Package ‘GPoM.FDLyapu’

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Type Package

Title Lyapunov Exponents and Kaplan-Yorke Dimension

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Description Estimation of the spectrum of Lyapunov Exponents and the Kaplan-Yorke dimension of any low-dimensional model of polynomial form.
It can be applied, for example, to systems such as the chaotic
Lorenz-1963 system or the hyperchaotic Rossler-1979 system.
It can also be applied to dynamical models in Ordinary Differential Equations (ODEs) directly obtained
from observational time series using the 'GPoM' package.
The approach used is semi-formal,
the Jacobian matrix being estimated automatically from the polynomial equations.
Two methods are made available; one introduced by Wolf et al.
(1985) <doi:10.1016/0167-2789(85)90011-9>
and the other one introduced by Grond et al. (2003) <doi:10.1016/S0960-0779(02)00479-4>.
The package is provided with an interface for a more intuitive usage,
it can also be run without the interface.
This platform is developed at the Centre d'Etudes Spatiales de la Biosphere (CESBIO),
UMR 5126 UPS/CNRS/CNES/IRD, 18 av. Edouard Belin, 31401 TOULOUSE, FRANCE.
The developments were funded by the French program Les Enveloppes Fluides
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Depends R (>= 2.10), GPoM, deSolve

Suggests knitr, rmarkdown, rgl, shiny

VignetteBuilder knitr

NeedsCompilation no

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`GPoM.FDLyapu-package` *GPoM.FDLyapu package: Lyapunov Exponents and Kaplan-Yorke Dimension*

Description

Estimation of the spectrum of Lyapunov Exponents and the Kaplan-Yorke dimension of any low-dimensional model of polynomial form. It can be applied, for example, to systems such as the chaotic Lorenz-1963 system or the hyperchaotic Rossler-1979 system. It can also be applied to dynamical models in Ordinary Differential Equations (ODEs) directly obtained from observational time series using the 'GPoM' package. The used approach is semi-formal, the Jacobian matrix being estimated automatically from the polynomial equations. Two methods are made available : one introduced by Wolf et al. (1985) [1] and the other one by Grond et al. (2003) [2].

Note

FOR USERS
This package was developped at Centre d'Etudes Spatiales de la Biosphere (Cesbio, UMR 5126, UPS-CNRS-CNES-IRD, http://www.cesbio.ups-tlse.fr). An important part of the developments were funded by the French program Les Enveloppes Fluides et l'Environnement (LEFE, MANU, projets GloMo, SpatioGloMo and MoMu). The French program Défi InFiNiTi (CNRS) and PNTS are also acknowledged (projects Crops'IChaos and Musc & SlowFast).

Author(s)

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References

[1] A. Wolf, J. B. Swift, H. L. Swinney & J. A. Vastano, Determining Lyapunov exponents from a time series, Physica D, 285-317, 1985.
[2] F. Grond, H. H. Diebner, S. Sahle, A. Mathias, S. Fischer, O. E. Rossler, A robust, locally interpretable algorithm for Lyapunov exponents, Chaos, Solitons \& Fractals, 16, 841-852 (2003).

DkyCalc — *DkyCalc : computes the Kaplan-Yorke dimension*

Description

Computes the Kaplan-Yorke dimension from the Lyapunov exponents (Kaplan and Yorke 1979).

Usage

```
DkyCalc(methodName, nVar, lyapExp)
```

Arguments

`methodName`	The method that was used to compute the lyapunov exponents
`nVar`	The model dimension (which corresponds to the number of exponents)
`lyapExp`	Time series of the local Lyapunov exponents spectrum (one column for each exponent)

References

Kaplan, J. & Yorke, J., Chaotic behavior of multidimensional difference equations. In: Peitgen H. O. and Walther H. O., "Functional Differential Equations and the Approximation of Fixed Points", Lecture Notes in Mathematics. 730. Berlin: Springer. p. 204-227, 1979.

