

Package ‘PlaneGeometry’

October 12, 2022

Type Package

Title Plane Geometry

Version 1.3.1

Author Stéphane Laurent

Maintainer Stéphane Laurent <laurent_step@outlook.fr>

Description An extensive set of plane geometry routines. Provides R6 classes representing triangles, circles, circular arcs, ellipses, elliptical arcs and lines, and their plot methods. Also provides R6 classes representing transformations: rotations, reflections, homotheties, scalings, general affine transformations, inversions, Möbius transformations.

License GPL-3

Encoding UTF-8

Imports R6, methods, graphics, stringr, gsl, uniformly, sdpt3r, fitConic

RoxygenNote 7.1.2

Suggests testthat, knitr, rmarkdown, viridisLite, sets, ellipse, freegroup, elliptic, rgl

VignetteBuilder knitr

URL <https://github.com/stla/PlaneGeometry>

BugReports <https://github.com/stla/PlaneGeometry/issues>

NeedsCompilation no

Repository CRAN

Date/Publication 2022-01-13 18:22:42 UTC

R topics documented:

Affine	3
AffineMappingEllipse2Ellipse	5
AffineMappingThreePoints	5
Arc	6
Circle	8

CircleAB	15
CircleOA	16
crossRatio	16
draw	17
ellint2	18
Ellipse	19
EllipseEquationFromFivePoints	27
EllipseFromCenterAndMatrix	28
EllipseFromEquation	28
EllipseFromFivePoints	29
EllipseFromFociAndOnePoint	30
EllipseFromThreeBoundaryPoints	30
EllipticalArc	31
fitEllipse	33
GaussianEllipse	34
Homothety	34
intersectionCircleCircle	36
intersectionCircleLine	37
intersectionEllipseLine	38
intersectionLineLine	38
Inversion	39
inversionFixingThreeCircles	43
inversionFixingTwoCircles	43
inversionFromCircle	44
inversionKeepingCircle	44
inversionSwappingTwoCircles	45
Line	45
LownerJohnEllipse	50
midCircles	51
Mobius	52
MobiusMappingCircle	55
MobiusMappingThreePoints	56
MobiusSwappingTwoPoints	56
Projection	57
radicalCenter	59
Reflection	60
Rotation	62
Scaling	66
ScalingXY	68
Shear	70
soddyCircle	72
SteinerChain	73
Translation	74
Triangle	76
TriangleThreeLines	90
unitCircle	90

Affine

R6 class representing an affine map.

Description

An affine map is given by a 2x2 matrix (a linear transformation) and a vector (the "intercept").

Active bindings

A get or set the matrix A

b get or set the vector b

Methods

Public methods:

- `Affine$new()`
- `Affine$print()`
- `Affine$get3x3matrix()`
- `Affine$inverse()`
- `Affine$compose()`
- `Affine$transform()`
- `Affine$transformLine()`
- `Affine$transformEllipse()`
- `Affine$clone()`

Method `new()`: Create a new Affine object.

Usage:

```
Affine$new(A, b)
```

Arguments:

A the 2x2 matrix of the affine map

b the shift vector of the affine map

Returns: A new Affine object.

Method `print()`: Show instance of an Affine object.

Usage:

```
Affine$print(...)
```

Arguments:

... ignored

Examples:

```
Affine$new(rbind(c(3.5,2),c(0,4)), c(-1, 1.25))
```

Method `get3x3matrix()`: The 3x3 matrix representing the affine map.

Usage:

`Affine$get3x3matrix()`

Method `inverse()`: The inverse affine transformation, if it exists.

Usage:

`Affine$inverse()`

Method `compose()`: Compose the reference affine map with another affine map.

Usage:

`Affine$compose(transfo, left = TRUE)`

Arguments:

`transfo` an Affine object

`left` logical, whether to compose at left or at right (i.e. returns $f_1 \circ f_0$ or $f_0 \circ f_1$)

Returns: An Affine object.

