

# Package ‘MultiStatM’

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**Type** Package

**Title** Multivariate Statistical Methods

**Version** 1.1.0

**Imports** arrangements, primes, Matrix, MASS, stats

**Description** Algorithms to build set partitions and commutator matrices and their use in the construction of multivariate d-Hermite polynomials; estimation and derivation of theoretical vector moments and vector cumulants of multivariate distributions; conversion formulae for multivariate moments and cumulants. Applications to estimation and derivation of multivariate measures of skewness and kurtosis; estimation and derivation of asymptotic covariances for d-variate Hermite polynomials, multivariate moments and cumulants and measures of skewness and kurtosis. The formulae implemented are discussed in Terdik (2021, ISBN:9783030813925), “Multivariate Statistical Methods”.

**License** GPL-3

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**Suggests** rmarkdown, knitr

**VignetteBuilder** knitr

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---

<code>conv_Cum2Mom</code>	<i>Convert cumulants to moments (univariate)</i>
---------------------------	--

---

### Description

Obtains a vector of univariate moments from a vector of univariate cumulants

### Usage

```
conv_Cum2Mom(cum_x)
```

### Arguments

<code>cum_x</code>	the r-vector of cumulants starting from the first - the mean - and arriving to the r-th order cumulant
--------------------	--

### Value

`mu_x` the vector of univariate moments

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 3.4 formula 3.23

### See Also

Other Moments and cumulants: [conv\\_Cum2MomMulti\(\)](#), [conv\\_Mom2CumMulti\(\)](#), [conv\\_Mom2Cum\(\)](#)

### Examples

```
cum_x<- c(1,2,3,4,5,6,7,8)
conv_Cum2Mom(cum_x)
```

---

conv\_Cum2MomMulti      *Convert T-cumulants to T-moments (multivariate)*

---

### Description

Obtains a vector of d-variate moments from a vector of d-variate cumulants

### Usage

```
conv_Cum2MomMulti(cum)
```

### Arguments

`cum`                    the list of r d-variate cumulants in vector form starting from the first cumulant - the vector of means - and arriving to the r-th order d-variate cumulant in vector form

### Value

Mom the list of r vectors of d-variate moments

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 3.4 formula 3.40

### See Also

Other Moments and cumulants: [conv\\_Cum2Mom\(\)](#), [conv\\_Mom2CumMulti\(\)](#), [conv\\_Mom2Cum\(\)](#)

### Examples

```
#cum contains the T-vector cumulants up to the fifth order of the bivariate
#standard normal distribution
cum<-list(c(0,0),c(1,0,0,1),c(0,0,0,0,0,0,0,0),c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),
c(rep(0,32)))
conv_Cum2MomMulti(cum)
```

---

conv_Mom2Cum	<i>Convert moments to cumulants (univariate)</i>
--------------	--

---

**Description**

Obtains a vector of univariate cumulants from a vector of univariate moments

**Usage**

```
conv_Mom2Cum(mu_x)
```

**Arguments**

mu\_x            the r-vector of moments starting from the first moment - the mean - and arriving to the r-th order moment

**Value**

cum\_x the vector of univariate cumulants

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 3.4 formula 3.20

**See Also**

Other Moments and cumulants: [conv\\_Cum2MomMulti\(\)](#), [conv\\_Cum2Mom\(\)](#), [conv\\_Mom2CumMulti\(\)](#)

**Examples**

```
mu_x<- c(1,2,3,4,5,6,7,8)
conv_Mom2Cum(mu_x)
```

---

conv_Mom2CumMulti	<i>Convert T-moments to T-cumulants (multivariate)</i>
-------------------	--

---

**Description**

Obtains a vector of d-variate cumulants from a vector of d-variate moments

**Usage**

```
conv_Mom2CumMulti(mu)
```

**Arguments**

mu                    the list of r d-variate moments in vector form starting from the first moment - the vector of means - and arriving to the r-th order d-variate moment in vector form

**Value**

Cum the list of n vectors of d-variate cumulants

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 3.4 formula 3.29

**See Also**

Other Moments and cumulants: [conv\\_Cum2MomMulti\(\)](#), [conv\\_Cum2Mom\(\)](#), [conv\\_Mom2Cum\(\)](#)

**Examples**

```
#Mu contains the T-vector moments up to the fifth order of the bivariate
#standard normal distribution
mu<-list(c(0,0),c(1,0,0,1),c(0,0,0,0,0,0,0,0),c(3,0,0,1,0,1,1,0,0,1,1,0,1,0,0,3),
c(rep(0,32)))
conv_Mom2CumMulti(mu)
```

---

conv\_Stand\_Multi            *Standardize multivariate data*

---

**Description**

For data formed by d-variate vectors x with sample covariance S and sample mean M, it computes the values  $z = S^{-1/2}(x - M)$

**Usage**

```
conv_Stand_Multi(x)
```

**Arguments**

x                    a multivariate data matrix, sample size is the number of rows

**Value**

a matrix of multivariate data with null mean vector and identity sample covariance matrix

**Examples**

```
x<-MASS::mvrnorm(1000,c(0,0,1,3),diag(4))
z<-conv_Stand_Multi(x)
mu_z<- apply(z,2,mean)
cov_z<- cov(z)
```

---

distr\_CFUSN\_MomCum\_Th *Moments and cumulants CFUSN*

---

**Description**

Provides the theoretical cumulants of the multivariate Canonical Fundamental Skew Normal distribution

**Usage**

```
distr_CFUSN_MomCum_Th(r, d, p, Delta, nMu = FALSE)
```

**Arguments**

r	The highest cumulant order
d	The multivariate dimension and number of rows of the skewness matrix Delta
p	The number of cols of the skewness matrix Delta
Delta	The skewness matrix
nMu	If set to TRUE, the list of the first r d-variate moments is provided

**Value**

The list of theoretical cumulants in vector form

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, Lemma 5.3 p.251

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```

r <- 4; d <- 2; p <- 3
Lamd <- matrix(sample(1:50-25, d*p), nrow=d)
ieg<- eigen(diag(p)+t(Lamd)%*%Lamd)
V <- ieg$vector
Delta <-Lamd %*% V %*% diag(1/sqrt(ieg$values)) %*% t(V)
MomCum_CFUSN <- distr_CFUSN_MomCum_Th (r,d,p,Delta)

```

---

distr_CFUSN_Rand	<i>Random multivariate CFUSN</i>
------------------	----------------------------------

---

**Description**

Generate random  $d$ -vectors from the multivariate Canonical Fundamental Skew-Normal (CFUSN) distribution

**Usage**

