediction and prediction uncertainty quantification. It also uses
the @data slot of SRE object to plot the training data set(s), and generates informative, latex-style
legend labels for each of the plots.

Usage

```
plot(x, y, ...)

## S4 method for signature 'SRE,list'
plot(x, y, ...)

## S4 method for signature 'SRE,STFDF'
plot(x, y, ...)
```

```
## S4 method for signature 'SRE,SpatialPointsDataFrame'
plot(x, y, ...)

## S4 method for signature 'SRE,SpatialPixelsDataFrame'
plot(x, y, ...)

## S4 method for signature 'SRE,SpatialPolygonsDataFrame'
plot(x, y, ...)
```

Arguments

x	object of class SRE
y	the Spatial*DataFrame or STFDF object resulting from the call predict(x). Keep in mind that predict() returns a list when method = "TMB"; the element $newdata contains the required Spatial/ST object. If the list itself is passed, you will receive the error: "x" and "y" lengths differ.
...	optional arguments passed on to plot_spatial_or_ST

Value

A list of ggplot objects consisting of the observed data, predictions, and standard errors. This list can then be supplied to, for example, ggpubr::ggarrange().

Examples

```
## See example in the help file for SRE
```

---

plotting-themes   *Plotting themes*

Description

Formats a ggplot object for neat plotting.

Usage

```
LinePlotTheme()

EmptyTheme()
```

Details

LinePlotTheme() creates ggplot object with a white background, a relatively large font, and grid lines. EmptyTheme() on the other hand creates a ggplot object with no axes or legends.

Value

Object of class ggplot

Examples

```
## Not run:
X <- data.frame(x=runif(100),y = runif(100), z = runif(100))
LinePlotTheme() + geom_point(data=X,aes(x,y,colour=z))
EmptyTheme() + geom_point(data=X,aes(x,y,colour=z))
## End(Not run)
```

---

plot_spatial_or_ST   *Plot a Spatial*DataFrame or STFDF object*

Description

Takes an object of class Spatial*DataFrame or STFDF, and plots requested data columns using ggplot2

Usage

```
plot_spatial_or_ST(
  newdata,
  column_names,
  map_layer = NULL,
  subset_time = NULL,
  palette = "Spectral",
  plot_over_world = FALSE,
  labels_from_coordnames = TRUE,
  ...
)

## S4 method for signature 'STFDF'
plot_spatial_or_ST(
  newdata,
  column_names,
  map_layer = NULL,
  subset_time = NULL,
  palette = "Spectral",
  plot_over_world = FALSE,
  labels_from_coordnames = TRUE,
  ...
)

## S4 method for signature 'SpatialPointsDataFrame'
plot_spatial_or_ST(
  newdata,
  column_names,
  map_layer = NULL,
  subset_time = NULL,
  palette = "Spectral",
```

```
  plot_over_world = FALSE,
  labels_from_coordnames = TRUE,
  ...
)

## S4 method for signature 'SpatialPixelsDataFrame'
plot_spatial_or_ST(
  newdata,
  column_names,
  map_layer = NULL,
  subset_time = NULL,
  palette = "Spectral",
  plot_over_world = FALSE,
  labels_from_coordnames = TRUE,
  ...
)

## S4 method for signature 'SpatialPolygonsDataFrame'
plot_spatial_or_ST(
  newdata,
  column_names,
  map_layer = NULL,
  subset_time = NULL,
  palette = "Spectral",
  plot_over_world = FALSE,
  labels_from_coordnames = TRUE,
  ...
)
```

Arguments

`newdata`	an object of class `Spatial*DataFrame` or `STFDF`
`column_names`	a vector of strings indicating the columns of the data to plot
`map_layer`	(optional) a ggplot layer or object to add below the plotted layer, often a map
`subset_time`	(optional) a vector of times to be included; applicable only for STFDF objects
`palette`	the palette supplied to the argument `palette` of `scale_*_distiller()`. Alternatively, if `palette` = "nasa", a vibrant colour palette is created using `scale_*_gradientn()`
`plot_over_world`	
	logical; if TRUE, `coord_map("mollweide")` and [draw_world](#) are used to plot over the world
`labels_from_coordnames`	
	logical; if TRUE, the coordinate names of `newdata` (i.e., `coordnames(newdata)`) are used as the horizontal- and vertical-axis labels. Otherwise, generic names, s_1 and s_2, are used
`...`	optional arguments passed on to whatever geom is appropriate for the `Spatial*DataFrame` or STFDF object (geom_point, geom_tile, geom_raster, or geom_polygon)

Value

A list of ggplot objects corresponding to the provided column_names. This list can then be supplied to, for example, ggpubr::ggarrange().