Method `transform()`: Transform a point or several points by the reference affine map.

Usage:

`Affine$transform(M)`

Arguments:

`M` a point or a two-column matrix of points, one point per row

Method `transformLine()`: Transform a line by the reference affine transformation (only for invertible affine maps).

Usage:

`Affine$transformLine(line)`

Arguments:

`line` a Line object

Returns: A Line object.

Method `transformEllipse()`: Transform an ellipse by the reference affine transformation (only for an invertible affine map). The result is an ellipse.

Usage:

`Affine$transformEllipse(ell)`

Arguments:

`ell` an Ellipse object or a Circle object

Returns: An Ellipse object.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

`Affine$clone(deep = FALSE)`

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## -----
## Method `Affine$print`
## -----

Affine$new(rbind(c(3.5,2),c(0,4)), c(-1, 1.25))
```

AffineMappingEllipse2Ellipse

Affine transformation mapping a given ellipse to a given ellipse

Description

Return the affine transformation which transforms ell1 to ell2.

Usage

```
AffineMappingEllipse2Ellipse(ell1, ell2)
```

Arguments

ell1, ell2 Ellipse or Circle objects

Value

An Affine object.

Examples

```
ell1 <- Ellipse$new(c(1,1), 5, 1, 30)
( ell2 <- Ellipse$new(c(4,-1), 3, 2, 50) )
f <- AffineMappingEllipse2Ellipse(ell1, ell2)
f$transformEllipse(ell1) # should be ell2
```

AffineMappingThreePoints

Affine transformation mapping three given points to three given points

Description

Return the affine transformation which sends P1 to Q1, P2 to Q2 and P3 to Q3.

Usage

```
AffineMappingThreePoints(P1, P2, P3, Q1, Q2, Q3)
```

Arguments

P1, P2, P3 three non-collinear points
 Q1, Q2, Q3 three non-collinear points

Value

An Affine object.

Arc

R6 class representing a circular arc

Description

An arc is given by a center, a radius, a starting angle and an ending angle. They are respectively named center, radius, alpha1 and alpha2.

Active bindings

center get or set the center
 radius get or set the radius
 alpha1 get or set the starting angle
 alpha2 get or set the ending angle
 degrees get or set the degrees field

Methods**Public methods:**

- [Arc\\$new\(\)](#)
- [Arc\\$print\(\)](#)
- [Arc\\$startingPoint\(\)](#)
- [Arc\\$endingPoint\(\)](#)
- [Arc\\$isEqual\(\)](#)
- [Arc\\$complementaryArc\(\)](#)
- [Arc\\$path\(\)](#)
- [Arc\\$clone\(\)](#)

Method new(): Create a new Arc object.

Usage:

Arc\$new(center, radius, alpha1, alpha2, degrees = TRUE)

Arguments:

center the center
 radius the radius
 alpha1 the starting angle

alpha2 the ending angle
degrees logical, whether alpha1 and alpha2 are given in degrees

Returns: A new Arc object.

Examples:

```
arc <- Arc$new(c(1,1), 1, 45, 90)
arc
arc$center
arc$center <- c(0,0)
arc
```

Method print(): Show instance of an Arc object.

Usage:

```
Arc$print(...)
```

Arguments:

... ignored

Examples:

```
Arc$new(c(0,0), 2, pi/4, pi/2, FALSE)
```

Method startingPoint(): Starting point of the reference arc.

Usage:

```
Arc$startingPoint()
```

Method endingPoint(): Ending point of the reference arc.

Usage:

```
Arc$endingPoint()
```

Method isEqual(): Check whether the reference arc equals another arc.

Usage:

```
Arc$isEqual(arc)
```

Arguments:

arc an Arc object

Method complementaryArc(): Complementary arc of the reference arc.

Usage:

```
Arc$complementaryArc()
```

Examples:

```
arc <- Arc$new(c(0,0), 1, 30, 60)
plot(NULL, type = "n", asp = 1, xlim = c(-1,1), ylim = c(-1,1),
      xlab = NA, ylab = NA)
draw(arc, lwd = 3, col = "red")
draw(arc$complementaryArc(), lwd = 3, col = "green")
```

Method path(): The reference arc as a path.

Usage:

```
Arc$path(npoints = 100L)
```

Arguments:

`npoints` number of points of the path

Returns: A matrix with two columns `x` and `y` of length `npoints`. See "Filling the lapping area of two circles" in the vignette for an example.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Arc$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## -----
## Method `Arc$new`
## -----

arc <- Arc$new(c(1,1), 1, 45, 90)
arc
arc$center
arc$center <- c(0,0)
arc

## -----
## Method `Arc$print`
## -----

Arc$new(c(0,0), 2, pi/4, pi/2, FALSE)

## -----
## Method `Arc$complementaryArc`
## -----

arc <- Arc$new(c(0,0), 1, 30, 60)
plot(NULL, type = "n", asp = 1, xlim = c(-1,1), ylim = c(-1,1),
      xlab = NA, ylab = NA)
draw(arc, lwd = 3, col = "red")
draw(arc$complementaryArc(), lwd = 3, col = "green")
```

Circle

R6 class representing a circle

Description

A circle is given by a center and a radius, named `center` and `radius`.

Active bindings

center get or set the center
radius get or set the radius

Methods**Public methods:**

- `Circle$new()`
- `Circle$print()`
- `Circle$pointFromAngle()`
- `Circle$diameter()`
- `Circle$tangent()`
- `Circle$tangentsThroughExternalPoint()`
- `Circle$isEqual()`
- `Circle$isDifferent()`
- `Circle$isOrthogonal()`
- `Circle$angle()`
- `Circle$includes()`
- `Circle$orthogonalThroughTwoPointsOnCircle()`
- `Circle$orthogonalThroughTwoPointsWithinCircle()`
- `Circle$power()`
- `Circle$radicalCenter()`
- `Circle$radicalAxis()`
- `Circle$rotate()`
- `Circle$translate()`
- `Circle$invert()`
- `Circle$asEllipse()`
- `Circle$randomPoints()`
- `Circle$clone()`

Method `new()`: Create a new Circle object.

Usage:

```
Circle$new(center, radius)
```

Arguments:

center the center

radius the radius

Returns: A new Circle object.

Examples:

```
circ <- Circle$new(c(1,1), 1)
circ
circ$center
circ$center <- c(0,0)
circ
```

Method `print()`: Show instance of a circle object.

Usage:

```
Circle$print(...)
```

Arguments:

... ignored

Examples:

```
Circle$new(c(0,0), 2)
```

Method `pointFromAngle()`: Get a point on the reference circle from its polar angle.

Usage:

```
Circle$pointFromAngle(alpha, degrees = TRUE)
```

Arguments:

alpha a number, the angle

degrees logical, whether alpha is given in degrees

Returns: The point on the circle with polar angle alpha.

Method `diameter()`: Diameter of the reference circle for a given polar angle.

Usage:

```
Circle$diameter(alpha)
```

Arguments:

alpha an angle in radians, there is one diameter for each value of alpha modulo pi

Returns: A segment (Line object).

Examples:

```
circ <- Circle$new(c(1,1), 5)
diams <- lapply(c(0, pi/3, 2*pi/3), circ$diameter)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-5,7),
      xlab = NA, ylab = NA)
draw(circ, lwd = 2, col = "yellow")
invisible(lapply(diams, draw, col = "blue"))
```

Method `tangent()`: Tangent of the reference circle at a given polar angle.