```
distr_CFUSN_Rand(n, Delta)
```

**Arguments**

n	The number of variates to be generated
Delta	Correlation matrix, the skewness matrix Delta

**Value**

A random matrix  $n \times d$

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 (5.5) p.247  
 S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644.

**See Also**

Other Random generation: [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_Uni\\_Rand\(\)](#)  
 Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)



**Examples**

```
d <- 2; p <- 3
Lamd <- matrix(sample(1:50-25, d*p), nrow=d)
ieg<- eigen(diag(p)+t(Lamd)%*%Lamd)
V <- ieg$vector
Delta <-Lamd %*% V %*% diag(1/sqrt(ieg$values)) %*% t(V)
x<-distr_CFUSN_Rand(20,Delta)
```

---

distr\_CFUSSD\_Rand      *Random multivariate CFUSSD*

---

**Description**

Generate random  $d$ -vectors from the multivariate Canonical Fundamental Skew-Spherical distribution (CFUSSD) with Gamma generator

**Usage**

```
distr_CFUSSD_Rand(n, d, p, a, b, Delta)
```

**Arguments**

<code>n</code>	sample size
<code>d</code>	dimension
<code>p</code>	dimension of the first term of (5.5)
<code>a</code>	shape parameter of the Gamma generator
<code>b</code>	scale parameter of the Gamma generator
<code>Delta</code>	skewness matrix

**Value**

A matrix of  $n \times d$  random numbers

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, (5.36) p. 266, (see p.247 for Delta)

**See Also**

Other Random generation: [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_Uni\\_Rand\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```
n <- 10^3; d <- 2; p <- 3 ; a <- 1; b <- 1
Lamd <- matrix(sample(1:50-25, d*p), nrow=d)
ieg<- eigen(diag(p)+t(Lamd)%*%Lamd)
V <- ieg$eigenvectors
Delta <-Lamd %*% V %*% diag(1/sqrt(ieg$values)) %*% t(V)
distr_CFUSSD_Rand(20,d,p,1,1,Delta)
```

---

distr\_SkewNorm\_EVSK\_Th

*EVSK multivariate Skew Normal*


---

**Description**

Computes the theoretical values of the mean vector, covariance, skewness vector, total skewness, kurtosis vector and total kurtosis for the multivariate Skew Normal distribution

**Usage**

```
distr_SkewNorm_EVSK_Th(omega, alpha)
```

**Arguments**

omega	A $d \times d$ correlation matrix
alpha	shape parameter d-vector

**Value**

A list of theoretical values for the mean vector, covariance, skewness vector, total skewness, kurtosis vector and total kurtosis

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 (5.5) p.247  
S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644.

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```
alpha<-c(10,5,0)
omega<-diag(3)
distr_SkewNorm_EVSK_Th(omega,alpha)
```

---

distr\_SkewNorm\_MomCum\_Th

*Moments and cumulants d-variate Skew Normal*

---

**Description**

Computes the theoretical values of moments and cumulants up to the  $r$ -th order. Warning: if `nMu = TRUE` it can be very slow

**Usage**

```
distr_SkewNorm_MomCum_Th(r = 4, omega, alpha, nMu = FALSE)
```

**Arguments**

<code>r</code>	the highest moment and cumulant order
<code>omega</code>	A $d \times d$ correlation matrix
<code>alpha</code>	shape parameter d-vector
<code>nMu</code>	if it is TRUE then moments are calculated as well

**Value**

A list of theoretical moments and cumulants

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 (5.5) p.247, Lemma 5.1 p. 246

S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644.

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```
alpha<-c(10,5,0)
omega<-diag(3)
distr_SkewNorm_MomCum_Th(r=4,omega,alpha)
```

---

distr\_SkewNorm\_Rand     *Random Multivariate Skew Normal*

---

**Description**

Generate random  $d$ -vectors from the multivariate Skew Normal distribution

**Usage**

```
distr_SkewNorm_Rand(n, omega, alpha)
```

**Arguments**

n	sample size
omega	correlation matrix with $d$ dimension
alpha	shape parameter vector of dimension $d$

**Value**

A random matrix  $n \times d$

**References**

Azzalini, A. with the collaboration of Capitanio, A. (2014). The Skew-Normal and Related Families. Cambridge University Press, IMS Monographs series.

Gy.H.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, Section 5.1.2

**See Also**

Other Random generation: [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_Uni\\_Rand\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```
alpha<-c(10,5,0)
omega<-diag(3)
x<-distr_SkewNorm_Rand(20,omega,alpha)
```

---

distr\_UniAbs\_EVSK\_Th    *Moments of the modulus of the Uniform distribution on the sphere*

---

### Description

Moments (up to the 4th order) of the modulus of the d-variate Uniform distribution on the sphere on (d-1)

### Usage

```
distr_UniAbs_EVSK_Th(d, nCum = FALSE)
```

### Arguments

d	vector-dimension
nCum	if it is TRUE then cumulants, skewness and kurtosis are calculated

### Value

The list of the first four moments in vector form

### References

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, Lemma 5.12 p.298

### See Also

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

---

distr\_Uni\_EVSK\_Th    *EVSK Uniform on the sphere*

---

### Description

Computes the theoretical values of the mean vector, covariance, skewness vector, total skewness, kurtosis vector and total kurtosis for the Uniform distribution on the sphere. Note that Skewness is ZERO

**Usage**

```
distr_Uni_EVSK_Th(d, nCum = TRUE)
```

**Arguments**

d	dimensions
nCum	if it is TRUE then cumulants, skewness and kurtosis are calculated

**Value**

The list with mean vector, covariance, skewness vector, total skewness, kurtosis vector and total kurtosis

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 Proposition 5.3 p.297

S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644.

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

---

distr\_Uni\_MomCum\_Th    *Moments and cumulants Uniform Distribution on the Sphere*

---

**Description**

By default, only moments are provided

**Usage**

