Package ‘IRTest’

October 12, 2022

Type Package

Title Parameter Estimation of Item Response Theory with Estimation of Latent Distribution

Version 0.0.2

Description Item response theory (IRT) parameter estimation using marginal maximum likelihood and expectation-maximization algorithm (Bock & Aitkin, 1981 <doi:10.1007/BF02293801>). Within parameter estimation algorithm, several methods for latent distribution estimation are available (Li, 2022 <https://www.riss.kr/search/detail/DetailView.do?p_mat_type=be54d9b8bc7cdb09&control_no=9a95f68e2c1126c5ffe0bdc3ef48d419>). Reflecting some features of the true latent distribution, these latent distribution estimation methods can possibly free the normality assumption on the latent distribution.

License GPL (>= 3)

Encoding UTF-8

LazyData true

RoxygenNote 7.2.1

URL https://github.com/SeewooLi/IRTest

BugReports https://github.com/SeewooLi/IRTest/issues

Suggests knitr, rmarkdown, testthat (>= 3.0.0)

VignetteBuilder knitr

Imports betafunctions, dcurver, ggplot2

Depends R (>= 2.10)

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R topics documented:

`DataGeneration` *Generating artificial item response data*

Description

This function generates artificial item response data with users specified item types, details of item parameters, and latent distribution.

Usage

```
DataGeneration(
  seed = 1,
  N = 2000,
  nitem_D = NULL,
  nitem_P = NULL,
  model_D,
  model_P = "GPCM",
  latent_dist = "Mixture",
  prob = 0.5,
  d = 1.7,
  sd_ratio = 1,
  a_l = 0.8,
  a_u = 2.5,
  c_l = 0,
  c_u = 0.2,
  categ
)
```

Arguments

`seed`	A numeric value that is used on random sampling. The seed number can guarantee the replicability of the result.
`N`	A numeric value. The number of examinees.
`nitem_D`	A numeric value. The number of dichotomous items.

nitem_P	A numeric value. The number of polytomous items.
model_D	A vector of length nitem_D. The *i*th element is the probability model for the *i*th dichotomous item.
model_P	A vector of length nitem_P. The *i*th element is the probability model for the *i*th polytomous item.
latent_dist	A character string that determines the type of latent distribution. Currently available options are "beta" (four-parameter beta distribution; rBeta.4P), "chi" (χ^2 distribution; rchisq), "normal" (standard normal distribution; rnorm), and "Mixture" (two-component Gaussian mixture distribution; see Li (2021) for details.)
prob	A numeric value required when latent_dist = "Mixture". It is a $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
d	A numeric value required when latent_dist = "Mixture". It is a $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
sd_ratio	A numeric value required when latent_dist = "Mixture". It is a $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
a_l	A numeric value. The lower bound of item discrimination parameters (*a*).
a_u	A numeric value. The upper bound of item discrimination parameters (*a*).
c_l	A numeric value. The lower bound of item guessing parameters (*c*).
c_u	A numeric value. The lower bound of item guessing parameters (*c*).
categ	A numeric vector of length nitem_P. The *i*th element equals the number of categories of the *i*th polyotomous item.

Value

This function returns a list which contains several objects:

theta	A vector of ability parameters (θ).
item_D	A matrix of dichotomous item parameters.
initialitem_D	A matrix that contains initial item parameter values for dichotomous items.
data_D	A matrix of dichotomous item responses where rows indicate examinees and columns indicate items.
item_P	A matrix of polytomous item parameters.
initialitem_P	A matrix that contains initial item parameter values for polytomous items.
data_P	A matrix of polytomous item responses where rows indicate examinees and columns indicate items.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