See Also

[plot](#)

Examples

```
## See example in the help file for FRK
```

---

real_line   *real line*

Description

Initialisation of the real-line (1D) manifold.

Usage

```
real_line(measure = Euclid_dist(dim = 1L))
```

Arguments

measure   an object of class measure

Details

A real line is initialised using a measure object. By default, the measure object (measure) describes the distance between two points as the absolute difference between the two coordinates.

Examples

```
R <- real_line()
print(type(R))
print(sp::dimensions(R))
```

---

remove_basis   *Removes basis functions*

Description

Takes an object of class `Basis` and returns an object of class `Basis` with selected basis functions removed

Usage

```
remove_basis(Basis, rmidx)

## S4 method for signature 'Basis,ANY'
remove_basis(Basis, rmidx)

## S4 method for signature 'Basis,SpatialPolygons'
remove_basis(Basis, rmidx)
```

Arguments

`Basis`	object of class `Basis`
`rmidx`	indices of basis functions to remove. Or a `SpatialPolygons` object; basis functions overlapping this `SpatialPolygons` object will be *retained*

See Also

[auto_basis](#) for automatically constructing basis functions and [show_basis](#) for visualising basis functions

Examples

```
library(sp)
df <- data.frame(x = rnorm(10),
                 y = rnorm(10))
coordinates(df) <- ~x+y
G <- auto_basis(plane(),df,nres=1)
data.frame(G) # Print info on basis

## Removing basis functions by index
G_subset <- remove_basis(G, 1:(nbasis(G)-1))
data.frame(G_subset)

## Removing basis functions using SpatialPolygons
x <- 1
poly <- Polygon(rbind(c(-x, -x), c(-x, x), c(x, x), c(x, -x), c(-x, -x)))
polys <- Polygons(list(poly), "1")
spatpolys <- SpatialPolygons(list(polys))
G_subset <- remove_basis(G, spatpolys)
data.frame(G_subset)
```

---

show_basis   *Show basis functions*

Description

Generic plotting function for visualising the basis functions.

Usage

```
show_basis(basis, ...)

## S4 method for signature 'Basis'
show_basis(basis, g = ggplot() + theme_bw() + xlab("") + ylab(""))

## S4 method for signature 'TensorP_Basis'
show_basis(basis, g = ggplot())
```

Arguments

basis	object of class Basis
...	not in use
g	object of class gg (a ggplot object) over which to overlay the basis functions (optional)

Details

The function show_basis adapts its behaviour to the manifold being used. With real_line, the 1D basis functions are plotted with colour distinguishing between the different resolutions. With plane, only local basis functions are supported (at present). Each basis function is shown as a circle with diameter equal to the scale parameter of the function. Linetype distinguishes the resolution. With sphere, the centres of the basis functions are shown as circles, with larger sizes corresponding to coarser resolutions. Space-time basis functions of subclass TensorP_Basis are visualised by showing the spatial basis functions and the temporal basis functions in two separate plots.

See Also

[auto_basis](#) for automatically constructing basis functions.

Examples

```
library(ggplot2)
library(sp)
data(meuse)
coordinates(meuse) = ~x+y # change into an sp object
G <- auto_basis(manifold = plane(),data=meuse,nres = 2,regular=2,prune=0.1,type = "bisquare")
## Not run: show_basis(G,ggplot()) + geom_point(data=data.frame(meuse),aes(x,y))
```

SpatialPolygonsDataFrame_to_df

SpatialPolygonsDataFrame to df

Description

Convert `SpatialPolygonsDataFrame` or `SpatialPixelsDataFrame` object to data frame.

Usage

```
SpatialPolygonsDataFrame_to_df(sp_polys, vars = names(sp_polys))
```

Arguments

sp_polys   object of class `SpatialPolygonsDataFrame` or `SpatialPixelsDataFrame`

vars   variables to put into data frame (by default all of them)

Details

This function is mainly used for plotting `SpatialPolygonsDataFrame` objects with `ggplot` rather than `spplot`. The coordinates of each polygon are extracted and concatenated into one long data frame. The attributes of each polygon are then attached to this data frame as variables that vary by polygon `id` (the rownames of the object).