```
distr_Uni_MomCum_Th(r, d, nCum = FALSE)
```

**Arguments**

r	highest order of moments and cumulants
d	dimension
nCum	if it is TRUE then cumulants are calculated

**Value**

The list of moments and cumulants in vector form

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 Proposition 5.3 p.297

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

**Examples**

```
# The first four moments for d=3
distr_Uni_MomCum_Th(4,3,nCum=0)
# The first four moments and cumulants for d=3
distr_Uni_MomCum_Th(4,3,nCum=4)
```

---

distr_Uni_Rand	<i>Random Uniform on the sphere</i>
----------------	-------------------------------------

---

**Description**

Generate random d-vectors from the Uniform distribution on the sphere

**Usage**

```
distr_Uni_Rand(n, d)
```

**Arguments**

n	sample size
d	dimension

**Value**

A random matrix  $n \times d$

**References**

- Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021
- S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644.

**See Also**

Other Random generation: [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

---

`distr_ZabsM_MomCum_Th` *Moments and cumulants multivariate central folded Normal distribution*

---

**Description**

Provides the theoretical moments and cumulants of the multivariate central Folded Normal distribution. By default only cumulants are provided.

**Usage**

```
distr_ZabsM_MomCum_Th(r, d, nMu = FALSE)
```

**Arguments**

<code>r</code>	The highest cumulant (moment) order
<code>d</code>	dimension
<code>nMu</code>	if True then moments are calculated as well.

**Value**

The list of cumulants (and moments) in vector form.

**References**

- Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, Lemma 5.2 p. 249

**See Also**

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_Zabs\\_MomCum\\_Th\(\)](#)



---

distr\_Zabs\_MomCum\_Th *Moments and cumulants Central folded Normal distribution*

---

### Description

Provides the theoretical moments and cumulants of the univariate Central Folded Normal distribution. By default only moments are provided.

### Usage

```
distr_Zabs_MomCum_Th(r, nCum = FALSE)
```

### Arguments

r	The highest moment (cumulant) order
nCum	if it is TRUE then cumulants are calculated

### Value

The list of moments and cumulants

### References

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021, Proposition 5.1 p.242 and formula: p. 301

### See Also

Other Multivariate distributions: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_CFUSN\\_Rand\(\)](#), [distr\\_CFUSSD\\_Rand\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_Rand\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_Uni\\_Rand\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#)

Other Theoretical Moments and Cumulants: [distr\\_CFUSN\\_MomCum\\_Th\(\)](#), [distr\\_SkewNorm\\_EVSK\\_Th\(\)](#), [distr\\_SkewNorm\\_MomCum\\_Th\(\)](#), [distr\\_UniAbs\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_EVSK\\_Th\(\)](#), [distr\\_Uni\\_MomCum\\_Th\(\)](#), [distr\\_ZabsM\\_MomCum\\_Th\(\)](#)

### Examples

```
# The first three moments
distr_Zabs_MomCum_Th(3, nCum = FALSE)
# The first three moments and cumulants
distr_Zabs_MomCum_Th(3, nCum = TRUE)
```

---

Esti_EVSK	<i>Estimation of multivariate Mean, Variance, T-Skewness and T-Kurtosis vectors</i>
-----------	---

---

**Description**

Provides estimates of mean, variance, skewness and kurtosis vectors for d-variate data

**Usage**

```
Esti_EVSK(X)
```

**Arguments**

X                    d-variate data vector

**Value**

The list of the estimated mean, variance, skewness and kurtosis vectors

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, Sections 6.4.1 and 6.5.1

**See Also**

Other Estimation: [Esti\\_Kurt\\_Variance\\_Th\(\)](#), [Esti\\_MMom\\_MCum\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#), [Esti\\_Skew\\_Variance\\_Th\(\)](#)

**Examples**

```
x<- MASS::mvrnorm(100,rep(0,3), 3*diag(rep(1,3)))
EVSK<-Esti_EVSK(x)
names(EVSK)
EVSK$estSkew
```

---

Esti_Hermite_Poly_HN_Multi	<i>Estimate the N-th d-variate Hermite polynomial</i>
----------------------------	---

---

**Description**

The vector x is standardized and the N-th d-variate polynomial is computed

**Usage**

```
Esti_Hermite_Poly_HN_Multi(x, N)
```

**Arguments**

x                    a d-variate data vector  
 N                    the order of the d-variate Hermite polynomial

**Value**

The vector of the N-th d-variate polynomial

**Examples**

```
x<-MASS::mvrnorm(100,rep(0,3),diag(3))
H3<-Esti_Hermite_Poly_HN_Multi(x,3)
```

---

Esti_Kurt_CMRSz	<i>Estimation of Cardoso, Mori, Rohatgi, Szekely (CMRSz's) kurtosis matrix</i>
-----------------	--

---

**Description**

Estimation of Cardoso, Mori, Rohatgi, Szekely (CMRSz's) kurtosis matrix

**Usage**

```
Esti_Kurt_CMRSz(x)
```

**Arguments**

x                    A matrix of multivariate data

**Value**

CMRSz.Kurtosis The kurtosis matrix

p.value The p-value for the hypothesis of symmetry under the Gaussian assumption

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Example 6.9

**See Also**

Other Indexes: [Esti\\_Kurt\\_Mardia\(\)](#), [Esti\\_Kurt\\_Total\(\)](#), [Esti\\_Skew\\_MRSz\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#)

Esti\_Kurt\_Mardia      *Estimation of Mardia's Kurtosis Index*

---

**Description**

Estimation of Mardia's Kurtosis Index

**Usage**

```
Esti_Kurt_Mardia(x)
```

**Arguments**

x                      A matrix of multivariate data

**Value**

Mardia.Kurtosis The kurtosis index  
p.value The p-value under the Gaussian hypothesis

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Example 6.1

**See Also**

Other Indexes: [Esti\\_Kurt\\_CMRSz\(\)](#), [Esti\\_Kurt\\_Total\(\)](#), [Esti\\_Skew\\_MRSz\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#)

---

Esti\_Kurt\_Total      *Estimation of the Total Kurtosis Index*

---

**Description**

Estimation of the Total Kurtosis Index

**Usage**