Usage:

```
Circle$tangent(alpha)
```

Arguments:

alpha an angle in radians, there is one tangent for each value of alpha modulo 2π

Examples:

```
circ <- Circle$new(c(1,1), 5)
tangents <- lapply(c(0, pi/3, 2*pi/3, pi, 4*pi/3, 5*pi/3), circ$tangent)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-5,7),
      xlab = NA, ylab = NA)
draw(circ, lwd = 2, col = "yellow")
invisible(lapply(tangents, draw, col = "blue"))
```

Method tangentsThroughExternalPoint(): Return the two tangents of the reference circle passing through an external point.

Usage:

Circle\$tangentsThroughExternalPoint(P)

Arguments:

P a point external to the reference circle

Returns: A list of two Line objects, the two tangents; the tangency points are in the B field of the lines.

Method isEqual(): Check whether the reference circle equals another circle.

Usage:

Circle\$isEqual(circ)

Arguments:

circ a Circle object

Method isDifferent(): Check whether the reference circle differs from another circle.

Usage:

Circle\$isDifferent(circ)

Arguments:

circ a Circle object

Method isOrthogonal(): Check whether the reference circle is orthogonal to a given circle.

Usage:

Circle\$isOrthogonal(circ)

Arguments:

circ a Circle object

Method angle(): Angle between the reference circle and a given circle, if they intersect.

Usage:

Circle\$angle(circ)

Arguments:

circ a Circle object

Method includes(): Check whether a point belongs to the reference circle.

Usage:

Circle\$includes(M)

Arguments:

M a point

Method orthogonalThroughTwoPointsOnCircle(): Orthogonal circle passing through two points on the reference circle.

Usage:

```
Circle$orthogonalThroughTwoPointsOnCircle(alpha1, alpha2, arc = FALSE)
```

Arguments:

alpha1, alpha2 two angles defining two points on the reference circle

arc logical, whether to return only the arc at the interior of the reference circle

Returns: A Circle object if arc=FALSE, an Arc object if arc=TRUE, or a Line object: the diameter of the reference circle defined by the two points in case when the two angles differ by pi.

Examples:

```
# hyperbolic triangle
circ <- Circle$new(c(5,5), 3)
arc1 <- circ$orthogonalThroughTwoPointsOnCircle(0, 2*pi/3, arc = TRUE)
arc2 <- circ$orthogonalThroughTwoPointsOnCircle(2*pi/3, 4*pi/3, arc = TRUE)
arc3 <- circ$orthogonalThroughTwoPointsOnCircle(4*pi/3, 0, arc = TRUE)
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type = "n", asp = 1, xlim = c(2,8), ylim = c(2,8))
draw(circ)
draw(arc1, col = "red", lwd = 2)
draw(arc2, col = "green", lwd = 2)
draw(arc3, col = "blue", lwd = 2)
par(opar)
```

Method orthogonalThroughTwoPointsWithinCircle(): Orthogonal circle passing through two points within the reference circle.

Usage:

```
Circle$orthogonalThroughTwoPointsWithinCircle(P1, P2, arc = FALSE)
```

Arguments:

P1, P2 two distinct points in the interior of the reference circle

arc logical, whether to return the arc joining the two points instead of the circle

Returns: A Circle object or an Arc object, or a Line object if the two points are on a diameter.

Examples:

```
circ <- Circle$new(c(0,0),3)
P1 <- c(1,1); P2 <- c(1, 2)
ocirc <- circ$orthogonalThroughTwoPointsWithinCircle(P1, P2)
arc <- circ$orthogonalThroughTwoPointsWithinCircle(P1, P2, arc = TRUE)
plot(0, 0, type = "n", asp = 1, xlab = NA, ylab = NA,
     xlim = c(-3, 4), ylim = c(-3, 4))
draw(circ, lwd = 2)
draw(ocirc, lty = "dashed", lwd = 2)
draw(arc, lwd = 3, col = "blue")
```

Method power(): Power of a point with respect to the reference circle.

Usage:

```
Circle$power(M)
```

Arguments:

M point

Returns: A number.

Method radicalCenter(): Radical center of two circles.