Examples

```
# Dichotomous item responses only

Alldata <- DataGeneration(seed = 1,
                          model_D = rep(3, 10),
                          N=500,
                          nitem_D = 10,
                          nitem_P = 0,
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_D
item <- Alldata$item_D
initialitem <- Alldata$initialitem_D
theta <- Alldata$theta


# Polytomous item responses only

Alldata <- DataGeneration(seed = 2,
                          N=1000,
                          nitem_D = 0,
                          nitem_P = 10,
                          categ = rep(3:7,each = 2),
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_P
item <- Alldata$item_P
initialitem <- Alldata$initialitem_P
theta <- Alldata$theta


# Mixed-format items

Alldata <- DataGeneration(seed = 2,
                        model_D = rep(1:2, each=10),# 1PL model is applied to item #1~10
                                                  # and 2PL model is applied to item #11~20.
                          N=1000,
                          nitem_D = 20,
                          nitem_P = 10,
```

```
                              categ = rep(3:7,each = 2),# 3 categories for item #21-22,
                                                        # 4 categories for item #23-24,
                                                        # ...,
                                                        # and 7 categories for item #29-30.
                              d = 1.664,
                              sd_ratio = 2,
                              prob = 0.3)

DataD <- Alldata$data_D
DataP <- Alldata$data_P
itemD <- Alldata$item_D
itemP <- Alldata$item_P
initialitemD <- Alldata$initialitem_D
initialitemP <- Alldata$initialitem_P
theta <- Alldata$theta
```

`dist2` *Re-parameterized two-component normal mixture distribution*

Description

Probability density for the re-parameterized two-component normal mixture distribution.

Usage

```
dist2(x, prob = 0.5, d = 0, sd_ratio = 1, overallmean = 0, overallsd = 1)
```

Arguments

`x`	A numeric vector. The location to evaluate the density function.
`prob`	A numeric value of $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
`d`	A numeric value of $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma}$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
`sd_ratio`	A numeric value of $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
`overallmean`	A numeric value of $\bar{\mu}$ that determines the overall mean of two-component Gaussian mixture distribution.
`overallsd`	A numeric value of $\bar{\sigma}$ that determines the overall standard deviation of two-component Gaussian mixture distribution.

Details

The overall mean and overall standard deviation can be expressed with original parameters of two-component Gaus

1) Overall mean ($\bar{\mu}$)

$$\bar{\mu} = \pi\mu_1 + (1-\pi)\mu_2$$

2) Overall standard deviation ($\bar{\sigma}$)

$$\bar{\sigma} = \sqrt{\pi\sigma_1^2 + (1-\pi)\sigma_2^2 + \pi(1-\pi)(\mu_2-\mu_1)^2}$$

Value

The evaluated probability density value(s).

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

GHc — *Gauss-Hermite constants*

Description

a `vector` that contains Gauss-Hermite constants

Usage

```
GHc
```

Format

An object of class `numeric` of length 21.

IRTest_Dich — *Item and ability parameters estimation for dichotomous items*

Description

This function estimates IRT item and ability parameters when all items are scored dichotomously. Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006).

Usage

```
IRTest_Dich(
  initialitem,
  data,
  range = c(-6, 6),
  q = 121,
  model,
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "SJ-ste",
  h = NULL
)
```

Arguments

`initialitem`	A matrix of initial item parameter values for starting the estimation algorithm
`data`	A matrix of item responses where responses are coded as 0 or 1. Rows and columns indicate examinees and items, respectively.
`range`	Range of the latent variable to be considered in the quadrature scheme. The default is from `-6` to `6`: `c(-6, 6)`.
`q`	A numeric value that represents the number of quadrature points. The default value is 121.
`model`	A vector that represents types of item characteristic functions applied to each item. Insert `1`, `"1PL"`, `"Rasch"`, or `"RASCH"` for one-parameter logistic model, `2`, `"2PL"` for two-parameter logistic model, and `3`, `"3PL"` for three-parameter logistic model.
`latent_dist`	A character string that determines latent distribution estimation method. Insert `"Normal"`, `"normal"`, or `"N"` to assume normal distribution on the latent distribution, `"EHM"` for empirical histogram method (Mislevy, 1984; Mislevy & Bock, 1985), `"Mixture"` for the method that uses two-component Gaussian mixture

distribution (Li, 2021; Mislevy, 1984), "DC" for Davidian-curve method (Woods & Lin, 2009), and "KDE" for kernel density estimation method (Li, 2022). The default value is set to "Normal" to follow the conventional assumption on latent distribution.