```
Esti_Kurt_Total(x)
```

**Arguments**

x                      A matrix of multivariate data

**Value**

Total.Kurtosis The total kurtosis index  
p.value The p-value under the Gaussian hypothesis

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Example 6.1

**See Also**

Other Indexes: [Esti\\_Kurt\\_CMRSz\(\)](#), [Esti\\_Kurt\\_Mardia\(\)](#), [Esti\\_Skew\\_MRSz\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#)

---

Esti\_Kurt\_Variance\_Th *Asymptotic Variance of the estimated kurtosis vector*

---

**Description**

Asymptotic Variance of the estimated kurtosis vector

**Usage**

Esti\_Kurt\_Variance\_Th(cum)

**Arguments**

cum                    The theoretical/estimated cumulants up to the 8th order in vector form

**Value**

The matrix of theoretical/estimated variance

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Ch. 6, formula (6.26)

**See Also**

Other Estimation: [Esti\\_EVSK\(\)](#), [Esti\\_MMom\\_MCum\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#), [Esti\\_Skew\\_Variance\\_Th\(\)](#)

---

 Esti\_MMom\_MCum

*Estimation of multivariate T-Moments and T-Cumulants*


---

### Description

Provides estimates of univariate and multivariate moments and cumulants up to order  $r$ . By default data are standardized; using only demeaned or raw data is also possible.

### Usage

```
Esti_MMom_MCum(X, r, centering = FALSE, scaling = TRUE)
```

### Arguments

<code>X</code>	d-vector data
<code>r</code>	The highest moment order ( $r > 2$ )
<code>centering</code>	set to T (and scaling = F) if only centering is needed
<code>scaling</code>	set to T (and centering=F) if standardization of multivariate data is needed

### Value

`estMu.r`: the list of the multivariate moments up to order  $r$   
`estCum.r`: the list of the multivariate cumulants up to order  $r$

### References

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021.

### See Also

Other Estimation: [Esti\\_EVSK\(\)](#), [Esti\\_Kurt\\_Variance\\_Th\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#), [Esti\\_Skew\\_Variance\\_Th\(\)](#)

### Examples

```
## generate random data from a 3-variate skew normal distribution
alpha<-c(10,5,0)
omega<-diag(3)
x<-distr_SkewNorm_Rand(50,omega,alpha)
## estimate the first three moments and cumulants from raw (uncentered and unstandardized) data
Esti_MMom_MCum(x,3,centering=FALSE,scaling=FALSE)
## estimate the first three moments and cumulants from standardized data
Esti_MMom_MCum(x,3,centering=FALSE,scaling=TRUE)
```

---

Esti_Skew_Mardia	<i>Estimation of Mardia's Skewness index</i>
------------------	--

---

**Description**

Compute the multivariate Mardia's skewness index and provides the p-value for the hypothesis of zero symmetry under the Gaussian assumption

**Usage**

```
Esti_Skew_Mardia(x)
```

**Arguments**

x                    A matrix of multivariate data

**Value**

Mardia.Skewness The skewness index  
p.value The p-value under the Gaussian hypothesis

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Example 6.1

**See Also**

Other Indexes: [Esti\\_Kurt\\_CMRSz\(\)](#), [Esti\\_Kurt\\_Mardia\(\)](#), [Esti\\_Kurt\\_Total\(\)](#), [Esti\\_Skew\\_MRSz\(\)](#)  
Other Estimation: [Esti\\_EVSK\(\)](#), [Esti\\_Kurt\\_Variance\\_Th\(\)](#), [Esti\\_MMom\\_MCum\(\)](#), [Esti\\_Skew\\_Variance\\_Th\(\)](#)

---

Esti_Skew_MRSz	<i>Estimation of Mori, Rohatgi, Szekely (MRSz's) skewness vector</i>
----------------	--

---

**Description**

Estimation of Mori, Rohatgi, Szekely (MRSz's) skewness vector

**Usage**

```
Esti_Skew_MRSz(x)
```

**Arguments**

x                    A matrix of multivariate data

**Value**

MRSz.Skewness.Vector The skewness vector

MRSz.Skewness.Index The skewness index

p.value The p-value for the hypothesis of symmetry under the Gaussian assumption

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Example 6.2

S. R. Jammalamadaka, E. Taufer, Gy. Terdik. On multivariate skewness and kurtosis. Sankhya A, 83(2), 607-644

**See Also**

Other Indexes: [Esti\\_Kurt\\_CMRSz\(\)](#), [Esti\\_Kurt\\_Mardia\(\)](#), [Esti\\_Kurt\\_Total\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#)

---

Esti\_Skew\_Variance\_Th *Asymptotic Variance of the estimated skewness vector*

---

**Description**

Asymptotic Variance of the estimated skewness vector

**Usage**

Esti\_Skew\_Variance\_Th(cum)

**Arguments**

cum                    The theoretical/estimated cumulants up to order 6 in vector form

**Value**

The matrix of theoretical/estimated variance

**References**

Gy.Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021. Ch.6, formula (6.13)

**See Also**

Other Estimation: [Esti\\_EVSK\(\)](#), [Esti\\_Kurt\\_Variance\\_Th\(\)](#), [Esti\\_MMom\\_MCum\(\)](#), [Esti\\_Skew\\_Mardia\(\)](#)



**Examples**

```
alpha<-c(10,5)
omega<-diag(rep(1,2))
MC <- distr_SkewNorm_MomCum_Th(r = 6,omega,alpha)
cum <- MC$CumX
VS <- Esti_Skew_Variance_Th(cum)
```

---

Esti\_Variance\_Skew\_Kurt

*Estimated Variance of skewness and kurtosis vectors*

---

**Description**

Provides the estimated covariance matrices of the data-estimated skewness and kurtosis vectors.

**Usage**

```
Esti_Variance_Skew_Kurt(X)
```

**Arguments**

X                      A matrix of d-variate data

**Value**

The list of covariance matrices of the skewness and kurtosis vectors

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021.

---

Hermite\_Coeff

*Coefficients of univariate Hermite polynomials*

---

**Description**

Provides the vector of coefficients of the univariate Hermite polynomial  $H_N(x)$  with variance 1 and order N.

**Usage**