Examples

```
#' Load the global model (here for Ebola Virus Diesease)
data(Ebola)
nVar = dim(Ebola$KL)[2]
pMax = dim(Ebola$KL)[1]
dMax = p2dMax(nVar, pMax)
#' Compute the time series of Lyapunov exponents
outLyapFD <- NULL
outLyapFD$Wolf <- lyapFDWolf(outLyapFD$Wolf, nVar= nVar, dMax = dMax,
                             coeffF = Ebola$KL,
```

```
                              tDeb = 0, dt = 0.01, tFin = 2,
                              yDeb = Ebola$yDeb)
#' estimate the Kaplan-Yorke dimension
DkyCalc(methodName = "Wolf", nVar= 3, lyapExp = outLyapFD$Wolf$lyapExpLoc)
```

Ebola *Ebola model for the epidemic of Ebola Virus Disease*

Description

The Ebola model is a four-dimensional model obtained by the global modelling technique from a set of two observational variables. The model reads $dX1/dt = a1Y1Y3 + a2Y2 - a3X1Y1$ $dY1/dt = Y2$ $dY2/dt = Y3$ $dY3/dt = b1+b2Y3+b3Y3^2-b4Y2-b5Y2^2+b6Y1-b7Y1Y3+b8Y1Y2-b9Y2-b10X1-b11X1Y3-b12X1Y2+b13X1Y1+b14X1^2$. It produces a chaotic behavior and its attractor has a fractal dimension Dky = 3.03. Its formulation can be visualized using visuEq(nVar = 4, dMax = 2, K = Ebola$KL, approx = 2) (the number of digit is truncated)

Usage

```
Ebola
```

Format

An object of class `list` of length 2.

Author(s)

Sylvain Mangiarotti <sylvain.mangiarotti@cesbio.cnes.fr>

References

[1] Sylvain Mangiarotti, Marisa Peyre, and Mireille Huc, A chaotic model for the epidemic of Ebola virus disease in West Africa (2013-2016), Chaos, 26, 113112, 2016.

GSproc *GSproc : Gram-Schmidt procedure*

Description

Computes regressors coefficients using the Gram-Schmidt procedure.

Usage

```
GSproc(polyK, ivec, weight = NULL)
```

Arguments

polyK	One list including $Y and $phy with: $Y a matrix for which the ith column will be used to add one orthogonal vector to the (i-1)th vectors of the current orthogonal base; and $phy such as the current orthogonal base is given by the (i-1)th first columns of matrix polyK$phy.
ivec	Defines i, the current vector of polyK$Y and the current orthogonal base of pParam$phy.
weight	The weighing vector.

Value

uNew The model parameterization, that is: The residual orthogonal vector that can be included into the current orthogonal base. If the current base is empty, uNew is equal to the input vector of $Y; if the base is complete, uNew equals 0.

Author(s)

Sylvain Mangiarotti

#@export

Lorenz63 *Lorenz-1963 system*

Description

The Lorenz-1963 model is a three-dimensional model obtained by Edouard N. Lorenz in 1963. The system reads $dX/dt = -\sigma Y - \sigma Z$ $dY/dt = X(\rho - Z) - Y$ $dZ/dt = XY - \beta Z$. For $(\sigma, \rho, \beta) = (10, 28, 8/3)$, it produces a chaotic behavior. The chaotic attractor reached at the convergence has become paradigmatic of chaos. Its formulation can be visualized using visuEq(nVar = 4, dMax = 2, K = Lrz63$KL, approx = 2)

Usage

```
Lorenz63
```

Format

An object of class `list` of length 2.

Author(s)

Sylvain Mangiarotti <sylvain.mangiarotti@cesbio.cnes.fr>

References

Lorenz, Edward Norton (1963). "Deterministic nonperiodic flow". Journal of the Atmospheric Sciences. 20 (2): 130–141.

`lyapFDGrond` *lyapFDGrond : Computes the Lyapunov spectrum (with compelled flow direction)*

Description

Computes all the Lyapunov exponents based on Gram-Schmidt procedure with zero-Lyapunov exponent compelled to the flow direction (Grond et al. 1985). The Jacobian matrix is computed from the original model by semi-Formal Derivation.

Usage

```
lyapFDGrond(outLyapFD = NULL, nVar, dMax, coeffF, intgrMthod = "rk4",
  tDeb = 0, dt, tFin, yDeb, Ddeb = NULL, nIterMin = 1,
  nIterStats = 50)
```

Arguments

`outLyapFD`	List of output data that can be used as an input in order to extend the computation
`nVar`	Model dimension
`dMax`	Maximum degree of the polynomial formulation
`coeffF`	Model matrix. Each column correspond to one equation. Lines provide the coefficients for each polynomial term which order is defined with function `poLabs(nVar, dMax)` in package `GPoM`)
`intgrMthod`	Numerical integration method ('rk4' by default)
`tDeb`	Initial integration time (0 by default)
`dt`	Integration time step
`tFin`	Final integration time
`yDeb`	Model initial conditions
`Ddeb`	Jacobian initial conditions (optional).
`nIterMin`	Minimum number of iterations (nIterMin= 1 by default)
`nIterStats`	Number of iterations used in the statistics computation

Value

List of output data

References

F. Grond, H. H. Diebner, S. Sahle, A. Mathias, S. Fischer, O. E. Rossler, A robust, locally interpretable algorithm for Lyapunov exponents, Chaos, Solitons \& Fractals, 16, 841-852 (2003).