max_iter
: A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.

threshold
: A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.

bandwidth
: A character value is needed when "KDE" is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for "KDE". The default value is "SJ-ste". See density for possible options.

h
: A natural number less than or equal to 10 is needed when "DC" is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

The probabilities for correct response ($u = 1$) in one-, two-, and three-parameter logistic models can be expressed as

1) One-parameter logistic (1PL) model

$$P(u = 1|\theta, b) = \frac{\exp{(\theta - b)}}{1 + \exp{(\theta - b)}}$$

2) Two-parameter logistic (2PL) model

$$P(u = 1|\theta, a, b) = \frac{\exp{(a(\theta - b))}}{1 + \exp{(a(\theta - b))}}$$

3) Three-parameter logistic (3PL) model

$$P(u = 1|\theta, a, b, c) = c + (1 - c)\frac{\exp{(a(\theta - b))}}{1 + \exp{(a(\theta - b))}}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta = X_k) = A(X_k)$$

where $k = 1, 2, ..., q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q - 1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta = X) = \pi\phi(X; \mu_1, \sigma_1) + (1 - \pi)\phi(X; \mu_2, \sigma_2)$$

where $\phi(X; \mu, \sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta = X) = \left\{ \sum_{\lambda=0}^{h} m_\lambda X^\lambda \right\}^2 \phi(X; 0, 1)$$

where h corresponds to the argument `h` and determines the degree of the polynomial.
4) Kernel density estimation method

$$P(\theta = X) = \frac{1}{Nh} \sum_{j=1}^{N} K\left(\frac{X - \theta_j}{h}\right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument `bandwidth`, and $K(\cdot)$ is a kernel function. The Gaussian kernel is used in this function.

Value

This function returns a `list` which contains several objects:

`par_est`	The item parameter estimates.
`se`	The standard errors for item parameter estimates.
`fk`	The estimated frequencies of examinees at each quadrature points.
`iter`	The number of EM-MML iterations required for the convergence.
`prob`	The estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
`d`	The estimated $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
`sd_ratio`	The estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
`quad`	The location of quadrature points.
`diff`	The final value of the monitored maximum item parameter change.
`Ak`	The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.
`Pk`	The posterior probabilities for each examinees at each quadrature points.
`theta`	The estimated ability parameter values. Expected *a posteriori* (EAP) is used for ability parameter estimation.
`logL`	The deviance (i.e., -2*log*L).
`bw`	The bandwidth used.
`Options`	A replication of input arguments.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. *Psychometrika, 46*(4), 443-459.

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

Li, S. (2022). *The effect of estimating latent distribution using kernel density estimation method on the accuracy and efficiency of parameter estimation of item response models* [Master's thesis, Yonsei University, Seoul]. Yonsei University Library.

Mislevy, R. J. (1984). Estimating latent distributions. *Psychometrika, 49*(3), 359-381.

Mislevy, R. J., & Bock, R. D. (1985). Implementation of the EM algorithm in the estimation of item parameters: The BILOG computer program. In D. J. Weiss (Ed.). *Proceedings of the 1982 item response theory and computerized adaptive testing conference* (pp. 189-202). University of Minnesota, Department of Psychology, Computerized Adaptive Testing Conference.

Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
# A preparation of dichotomous item response data

Alldata <- DataGeneration(seed = 1,
                          model_D = rep(1, 10),
                          N=500,
                          nitem_D = 10,
                          nitem_P = 0,
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_D
item <- Alldata$item_D
initialitem <- Alldata$initialitem_D
theta <- Alldata$theta

# Analysis

M1 <- IRTest_Dich(initialitem = initialitem,
                  data = data,
                  model = rep(1,10),
                  latent_dist = "KDE",
              bandwidth = "SJ-ste", # an argument required only when "latent_dist = 'KDE'"
                  max_iter = 200,
                  threshold = .001,
```