```
Hermite_Coeff(N)
```

**Arguments**

N                      The order of polynomial

**Value**

The vector of coefficients of  $x^N, x^{N-2}...$

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.4 (4.24)

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

---

Hermite_CoeffMulti	<i>Coefficients of multivariate T-Hermite polynomials for standardized variate</i>
--------------------	--

---

**Description**

Provides the matrix of coefficients of  $x^{\otimes N}, \kappa_2^{\otimes} x^{\otimes(N-2)}...$  for the d-variate T-Hermite polynomials up to order N.

**Usage**

```
Hermite_CoeffMulti(N, d)
```

**Arguments**

N	the maximum order of polynomials
d	the dimension of d-variate X

**Value**

The list of matrices of coefficients for the d-variate polynomials from 1 to N

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.6.2, p. 223, Remark 4.8,

**See Also**

Other Hermite: [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

**Examples**

```

N <- 5; d <- 3
H_N_Xc <- Hermite_CoeffMulti(N,d) # coefficients
X <- c(1:3);
X3 <- kronecker(X,kronecker(X,X));
X5 <- kronecker(X3,kronecker(X,X))
Idv <- as.vector(diag(d)) # vector of variance matrix
# value of H5 at X is
vH5<-H_N_Xc[[1]] %**% X5 + H_N_Xc[[2]] %**%kronecker(Idv,X3) +
  H_N_Xc[[3]] %**%kronecker(kronecker(Idv,Idv),X)

```

---

Hermite\_Nth

*T-Hermite polynomial with order N at standardized vector x*


---

**Description**

Computes the N-th d-variate T-Hermite polynomial at standardized vector x

**Usage**

```
Hermite_Nth(x, N)
```

**Arguments**

x	multivariate data of size d
N	degree of T-Hermite polynomial

**Value**

d-variate T-Hermite polynomial of order N evaluated at vector x

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.6.2, (4.73), p.223

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

---

Hermite\_N\_Cov\_X1\_X2     *Covariance matrix for multivariate T-Hermite polynomials*

---

### Description

Computation of the covariance matrix between d-variate T-Hermite polynomials  $H_N(X_1)$  and  $H_N(X_2)$ .

### Usage

```
Hermite_N_Cov_X1_X2(SigX12, N)
```

### Arguments

SigX12	Covariance matrix of the Gaussian vectors X1 and X2 respectively of dimensions d1 and d2
N	Common degree of the multivariate Hermite polynomials

### Value

Covariance matrix of  $H_N(X_1)$  and  $H_N(X_2)$

### References

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. (4.59), (4.66),

### See Also

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

### Examples

```
Covmat<-matrix(c(1,0.8,0.8,1),2,2)
Cov_X1_X2 <- Hermite_N_Cov_X1_X2(Covmat,3)
```

---

Hermite\_Poly\_HN      *Univariate Hermite polynomials*

---

**Description**

Provides the vector of univariate Hermite polynomials up to order N evaluated at x

**Usage**

```
Hermite_Poly_HN(x, N, sigma2 = 1)
```

**Arguments**

x	A scalar at which to evaluate the Hermite polynomials
N	The maximum order of the polynomials
sigma2	The variance, by default is set to 1

**Value**

H\_N\_x The vector of Hermite polynomials with degrees from 1 to N evaluated at x

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.1

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

---

Hermite\_Poly\_HN\_Multi      *Multivariate T-Hermite polynomials*

---

**Description**

Computes the multivariate T-Hermite polynomials up to order N at vector variate x with covariance matrix Sig2

**Usage**

```
Hermite_Poly_HN_Multi(x, N, Sig2 = diag(length(x)))
```

**Arguments**

x	the d-vector of values at which to evaluate the polynomials
N	the maximum order of polynomials
Sig2	the covariance matrix; default value is the unit matrix <code>diag(length(x))</code>

**Value**

The list of d-variate polynomials of order from 1 to N evaluated at vector x

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.6.2, (4.73), p.223

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

**Examples**

```
x<-c(1,3)
N<-3
Sig2<- diag(length(x)) # matrix(c(1,0,0,1),2,2,byrow = T)
Hermite_Poly_HN_Multi(x,N)
```

---

Hermite\_Poly\_NH\_Inv     *Inverse univariate Hermite polynomial*

---

**Description**

Inverse univariate Hermite polynomial

**Usage**

```
Hermite_Poly_NH_Inv(H_N_x, sigma2 = 1)
```

**Arguments**

H_N_x	The vector of Hermite Polynomials from 1 to N evaluated at x
sigma2	The variance, by default is set to 1

**Value**

The vector of x powers:  $x^n, n = 1 : N$

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.4, (4.23), p.198

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Multi\\_Inv\(\)](#)

---

Hermite\_Poly\_NH\_Multi\_Inv

*Inverse of d-variate T-Hermite Polynomial*

---

**Description**

Compute the powers of vector variate x when Hermite polynomials are given

**Usage**

```
Hermite_Poly_NH_Multi_Inv(H_N_X, N, Sig2 = diag(length(H_N_X[[1]])))
```

**Arguments**

H_N_X	The list of d-variate T-Hermite Polynomials of order from 1 to N evaluated at X
N	the highest polynomial order
Sig2	The variance matrix of x, the default is set to unit matrix

**Value**

The list of  $x$ ,  $x^{\otimes 2}$ , ...  $x^{\otimes N}$

**References**

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 4.6.2, (4.72), p.223

**See Also**

Other Hermite: [Hermite\\_CoeffMulti\(\)](#), [Hermite\\_Coeff\(\)](#), [Hermite\\_N\\_Cov\\_X1\\_X2\(\)](#), [Hermite\\_Nth\(\)](#), [Hermite\\_Poly\\_HN\\_Multi\(\)](#), [Hermite\\_Poly\\_HN\(\)](#), [Hermite\\_Poly\\_NH\\_Inv\(\)](#)

**Examples**

```
x<-c(1,3)
Sig2 <- diag(length(x)) # matrix(c(1,0,0,1),2,2,byrow=T)
N<-4
H_N_X<-Hermite_Poly_HN_Multi(x,N,Sig2)
x_ad_n <- Hermite_Poly_NH_Multi_Inv(H_N_X,N,Sig2)
```

---

indx\_Commutator\_Kmn     *Index vector for commutation of T-products of two vectors*

---

### Description

Transforms vec A to vec of the transposed A. Same results as matr\_Commutator\_Kmn.

### Usage

```
indx_Commutator_Kmn(m, n)
```

### Arguments

m	Row-dimension
n	Col-dimension

### Value

A vector of indexes to provide the commutation

### References

Gy. Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 (p.8, (1.12)).

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
A<-1:6
A[indx_Commutator_Kmn(3,2)]
## Same as
as.vector(matr_Commutator_Kmn(3,2)**%A)
```



indx\_Commutator\_Kperm *Index vector for commutation of T-products of any number of vectors*

### Description

Produces any permutation of kronecker products of vectors of any length. Same results as matr\_Commutator\_Kperm.