F. Grond \& H. H. Diebner: Local Lyapunov exponents for dissipative continuous systems. Chaos, Solitons \& Fractals, 23, 1809-1817 (2005).

Examples

```
data(Ebola)
nVar = dim(Ebola$KL)[2]
pMax = dim(Ebola$KL)[1]
dMax = p2dMax(nVar, pMax)
outLyapFD <- NULL
outLyapFD$Grond <- lyapFDGrond(outLyapFD$Grond, nVar= nVar, dMax = dMax, coeffF = Ebola$KL,
                               tDeb = 0, dt = 0.01, tFin = 2, yDeb = Ebola$yDeb)
```

`lyapFDWolf` *lyapFDWolf : computes Lyapunov spectrum with Wolf method*

Description

Computes all the Lyapunov exponents based on Gram-Schmidt procedure (Wolf et al. 1985). The Jacobian matrix is computed from the original model by semi-Formal Derivation.

Usage

```
lyapFDWolf(outLyapFD = NULL, nVar, dMax, coeffF, intgrMthod = "rk4",
  tDeb = 0, dt, tFin, yDeb, Ddeb = NULL, nIterMin = 1,
  nIterStats = 50)
```

Arguments

`outLyapFD`	List of output data that can be used as an input in order to extend the computation
`nVar`	Model dimension
`dMax`	Maximum degree of the polynomial formulation
`coeffF`	Model matrix. Each column correspond to one equation. Lines provide the coefficients for each polynomial term which order is defined with function `poLabs(nVar, dMax)` in package `GPoM`)
`intgrMthod`	Numerical integration method ('rk4' by default)
`tDeb`	Initial integration time (0 by default)
`dt`	Integration time step
`tFin`	Final integration time
`yDeb`	Model initial conditions
`Ddeb`	Jacobian initial conditions (optional).
`nIterMin`	Minimum number of iterations (nIterMin= 1 by default)
`nIterStats`	Number of iterations used in the statistics computation

Value

List of output data

References

A. Wolf, J. B. Swift, H. L. Swinney & J. A. Vastano, Determining Lyapunov exponents from a time series, Physica D, 285-317, 1985.

Examples

```
data(Ebola)
nVar = dim(Ebola$KL)[2]
pMax = dim(Ebola$KL)[1]
dMax = p2dMax(nVar, pMax)
outLyapFD <- NULL
outLyapFD$Wolf <- lyapFDWolf(outLyapFD$Wolf, nVar= nVar, dMax = dMax,
                             coeffF = Ebola$KL,
                             tDeb = 0, dt = 0.01, tFin = 2,
                             yDeb = Ebola$yDeb)
```

`outLyapFD` *An output of FDLyapu run*

Description

This data set provides two examples of outputs (Rossler-1976 and Rossler-1979 systems) produced by the FDLyapu algorithm.

Usage

```
outLyapFD
```

Format

An object of class `list` of length 2.

Author(s)

Sylvain Mangiarotti <sylvain.mangiarotti@cesbio.cnes.fr>

Plague — *eco-epidemiology Plague model for the epidemic-epizootic of plague in Bombay (1896-1911)*

Description

The eco-epidemiolgy plague model is a three-dimensional model obtained using the global modelling technique from a set of three observational variables: - X the number of human cases - Y the number of contaminated brown rats - Z the number of contaminated black rats $dX/dt = fx(X, Y, Z)$ $dY/dt = fy(X, Y, Z)$ $dZ/dt = fz(X, Y, Z)$. It can produce a chaotic behavior by tuning its parameterization. Its formulation can be visualized using visuEq(K = GP_DRDT_1110_dM2$models$model93, approx = 2, substit = 1) (the number of digit is truncated)

Usage

```
Plague
```

Format

An object of class `list` of length 1.