```
h=4 # an argument required only when "latent_dist = 'DC'"
)
```

IRTest_Mix — *Item and ability parameters estimation for a mixed-format item response data*

Description

This function estimates IRT item and ability parameters when a test consists of mixed-format items (i.e., a combination of dichotomous and polytomous items). In educational context, the combination of these two item formats takes an advantage; Dichotomous item format expedites scoring and is conducive to cover broad domain, while Polytomous item format (e.g., free response item) encourages students to exert complex cognitive skills (Lee et al., 2020). Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006).

Usage

```
IRTest_Mix(
  initialitem_D,
  initialitem_P,
  data_D,
  data_P,
  range = c(-6, 6),
  q = 121,
  model_D,
  model_P = "GPCM",
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "nrd",
  h = NULL
)
```

Arguments

initialitem_D	A matrix of initial dichotomous item parameter values for starting the estimation algorithm.
initialitem_P	A matrix of initial polytomous item parameter values for starting the estimation algorithm.
data_D	A matrix of item responses where responses are coded as 0 or 1. Rows and columns indicate examinees and items, respectively.

data_P	A matrix of polytomous item responses where responses are coded as `0, 1, ...,` or `m` for an `m+1` category item. Rows and columns indicate examinees and items, respectively.
range	Range of the latent variable to be considered in the quadrature scheme. The default is from `-6` to `6`: `c(-6, 6)`.
q	A numeric value that represents the number of quadrature points. The default value is 121.
model_D	A vector that represents types of item characteristic functions applied to each item. Insert `1`, `"1PL"`, `"Rasch"`, or `"RASCH"` for one-parameter logistic model, `2`, `"2PL"` for two-parameter logistic model, and `3`, `"3PL"` for three-parameter logistic model.
model_P	Currently, only the default (`"GPCM"`) is available.
latent_dist	A character string that determines latent distribution estimation method. Insert `"Normal"`, `"normal"`, or `"N"` to assume normal distribution on the latent distribution, `"EHM"` for empirical histogram method (Mislevy, 1984; Mislevy & Bock, 1985), `"Mixture"` for the method that uses two-component Gaussian mixture distribution (Li, 2021; Mislevy, 1984), `"DC"` for Davidian-curve method (Woods & Lin, 2009), and `"KDE"` for kernel density estimation method (Li, 2022). The default value is set to `"Normal"` to follow the conventional assumption on latent distribution.
max_iter	A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.
threshold	A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.
bandwidth	A character value is needed when `"KDE"` is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for `"KDE"`. The default value is `"SJ-ste"`. See `density` for possible options.
h	A natural number less than or equal to 10 is needed when `"DC"` is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

Dichotomous: The probabilities for correct response ($u=1$) in one-, two-, and three-parameter logistic models can b

1) One-parameter logistic (1PL) model

$$P(u=1|\theta,b)=\frac{\exp{(\theta-b)}}{1+\exp{(\theta-b)}}$$

2) Two-parameter logistic (2PL) model

$$P(u=1|\theta,a,b)=\frac{\exp{(a(\theta-b))}}{1+\exp{(a(\theta-b))}}$$

3) Three-parameter logistic (3PL) model

$$P(u=1|\theta,a,b,c)=c+(1-c)\frac{\exp{(a(\theta-b))}}{1+\exp{(a(\theta-b))}}$$

Polytomous: The probability for scoring k (i.e., $u = k; k = 0, 1, ..., m; m \geq 2$) in generalized partial credit model (GP

1) generalized partial credit model (GPCM)

$$P(u = 0|\theta, a, b_1, ..., b_m) = \frac{1}{1 + \sum_{c=1}^{m} \exp\left[\sum_{v=1}^{c} a(\theta - b_v)\right]}$$

$$P(u = 1|\theta, a, b_1, ..., b_m) = \frac{\exp\left(a(\theta - b_1)\right)}{1 + \sum_{c=1}^{m} \exp\left[\sum_{v=1}^{c} a(\theta - b_v)\right]}$$

$$\vdots$$

$$P(u = m|\theta, a, b_1, ..., b_m) = \frac{\exp\left[\sum_{v=1}^{m} a(\theta - b_v)\right]}{1 + \sum_{c=1}^{m} \exp\left[\sum_{v=1}^{c} a(\theta - b_v)\right]}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta = X_k) = A(X_k)$$

where $k = 1, 2, ..., q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q - 1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta = X) = \pi\phi(X; \mu_1, \sigma_1) + (1 - \pi)\phi(X; \mu_2, \sigma_2)$$

where $\phi(X; \mu, \sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta = X) = \left\{\sum_{\lambda=0}^{h} m_\lambda X^\lambda\right\}^2 \phi(X; 0, 1)$$