### Usage

```
indx_Commutator_Kperm(perm, dims)
```

### Arguments

perm	vector indicating the permutation of the order in the Kronecker product,
dims	vector indicating the dimensions of the vectors, use dims <- d if all dimensions are equal

### Value

An index vector to produce the permutation

### References

Holmquist B (1996) The d-variate vector Hermite polynomial of order. Linear Algebra and its Applications 237/238, 155-190.

Gy., Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, 1.2.4 Computing T-Products of Vectors.

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
a1<-c(1,2)
a2<-c(2,3,4)
a3<-c(1,3)
p1<-a1%x%a2%x%a3
p1[indx_Commutator_Kperm(c(3,1,2),c(2,3,2))]
## Same as
a3%x%a1%x%a2
## Same as
as.vector(matr_Commutator_Kperm(c(3,1,2),c(2,3,2))%*%p1)
```

---

indx_Elimination	<i>Distinct values selection vector</i>
------------------	---

---

### Description

Eliminates the duplicated/q-plicated elements in a T-vector of multivariate moments and cumulants. Produces the same results as `matr_Elimination`. Note `indx_Elimination` does not provide the same results as `unique()`

### Usage

```
indx_Elimination(d, q)
```

### Arguments

d	dimension of a vector x
q	power of the Kronecker product

### Value

A vector of indexes of the distinct elements in the T-vector

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.3.2 Multi-Indexing, Elimination, and Duplication, p.21,(1.32)

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
x<-c(1,0,3)
y<-kronecker(x,kronecker(x,x))
y[indx_Elimination(3,3)]
## Not the same results as
unique(y)
```

---

indx_Qplication	<i>Qplication vector</i>
-----------------	--------------------------

---

### Description

Restores the duplicated/q-plicated elements which are eliminated by `matr_Elimination` or `indx_Elimination` in a T-product of vectors of dimension `d`. It produces the same results as `matr_Qplication`.

### Usage

```
indx_Qplication(xelim, d, q)
```

### Arguments

<code>xelim</code>	the T-product vector with distinct elements
<code>d</code>	dimension of the vectors in the T-product
<code>q</code>	power of the Kronecker product

### Value

A vector (T-vector) with all elements previously eliminated by `indx_Elimination`

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, p.21, (1.31)

### See Also

Other Matrices and commutators: `indx_Commutator_Kmn()`, `indx_Commutator_Kperm()`, `indx_Elimination()`, `indx_Symmetry()`, `indx_UnivMomCum()`, `matr_Commutator_Kmn()`, `matr_Commutator_Kperm()`, `matr_Commutator_Mixing()`, `matr_Commutator_Moment()`, `matr_Elimination()`, `matr_Qplication()`, `matr_Symmetry()`

### Examples

```
x<-c(1,2,3)
y<-kronecker(kronecker(x,x),x)
## Distinct elements of y
z<-y[indx_Elimination(3,3)]
## Restore eliminated elements in z
indx_Qplication(z,3,3)
```

---

`indx_Symmetry`*Symmetrizing vector*

---

**Description**

Vector symmetrizing a T-product of vectors of the same dimension  $d$ . Produces the same results as `matr_Symmetry`

**Usage**

```
indx_Symmetry(x, d, n)
```

**Arguments**

<code>x</code>	the vector to be symmetrized of dimension $d^n$
<code>d</code>	size of the single vectors in the product
<code>n</code>	power of the T-product

**Value**

A vector with the symmetrized version of  $x$  of dimension  $d^n$

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.3.1 Symmetrization, p.14. (1.29)

**See Also**

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

**Examples**

```
a<-c(1,2)
b<-c(2,3)
c<-kronecker(kronecker(a,a),b)
## The symmetrized version of c is
indx_Symmetry(c,2,3)
```

---

indx_UnivMomCum	<i>Univariate moments and cumulants from T-vectors</i>
-----------------	--

---

### Description

A vector of indexes to select the moments and cumulants of the single components of the random vector X for which a T-vector of moments and cumulants is available

### Usage

```
indx_UnivMomCum(d, q)
```

### Arguments

d	dimension of a vector X
q	power of the Kronecker product

### Value

A vector of indexes

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
## For a 3-variate skewness and kurtosis vectors estimated from data, extract
## the skewness and kurtosis estimates for each of the single components of the vector
alpha<-c(10,5,0)
omega<-diag(rep(1,3))
X<-distr_SkewNorm_Rand(200, omega, alpha)
EVSK<-Esti_EVSK(X)
## Get the univariate skewness and kurtosis for X1,X2,X3
EVSK$estSkew[indx_UnivMomCum(3,3)]
EVSK$estKurt[indx_UnivMomCum(3,4)]
```

---

matr\_Commutator\_Kmn     *Commutation matrix*

---

### Description

Transforms vec A to vec of the transposed A. An option for sparse matrix is provided, by default a non-sparse matrix is produced. Using sparse matrices increases computation times, but far less memory is required

### Usage

```
matr_Commutator_Kmn(m, n, useSparse = FALSE)
```

### Arguments

m	Row-dimension
n	Col-dimension
useSparse	T or F.

### Value

A commutation matrix matrix of dimension  $mn \times mn$ . If useSparse=TRUE an object of the class "dgCMatrix" is produced.

### References

Gy. Terdik, Multivariate statistical methods - Going beyond the linear, Springer 2021 (p.8, (1.12)).

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
A<-matrix(1:6,3,2)
as.vector(matr_Commutator_Kmn(3,2)%*%c(A))
```

---

matr\_Commutator\_Kperm *Commutator for T-products of vectors*

---

### Description

Produces any permutation of kronecker products of vectors of any length. An option for sparse matrix is provided, by default a non-sparse matrix is produced. Using sparse matrices increases computation times, but far less memory is required.