Examples

```
library(sp)
library(ggplot2)
opts_FRK$set("parallel",0L)
df <- data.frame(id = c(rep(1,4),rep(2,4)),
                 x = c(0,1,0,0,2,3,2,2),
                 y=c(0,0,1,0,0,1,1,0))
pols <- df_to_SpatialPolygons(df,"id",c("x","y"),CRS())
polsdf <- SpatialPolygonsDataFrame(pols,data.frame(p = c(1,2),row.names=row.names(pols)))
df2 <- SpatialPolygonsDataFrame_to_df(polsdf)
## Not run: ggplot(df2,aes(x=x,y=y,group=id)) + geom_polygon()
```

---

sphere   *sphere*

Description

Initialisation of the 2-sphere, S2.

Usage

```
sphere(radius = 6371)
```

Arguments

radius   radius of sphere

Details

The 2D surface of a sphere is initialised using a radius parameter. The default value of the radius R is R=6371 km, Earth's radius, while the measure used to compute distances on the sphere is the great-circle distance on a sphere of radius R.

Examples

```
S <- sphere()
print(sp::dimensions(S))
```

SRE-class   *Spatial Random Effects class*

Description

This is the central class definition of the FRK package, containing the model and all other information required for estimation and prediction.

Details

The spatial random effects (SRE) model is the model employed in Fixed Rank Kriging, and the SRE object contains all information required for estimation and prediction from spatial data. Object slots contain both other objects (for example, an object of class Basis) and matrices derived from these objects (for example, the matrix S) in order to facilitate computations.

Slots

f formula used to define the SRE object. All covariates employed need to be specified in the object BAUs

data the original data from which the model's parameters are estimated

basis object of class Basis used to construct the matrix S

BAUs object of class SpatialPolygonsDataFrame, SpatialPixelsDataFrame of STFDF that contains the Basic Areal Units (BAUs) that are used to both (i) project the data onto a common discretisation if they are point-referenced and (ii) provide a BAU-to-data relationship if the data has a spatial footprint

S matrix constructed by evaluating the basis functions at all the data locations (of class Matrix)

S0 matrix constructed by evaluating the basis functions at all BAUs (of class Matrix)

D_basis list of distance-matrices of class Matrix, one for each basis-function resolution

Ve measurement-error variance-covariance matrix (typically diagonal and of class Matrix)

Vfs fine-scale variance-covariance matrix at the data locations (typically diagonal and of class Matrix) up to a constant of proportionality estimated using the EM algorithm

Vfs_BAUs fine-scale variance-covariance matrix at the BAU centroids (typically diagonal and of class Matrix) up to a constant of proportionality estimated using the EM algorithm

Qfs_BAUs fine-scale precision matrix at the BAU centroids (typically diagonal and of class Matrix) up to a constant of proportionality estimated using the EM algorithm

Z vector of observations (of class Matrix)

Cmat incidence matrix mapping the observations to the BAUs

X matrix of covariates at all the data locations

K_type type of prior covariance matrix of random effects. Can be "block-exponential" (correlation between effects decays as a function of distance between the basis-function centroids), "unstructured" (all elements in K are unknown and need to be estimated), or "neighbour" (a sparse precision matrix is used, whereby only neighbouring basis functions have non-zero precision matrix elements).

mu_eta updated expectation of the basis function random effects (estimated)

S_eta updated covariance matrix of random effects (estimated)

Q_eta updated precision matrix of random effects (estimated)

Khat prior covariance matrix of random effects (estimated)

Khat_inv prior precision matrix of random effects (estimated)

alphahat fixed-effect regression coefficients (estimated)

sigma2fshat fine-scale variation scaling (estimated)

fs_model type of fine-scale variation (independent or CAR-based). Currently only "ind" is permitted

info_fit information on fitting (convergence etc.)

response A character string indicating the assumed distribution of the response variable

link A character string indicating the desired link function. Can be "log", "identity", "logit", "probit", "cloglog", "reciprocal", or "reciprocal-squared". Note that only sensible link-function and response-distribution combinations are permitted.

mu_xi updated expectation of the fine-scale random effects at all BAUs (estimated)

Q_posterior updated joint precision matrix of the basis function random effects and observed fine-scale random effects (estimated)

log_likelihood the log likelihood of the fitted model

method the fitting procedure used to fit the SRE model

phi the estimated dispersion parameter (assumed constant throughout the spatial domain)

k_Z vector of known size parameters at the data support (only applicable to binomial and negative-binomial response distributions)

k_BAU vector of known size parameters at the observed BAUs (only applicable to binomial and negative-binomial response distributions)

include_fs flag indicating whether the fine-scale variation should be included in the model

normalise_wts if TRUE, the rows of the incidence matrices C_Z and C_P are normalised to sum to 1, so that the mapping represents a weighted average; if false, no normalisation of the weights occurs (i.e., the mapping corresponds to a weighted sum)

fs_by_spatial_BAU if TRUE, then each BAU is associated with its own fine-scale variance parameter

obsidx indices of observed BAUs

simple_kriging_fixed logical indicating whether one wishes to commit to simple kriging at the fitting stage: If TRUE, model fitting is faster, but the option to conduct universal kriging at the prediction stage is removed

References

Zammit-Mangion, A. and Cressie, N. (2017). FRK: An R package for spatial and spatio-temporal prediction with large datasets. Journal of Statistical Software, 98(4), 1-48. doi:10.18637/jss.v098.i04.