where h corresponds to the argument `h` and determines the degree of the polynomial.

4) Kernel density estimation method

$$P(\theta = X) = \frac{1}{Nh}\sum_{j=1}^{N} K\left(\frac{X - \theta_j}{h}\right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument `bw`, and $K(\bullet)$ is a kernel function. The Gaussian kernel is used in this function.

Value

This function returns a `list` which contains several objects:

`par_est` The list item parameter estimates. The first object of `par_est` is the matrix of item parameter estimates for dichotomous items, and The second object is the matrix of item parameter estimates for polytomous items.

se	The standard errors for item parameter estimates. The first object of se is the matrix of standard errors for dichotomous items, and The second object is the matrix of standard errors for polytomous items.
fk	The estimated frequencies of examinees at each quadrature points.
iter	The number of EM-MML iterations required for the convergence.
prob	The estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
d	The estimated $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
sd_ratio	The estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
quad	The location of quadrature points.
diff	The final value of the monitored maximum item parameter change.
Ak	The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.
Pk	The posterior probabilities for each examinees at each quadrature points.
theta	The estimated ability parameter values. Expected *a posteriori* (EAP) is used for ability parameter estimation.
logL	The deviance (i.e., -2logL).
bw	The bandwidth used.
Options	A replication of input arguments.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. *Psychometrika, 46*(4), 443-459.

Lee, W. C., Kim, S. Y., Choi, J., & Kang, Y. (2020). IRT Approaches to Modeling Scores on Mixed-Format Tests. *Journal of Educational Measurement, 57*(2), 230-254.

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

Li, S. (2022). *The effect of estimating latent distribution using kernel density estimation method on the accuracy and efficiency of parameter estimation of item response models* [Master's thesis, Yonsei University, Seoul]. Yonsei University Library.

Mislevy, R. J. (1984). Estimating latent distributions. *Psychometrika, 49*(3), 359-381.

Mislevy, R. J., & Bock, R. D. (1985). Implementation of the EM algorithm in the estimation of item parameters: The BILOG computer program. In D. J. Weiss (Ed.). *Proceedings of the 1982 item response theory and computerized adaptive testing conference* (pp. 189-202). University of Minnesota, Department of Psychology, Computerized Adaptive Testing Conference.

Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
# A preparation of mixed-format item response data

Alldata <- DataGeneration(seed = 2,
                         model_D = rep(1:2, each=3),# 1PL model is applied to item #1~10
                                                    # and 2PL model is applied to item #11~20.
                          N=1000,
                          nitem_D = 6,
                          nitem_P = 5,
                          categ = rep(3:7,each = 1),# 3 categories for item #21-22,
                                                    # 4 categories for item #23-24,
                                                    # ...,
                                                    # and 7 categories for item #29-30.
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

DataD <- Alldata$data_D
DataP <- Alldata$data_P
itemD <- Alldata$item_D
itemP <- Alldata$item_P
initialitemD <- Alldata$initialitem_D
initialitemP <- Alldata$initialitem_P
theta <- Alldata$theta

# Analysis

M1 <- IRTest_Mix(initialitem_D = initialitemD,
                 initialitem_P = initialitemP,
                 data_D = DataD,
                 data_P = DataP,
                 model_D = rep(1:2, each=3),
                 latent_dist = "KDE",
                 bandwidth = "SJ-ste",
                 max_iter = 200,
                 threshold = .001,
                 h=9)
```

IRTest_Poly *Item and ability parameters estimation for polytomous items*

Description

This function estimates IRT item and ability parameters when all items are scored polytomously. Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006). Only generalized partial credit model (GPCM) is currently available.