### Usage

```
matr_Commutator_Kperm(perm, dims, useSparse = FALSE)
```

### Arguments

perm	vector indicating the permutation of the order in the Kronecker product,
dims	vector indicating the dimensions of the vectors, use <code>dims &lt;- d</code> if all dimensions are equal
useSparse	T or F.

### Value

A square permutation matrix of size `prod(dims)`. If `useSparse=TRUE` an object of the class "dgC-Matrix" is produced.

### References

Holmquist B (1996) The d-variate vector Hermite polynomial of order. *Linear Algebra and its Applications* 237/238, 155-190.

Gy., Terdik, *Multivariate statistical methods - going beyond the linear*, Springer 2021, 1.2.4 Computing T-Products of Vectors.

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
dims <- c(2,3,2)
perm <- c(1,3,2)
matr_Commutator_Kperm(perm,dims)
perm <- c(3,1,4,2)
dims <- 4 # All vectors with dimension 4
# If all dimensions are equal, using dims <- d instead of
# dims <- c(d,d,d,d,d,d,d,d) will be much faster.
```

```
# For example, for perm <- c(2,4,6,1,3,8,5,7) and d <- 3
# matr_Commutator_Kperm(c(2,4,6,1,3,8,5,7),3) ## requires 2.11 secs
# matr_Commutator_Kperm(c(2,4,6,1,3,8,5,7),c(3,3,3,3,3,3,3,3)) ## requires 1326.47 secs
```

---

matr\_Commutator\_Mixing

*Mixing commutator*

---

### Description

Used for the expected value of the T-product of two Hermite polynomials with dimensions d1 and d2 respectively. With an option for sparse matrices.

### Usage

```
matr_Commutator_Mixing(d1, d2, useSparse = FALSE)
```

### Arguments

d1	dimension of the first group of vectors
d2	dimension of the second group of vectors
useSparse	T or F.

### Value

A square matrix of dimension  $\text{prod}(d1) \times \text{prod}(d2)$ . If useSparse=TRUE an object of the class "dgCMatrix" is produced.

### References

Gy.Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Formula (4.58) p. 218.

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
d1 <- c(2, 3, 2)
d2 <- c(3, 2, 2)
MCM <- matr_Commutator_Mixing(d1, d2)
```



---

`matr_Commutator_Moment`*Commutator matrix for moment formulae*

---

**Description**

Commutator matrix for moment formulae

**Usage**

```
matr_Commutator_Moment(e1_rm, d, useSparse = FALSE)
```

**Arguments**

<code>e1_rm</code>	type of a partition
<code>d</code>	dimensions of the vector
<code>useSparse</code>	TRUE or FALSE

**Value**

A commutator matrix

**References**

Gy., Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, Section 2.4.3, p.100, Sect. A.2.1, p. 353., Corollary 2.6., p.95

**See Also**

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

**Examples**

```
n=4; r=2 ; m=1 ; d=2;
PTA<-Partition_Type_All(n)
e1_r<-PTA$eL_r[[r]][m,]
## e1_r is a given type (always a vector)
MC<- matr_Commutator_Moment(e1_r,d)
MC
```

---

matr\_Elimination      *Elimination Matrix*

---

### Description

Eliminates the duplicated/q-plicated elements in a T-vector of multivariate moments and cumulants.

### Usage

```
matr_Elimination(d, q, useSparse = FALSE)
```

### Arguments

d	dimension of a vector x
q	power of the Kronecker product
useSparse	TRUE or FALSE.

### Value

Elimination matrix of order  $\eta_{d,q} \times d^q = \binom{d+q-1}{q}$ . If useSparse=TRUE an object of the class "dgCMatrix" is produced.

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.3.2 Multi-Indexing, Elimination, and Duplication, p.21,(1.32)

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Qplication\(\)](#), [matr\\_Symmetry\(\)](#)

### Examples

```
x<-c(1,2,3)
y<-kronecker(kronecker(x,x),x)
## Distinct elements of y
z<-as.matrix(matr_Elimination(3,3))%*%y
## Restore eliminated elements in z
as.vector(matr_Qplication(3,3)%*%z)
```

---

matr_Qplication	<i>Qplication Matrix</i>
-----------------	--------------------------

---

### Description

Restores the duplicated/q-plicated elements which are eliminated by `matr_Elimination` in a T-product of vectors of dimension  $d$ .

### Usage

```
matr_Qplication(d, q, useSparse = FALSE)
```

### Arguments

<code>d</code>	dimension of a vector $x$
<code>q</code>	power of the Kronecker product
<code>useSparse</code>	TRUE or FALSE.

### Details

Note: since the algorithm of elimination is not unique, q-plication works together with the function `matr_Elimination` only.

### Value

Qplication matrix of order  $d^q \times \eta_{d,q}$ , see (1.30), p.15. If `useSparse=TRUE` an object of the class "dgCMatrix" is produced.

### References

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, p.21, (1.31)

### See Also

Other Matrices and commutators: `indx_Commutator_Kmn()`, `indx_Commutator_Kperm()`, `indx_Elimination()`, `indx_Qplication()`, `indx_Symmetry()`, `indx_UnivMomCum()`, `matr_Commutator_Kmn()`, `matr_Commutator_Kperm()`, `matr_Commutator_Mixing()`, `matr_Commutator_Moment()`, `matr_Elimination()`, `matr_Symmetry()`

### Examples

```
x<-c(1,2,3)
y<-kronecker(kronecker(x,x),x)
## Distinct elements of y
z<-as.matrix(matr_Elimination(3,3))%*%y
## Restore eliminated elements in z
as.vector(matr_Qplication(3,3)%*%z)
```

---

matr_Symmetry	<i>Symmetrizer Matrix</i>
---------------	---------------------------

---

### Description

Based on Chacon and Duong (2015) efficient recursive algorithms for functionals based on higher order derivatives. An option for sparse matrix is provided. By using sparse matrices far less memory is required and faster computation times are obtained

### Usage

```
matr_Symmetry(d, n, useSparse = FALSE)
```

### Arguments

d	dimension of a vector x
n	power of the Kronecker product
useSparse	TRUE or FALSE. If TRUE an object of the class "dgCMatrix" is produced.