Author(s)

Sylvain Mangiarotti <sylvain.mangiarotti@cesbio.cnes.fr>

References

[1] Sylvain Mangiarotti, Low dimensional chaotic models for the plague epidemic in Bombay (1896-1911), Chaos Solitons \& Fractals, 26, 113112, 2016.

plotConvergence — *plotConvergence*

Description

Plots the evolution of the standard deviations of either the Lyapunov exponents or the Kaplan-Yorke dimension

Usage

```
plotConvergence(nVar, outL, plotType = "exp", xlim = NULL,
  ylim = NULL, lastIter = NULL, dt = NULL, expList = NULL,
  legend = FALSE)
```

Arguments

`nVar`	Model dimension
`outL`	An input list issued from the FDLyapu algorithm updated at each call, with outL$t the time vector, outL$LyapExp the mean Lyapunov exponents outL$LyapExpLoc the local Lyapunov exponents, outL$DkyLoc the mean Kaplan-Yorke dimension. outL$DkyLoc the local Kaplan-Yorke dimension.
`plotType`	If equals to "exp" the Lyapunov exponents are plotted. Otherwise the Kaplan-Yorke dimension is plotted
`xlim`	The limits used for the x-axis. If NULL it is automatically adjusted to the minimum and maximum data values
`ylim`	The limits used for the u-axis. If NULL it is automatically adjusted to the minimum and maximum data values
`lastIter`	If not NULL indicates the last iterations used for the plot.
`dt`	Integration time step
`expList`	Indicates which exponents have to be plotted. If NULL all are plotted otherwise must be a vector of booleans
`legend`	Indicates if the legend has to appear (TRUE) or not (FALSE)

`plotLocalExponents` *plotLocalExponents*

Description

Plots the local Lyapunov exponents or the local Kaplan-Yorke dimension

Usage

```
plotLocalExponents(nVar, outL, plotType = "exp", xlim = NULL,
  ylim = NULL, lastIter = NULL, dt = NULL, expList = NULL,
  legend = FALSE)
```

Arguments

`nVar`	Model dimension
`outL`	An input list issued from the FDLyapu algorithm updated at each call, with outL$t the time vector, outL$LyapExp the mean Lyapunov exponents outL$LyapExpLoc the local Lyapunov exponents, outL$DkyLoc the mean Kaplan-Yorke dimension. outL$DkyLoc the local Kaplan-Yorke dimension.
`plotType`	If equals to "exp" the Lyapunov exponents are plotted. Otherwise the Kaplan-Yorke dimension is plotted
`xlim`	The limits used for the x-axis. If NULL it is automatically adjusted to the minimum and maximum data values

`ylim`	The limits used for the u-axis. If NULL it is automatically adjusted to the minimum and maximum data values
`lastIter`	If not NULL indicates the last iterations used for the plot.
`dt`	Integration time step
`expList`	Indicates which exponents have to be plotted. If NULL all are plotted otherwise must be a vector of booleans
`legend`	Indicates if the legend has to appear (TRUE) or not (FALSE)

`plotMeanExponents` *plotMeanExponents*

Description

Plots the averaged Lyapunov exponents or the averaged Kaplan-Yorke dimension

Usage

```
plotMeanExponents(nVar, outL, nIterStats = NULL, plotType = "exp",
  xlim = NULL, ylim = NULL, expList = NULL, legend = FALSE)
```

Arguments

`nVar`	Model dimension
`outL`	An input list issued from the FDLyapu algorithm updated at each call, with outL$t the time vector, outL$LyapExp the mean Lyapunov exponents outL$LyapExpLoc the local Lyapunov exponents, outL$DkyLoc the mean Kaplan-Yorke dimension. outL$DkyLoc the local Kaplan-Yorke dimension.
`nIterStats`	Number of iterations used in the statistics computation
`plotType`	If equals to "exp" the Lyapunov exponents are plotted. Otherwise the Kaplan-Yorke dimension is plotted
`xlim`	The limits used for the x-axis. If NULL it is automatically adjusted to the minimum and maximum data values
`ylim`	The limits used for the u-axis. If NULL it is automatically adjusted to the minimum and maximum data values
`expList`	Indicates which exponents have to be plotted. If NULL all are plotted otherwise must be a vector of booleans
`legend`	Indicates if the legend has to appear (TRUE) or not (FALSE)

Reit98	*The 9D model for a Rayleigh-Bénard convection in a square cell*

Description

The 9D model for a Rayleigh-Bénard convection in a square cell was introduced by Reiterer et al. in 1998. It can produce an apparently smooth transition from chaos to hyperchaos.

Usage

```
Reit98
```

Format

An object of class `matrix` with 55 rows and 9 columns.

Author(s)

Sylvain Mangiarotti <sylvain.mangiarotti@cesbio.cnes.fr>

References

P. Reiterer, C. Lainscsek, F. Schürrer, C. Letellier, and J. Maquet, A nine-dimensional Lorenz system to study high-dimensional chaos, Journal of Physics A, 31, 7121-7139, 1998.

Index