Usage

```
IRTest_Poly(
  initialitem,
  data,
  range = c(-6, 6),
  q = 121,
  model,
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "nrd",
  h = NULL
)
```

Arguments

`initialitem`	A matrix of initial item parameter values for starting the estimation algorithm. This matrix determines the number of categories for each item.
`data`	A matrix of item responses where responses are coded as `0, 1, ..., m` for an `m+1` category item. Rows and columns indicate examinees and items, respectively.
`range`	Range of the latent variable to be considered in the quadrature scheme. The default is from `-6` to `6`: `c(-6, 6)`.
`q`	A numeric value that represents the number of quadrature points. The default value is 121.
`model`	A vector that represents types of item characteristic functions applied to each item. However, only generalized partial credit model (`GPCM`) is currently available.
`latent_dist`	A character string that determines latent distribution estimation method. Insert `"Normal"`, `"normal"`, or `"N"` to assume normal distribution on the latent distribution, `"EHM"` for empirical histogram method (Mislevy, 1984; Mislevy & Bock,

1985), "Mixture" for the method that uses two-component Gaussian mixture distribution (Li, 2021; Mislevy, 1984), "DC" for Davidian-curve method (Woods & Lin, 2009), and "KDE" for kernel density estimation method (Li, 2022). The default value is set to "Normal" to follow the conventional assumption on latent distribution.

`max_iter` A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.

`threshold` A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.

`bandwidth` A character value is needed when "KDE" is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for "KDE". The default value is "SJ-ste". See density for possible options.

`h` A natural number less than or equal to 10 is needed when "DC" is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

The probability for scoring k (i.e., $u=k; k=0,1,...,m; m \geq 2$) in generalized partial credit model (GPCM) can be

1) generalized partial credit model (GPCM)

$$P(u=0|\theta, a, b_1, ..., b_m) = \frac{1}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

$$P(u=1|\theta, a, b_1, ..., b_m) = \frac{\exp\left(a(\theta-b_1)\right)}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

$$\vdots$$

$$P(u=m|\theta, a, b_1, ..., b_m) = \frac{\exp\left[\sum_{v=1}^{m}a(\theta-b_v)\right]}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta = X_k) = A(X_k)$$

where $k = 1, 2, ..., q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q-1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta = X) = \pi\phi(X; \mu_1, \sigma_1) + (1-\pi)\phi(X; \mu_2, \sigma_2)$$

where $\phi(X; \mu, \sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta = X) = \left\{\sum_{\lambda=0}^{h} m_\lambda X^\lambda\right\}^2 \phi(X; 0, 1)$$

where h corresponds to the argument h and determines the degree of the polynomial.

4) Kernel density estimation method

$$P(\theta = X) = \frac{1}{Nh}\sum_{j=1}^{N} K\left(\frac{X-\theta_j}{h}\right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument bw, and $K(\bullet)$ is a kernel function. The Gaussian kernel is used in this function.

Value

This function returns a list which contains several objects:

par_est	The item parameter estimates.
se	The standard errors for item parameter estimates.
fk	The estimated frequencies of examinees at each quadrature points.
iter	The number of EM-MML iterations required for the convergence.
prob	The estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
d	The estimated $\delta = \frac{\mu_2-\mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
sd_ratio	The estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
quad	The location of quadrature points.
diff	The final value of the monitored maximum item parameter change.
Ak	The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.
Pk	The posterior probabilities for each examinees at each quadrature points.
theta	The estimated ability parameter values. Expected *a posteriori* (EAP) is used for ability parameter estimation.
logL	The deviance (i.e., -2*log*L).
bw	The bandwidth used.
Options	A replication of input arguments.