### Value

A Symmetrizer matrix with order  $d^n \times d^n$ . If useSparse=TRUE an object of the class "dgCMatrix" is produced.

### References

Chacon, J. E., and Duong, T. (2015). Efficient recursive algorithms for functionals based on higher order derivatives of the multivariate Gaussian density. *Statistics and Computing*, 25(5), 959-974.

Gy. Terdik, *Multivariate statistical methods - going beyond the linear*, Springer 2021. Section 1.3.1 Symmetrization, p.14. (1.29)

### See Also

Other Matrices and commutators: [indx\\_Commutator\\_Kmn\(\)](#), [indx\\_Commutator\\_Kperm\(\)](#), [indx\\_Elimination\(\)](#), [indx\\_Qplication\(\)](#), [indx\\_Symmetry\(\)](#), [indx\\_UnivMomCum\(\)](#), [matr\\_Commutator\\_Kmn\(\)](#), [matr\\_Commutator\\_Kperm\(\)](#), [matr\\_Commutator\\_Mixing\(\)](#), [matr\\_Commutator\\_Moment\(\)](#), [matr\\_Elimination\(\)](#), [matr\\_Qplication\(\)](#)

### Examples

```
a<-c(1,2)
b<-c(2,3)
c<-kronecker(kronecker(a,a),b)
## The symmetrized version of c is
as.vector(matr_Symmetry(2,3)%*%c)
```

---

Partition\_2Perm      *Permutation of elements according to partition pU*

---

**Description**

Permutation of elements according to partition pU

**Usage**

Partition\_2Perm(pU)

**Arguments**

pU                      A partition matrix. For instance a matrix generated by Partition\_Type\_All.

**Value**

perm\_pU A vector with the elements 1 to N permuted according to pU. The numbers of 1 : N are listed in the order of their occurrence in the blocks of pU.

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.4.4

**See Also**

Other Partitions: [Partition\\_Indecomposable\(\)](#), [Partition\\_Pairs\(\)](#), [Partition\\_Type\\_All\(\)](#), [Permutation\\_Inverse\(\)](#)

**Examples**

```
PA<-Partition_Type_All(4)
Partition_2Perm(PA$Part.class[[3]])
```

---

Partition\_DiagramsClosedNoLoops  
*Closed Diagrams without Loops*

---

**Description**

Given a partition L looks for those partitions K of pairs which are indecomposable in accordance with L, (L, K) represent diagrams. It is used for getting higher order cumulants of Hermite polynomials defined by L, see Terdik (2021) Proposition 4.3, p.194.

**Usage**

Partition\_DiagramsClosedNoLoops(L)

**Arguments**

L a partition matrix

**Value**

The list of partition matrices indecomposable with respect to L

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.4.8.1

---

Partition\_Indecomposable

*Building indecomposable partitions*

---

**Description**

Produces the list of all indecomposable partitions with respect to the partition matrix L

**Usage**

Partition\_Indecomposable(L)

**Arguments**

L A partition matrix

**Value**

IndecompK2L A list of partition matrices indecomposable with respect to L

num\_by\_sizes A vector indicating the number of indecomposable partitions with respect to L by sizes

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.4.6

**See Also**

Other Partitions: [Partition\\_2Perm\(\)](#), [Partition\\_Pairs\(\)](#), [Partition\\_Type\\_All\(\)](#), [Permutation\\_Inverse\(\)](#)

**Examples**

```
L<-matrix(c(1,1,0,0,0,0,1,1),2,4,byrow=TRUE)
IP<-Partition_Indecomposable(L)
IP$IndecompK2L
IP$num_by_sizes
```

---

Partition_Pairs	<i>Partition into pairs of the set 1:N</i>
-----------------	--

---

**Description**

Partition into pairs of the set 1:N

**Usage**

```
Partition_Pairs(N)
```

**Arguments**

N                    The (integer) number of elements to be partitioned

**Value**

The list of partition matrices with blocks containing two elements. The list is empty if N is odd.

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Section 1.4.8

**See Also**

Other Partitions: [Partition\\_2Perm\(\)](#), [Partition\\_Indecomposable\(\)](#), [Partition\\_Type\\_All\(\)](#), [Permutation\\_Inverse\(\)](#)

**Examples**

```
PA<-Partition_Pairs(4)
```

---

Partition\_Type\_All      *Partitions, type and number of partitions*

---

**Description**

Generates all partitions of N numbers and classify them by type

**Usage**

Partition\_Type\_All(N)

**Arguments**

N                      The (integer) number of elements to be partitioned

**Value**

Part.class The list of all possible partitions given as partition matrices

S\_N\_r A vector with the number of partitions of size r=1, r=2, etc. (Stirling numbers of second kind )

eL\_r A list of partition types with respect to partitions of size r=1, r=2, etc.

S\_r\_j Vectors of number of partitions with given types grouped by partitions of size r=1, r=2, etc.

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021. Case 1.4, p.31 and Example 1.18, p.32.

**See Also**

Other Partitions: [Partition\\_2Perm\(\)](#), [Partition\\_Indecomposable\(\)](#), [Partition\\_Pairs\(\)](#), [Permutation\\_Inverse\(\)](#)

**Examples**

```
# See Example 1.18, p. 32, reference below
PTA<-Partition_Type_All(4)
# Partitions generated
PTA$Part.class
# Partitions of size 2 includes two types
PTA$eL_r[[2]]
# Number of partitions with r=1 blocks, r=2 blocks, etc-
PTA$S_N_r
# Number of different types collected by partitions of size r=1, r=2, etc.
PTA$S_r_j
# Partitions with size r=2, includes two types (above) each with number
PTA$S_r_j[[2]]
```



---

Permutation\_Inverse     *Inverse of a Permutation*

---

**Description**

Inverse of a Permutation

**Usage**

```
Permutation_Inverse(permutation0)
```

**Arguments**

permutation0     A permutation of numbers 1:n

**Value**

A vector containing the inverse permutation of permutation0

**References**

Gy. Terdik, Multivariate statistical methods - going beyond the linear, Springer 2021, Remark 1.1, p.2

**See Also**

Other Partitions: [Partition\\_2Perm\(\)](#), [Partition\\_Indecomposable\(\)](#), [Partition\\_Pairs\(\)](#), [Partition\\_Type\\_All\(\)](#)

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