See Also

[SRE](#) for details on how to construct and fit SRE models.

SRE.predict   *Deprecated: Please use* [predict](#)

Description

Deprecated: Please use [predict](#)

Usage

```
SRE.predict(...)
```

Arguments

...   (Deprecated)

STplane   *plane in space-time*

Description

Initialisation of a 2D plane with a temporal dimension.

Usage

```
STplane(measure = Euclid_dist(dim = 3L))
```

Arguments

measure   an object of class measure

Details

A 2D plane with a time component added is initialised using a `measure` object. By default, the measure object (`measure`) is the Euclidean distance in 3 dimensions, Euclid_dist.

Examples

```
P <- STplane()
print(type(P))
print(sp::dimensions(P))
```

STsphere   *Space-time sphere*

Description

Initialisation of a 2-sphere (S2) with a temporal dimension

Usage

```
STsphere(radius = 6371)
```

Arguments

radius   radius of sphere

Details

As with the spatial-only sphere, the sphere surface is initialised using a `radius` parameter. The default value of the radius R is R=6371, which is the Earth's radius in km, while the measure used to compute distances on the sphere is the great-circle distance on a sphere of radius R. By default Euclidean geometry is used to factor in the time component, so that $dist((s1,t1),(s2,t2)) = sqrt(gc\_dist(s1,s2)^2 + (t1 - t2)^2)$. Frequently this distance can be used since separate correlation length scales for space and time are estimated in the EM algorithm (that effectively scale space and time separately).

Examples

```
S <- STsphere()
print(sp::dimensions(S))
```

TensorP   *Tensor product of basis functions*

Description

Constructs a new set of basis functions by finding the tensor product of two sets of basis functions.

Usage

```
TensorP(Basis1, Basis2)

## S4 method for signature 'Basis,Basis'
TensorP(Basis1, Basis2)
```

Arguments

Basis1   first set of basis functions

Basis2   second set of basis functions

See Also

[auto_basis](#) for automatically constructing basis functions and [show_basis](#) for visualising basis functions.

Examples

```
library(spacetime)
library(sp)
library(dplyr)
sim_data <- data.frame(lon = runif(20,-180,180),
                       lat = runif(20,-90,90),
                       t = 1:20,
                       z = rnorm(20),
                       std = 0.1)
time <- as.POSIXct("2003-05-01",tz="") + 3600*24*(sim_data$t-1)
space <- sim_data[,c("lon","lat")]
coordinates(space) = ~lon+lat # change into an sp object
proj4string(space)=CRS("+proj=longlat +ellps=sphere")
STobj <- STIDF(space,time,data=sim_data)
G_spatial <- auto_basis(manifold = sphere(),
                        data=as(STobj,"Spatial"),
                        nres = 1,
                        type = "bisquare",
                        subsamp = 20000)
G_temporal <- local_basis(manifold=real_line(),loc = matrix(c(1,3)),scale = rep(1,2))
G <- TensorP(G_spatial,G_temporal)
# show_basis(G_spatial)
# show_basis(G_temporal)
```

---

type	*Type of manifold*

Description

Retrieve slot type from object

Usage

```
type(.Object)

## S4 method for signature 'manifold'
type(.Object)
```

Arguments

.Object   object of class Basis or manifold

See Also

[real_line](), [plane](), [sphere](), [STplane]() and [STsphere]() for constructing manifolds.

Examples

```
S <- sphere()
print(type(S))
```

---

worldmap	*World map*

Description

This world map was extracted from the package maps v.3.0.1 by running ggplot2::map_data("world").
To reduce the data size, only every third point of this data frame is contained in worldmap.

Usage

```
worldmap
```

Format

A data frame with 33971 rows and 6 variables:

long longitude coordinate

lat latitude coordinate

group polygon (region) number

order order of point in polygon boundary

region region name

subregion subregion name

References

Original S code by Becker, R.A. and Wilks, R.A. This R version is by Brownrigg, R. Enhancements have been made by Minka, T.P. and Deckmyn, A. (2015) maps: Draw Geographical Maps, R package version 3.0.1.

Index