Author(s)

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References

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Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
# A preparation of dichotomous item response data

Alldata <- DataGeneration(seed = 1,
                          model_P = "GPCM",
                          categ = rep(3:4, each = 4),
                          N=1000,
                          nitem_D = 0,
                          nitem_P = 8,
                          d = 1.414,
                          sd_ratio = 2,
                          prob = 0.5)

data <- Alldata$data_P
item <- Alldata$item_P
initialitem <- Alldata$initialitem_P
theta <- Alldata$theta


# Analysis

M1 <- IRTest_Poly(initialitem = initialitem,
                  data = data,
                  model = "GPCM",
                  latent_dist = "KDE",
              bandwidth = "SJ-ste", # an argument required only when "latent_dist = 'KDE'"
                  max_iter = 200,
                  threshold = .001,
                  h=4 # an argument required only when "latent_dist = 'DC'"
                  )
```

original_par_2GM *Recovering original parameters of two-component Gaussian mixture distribution from re-parameterized parameters*

Description

Recovering original parameters of two-component Gaussian mixture distribution from re-parameterized parameters

Usage

```
original_par_2GM(
  prob = 0.5,
  d = 0,
  sd_ratio = 1,
  overallmean = 0,
  overallsd = 1
)
```

Arguments

prob	A numeric value of $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
d	A numeric value of $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma}$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
sd_ratio	A numeric value of $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
overallmean	A numeric value of $\bar{\mu}$ that determines the overall mean of two-component Gaussian mixture distribution.
overallsd	A numeric value of $\bar{\sigma}$ that determines the overall standard deviation of two-component Gaussian mixture distribution.

Details

The original two-component Gaussian mixture distribution

$$f(x) = \pi \times \phi(x|\mu_1, \sigma_1) + (1 - \pi) \times \phi(x|\mu_2, \sigma_2)$$

, where ϕ is a Gaussian component.

The re-parameterized two-component Gaussian mixture distribution

$$f(x) = 2GM(x|\pi, \delta, \zeta, \bar{\mu}, \bar{\sigma})$$

, where $\bar{\mu}$ is overall mean and $\bar{\sigma}$ is overall standard deviation of the distribution.

The original parameters of two-component Gaussian mixture distribution can be retrieved as follows;

1) Mean of the first Gaussian component (m1).

$$\mu_1 = -(1-\pi)\delta\bar{\sigma} + \bar{\mu}$$

2) Mean of the second Gaussian component (m2).

$$\mu_2 = \pi\delta\bar{\sigma} + \bar{\mu}$$

3) Standard deviation of the first Gaussian component (s1).

$$\sigma_1^2 = \bar{\sigma}^2 \left(\frac{1-\pi(1-\pi)\delta^2}{\pi + (1-\pi)\zeta^2} \right)$$

4) Standard deviation of the second Gaussian component (s2).

$$\sigma_1^2 = \bar{\sigma}^2 \left(\frac{1-\pi(1-\pi)\delta^2}{\frac{1}{\zeta^2}\pi + (1-\pi)} \right) = \zeta^2\sigma_1^2$$

Value

This function returns a vector of length 4: `c(m1,m2,s1,s2)`.

m1	The location parameter (mean) of the first Gaussian component.
m2	The location parameter (mean) of the second Gaussian component.
s1	The scale parameter (standard deviation) of the first Gaussian component.
s2	The scale parameter (standard deviation) of the second Gaussian component.

Author(s)

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References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

plot_LD *Plotting the estimated latent distribution*

Description

This function draws a plot of the estimated latent distribution (the prior distribution of the latent variable).

Usage

```
plot_LD(model, xlim = c(-6, 6))
```

Arguments

model	An object obtained from either IRTest_Dich, IRTest_Poly, or IRTest_Mix.
xlim	A vector of length 2 which determines the range of the plot. The default is c(-6, 6)

Value

A plot of estimated latent distribution.

Author(s)

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Examples

```
# Data generation and model fitting

Alldata <- DataGeneration(seed = 1,
                          #model_D = rep(1, 10),
                          N=1000,
                          nitem_D = 0,
                          nitem_P = 8,
                          categ = rep(3:4,each = 4),
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_P
item <- Alldata$item_P
initialitem <- Alldata$initialitem_P
theta <- Alldata$theta

M1 <- IRTest_Poly(initialitem = initialitem,
                  data = data,
                  model = "GPCM",
                  latent_dist = "Mixture",
```

```
                  max_iter = 200,
                  threshold = .001,
                  )


# Plotting the latent distribution

plot_LD(model=M1, xlim = c(-6, 6))
```

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