Usage:

Circle\$radicalCenter(circ2)

Arguments:

circ2 a Circle object

Method radicalAxis(): Radical axis of two circles.

Usage:

Circle\$radicalAxis(circ2)

Arguments:

circ2 a Circle object

Returns: A Line object.

Method rotate(): Rotate the reference circle.

Usage:

Circle\$rotate(alpha, 0, degrees = TRUE)

Arguments:

alpha angle of rotation

0 center of rotation

degrees logical, whether alpha is given in degrees

Returns: A Circle object.

Method translate(): Translate the reference circle.

Usage:

Circle\$translate(v)

Arguments:

v the vector of translation

Returns: A Circle object.

Method invert(): Invert the reference circle.

Usage:

Circle\$invert(inversion)

Arguments:

inversion an Inversion object

Returns: A Circle object or a Line object.

Method asEllipse(): Convert the reference circle to an Ellipse object.

Usage:

Circle\$asEllipse()

Method `randomPoints()`: Random points on or in the reference circle.

Usage:

```
Circle$randomPoints(n, where = "in")
```

Arguments:

`n` an integer, the desired number of points

`where` "in" to generate inside the circle, "on" to generate on the circle

Returns: The generated points in a two columns matrix with `n` rows.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Circle$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

See Also

[radicalCenter](#) for the radical center of three circles.

Examples

```
## -----
## Method `Circle$new`
## -----

circ <- Circle$new(c(1,1), 1)
circ
circ$center
circ$center <- c(0,0)
circ

## -----
## Method `Circle$print`
## -----

Circle$new(c(0,0), 2)

## -----
## Method `Circle$diameter`
## -----

circ <- Circle$new(c(1,1), 5)
diams <- lapply(c(0, pi/3, 2*pi/3), circ$diameter)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-5,7),
      xlab = NA, ylab = NA)
draw(circ, lwd = 2, col = "yellow")
invisible(lapply(diams, draw, col = "blue"))

## -----
```

```

## Method `Circle$tangent`
## -----

circ <- Circle$new(c(1,1), 5)
tangents <- lapply(c(0, pi/3, 2*pi/3, pi, 4*pi/3, 5*pi/3), circ$tangent)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-5,7),
      xlab = NA, ylab = NA)
draw(circ, lwd = 2, col = "yellow")
invisible(lapply(tangents, draw, col = "blue"))

## -----
## Method `Circle$orthogonalThroughTwoPointsOnCircle`
## -----

# hyperbolic triangle
circ <- Circle$new(c(5,5), 3)
arc1 <- circ$orthogonalThroughTwoPointsOnCircle(0, 2*pi/3, arc = TRUE)
arc2 <- circ$orthogonalThroughTwoPointsOnCircle(2*pi/3, 4*pi/3, arc = TRUE)
arc3 <- circ$orthogonalThroughTwoPointsOnCircle(4*pi/3, 0, arc = TRUE)
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type = "n", asp = 1, xlim = c(2,8), ylim = c(2,8))
draw(circ)
draw(arc1, col = "red", lwd = 2)
draw(arc2, col = "green", lwd = 2)
draw(arc3, col = "blue", lwd = 2)
par(opar)

## -----
## Method `Circle$orthogonalThroughTwoPointsWithinCircle`
## -----

circ <- Circle$new(c(0,0),3)
P1 <- c(1,1); P2 <- c(1, 2)
ocirc <- circ$orthogonalThroughTwoPointsWithinCircle(P1, P2)
arc <- circ$orthogonalThroughTwoPointsWithinCircle(P1, P2, arc = TRUE)
plot(0, 0, type = "n", asp = 1, xlab = NA, ylab = NA,
      xlim = c(-3, 4), ylim = c(-3, 4))
draw(circ, lwd = 2)
draw(ocirc, lty = "dashed", lwd = 2)
draw(arc, lwd = 3, col = "blue")

```

CircleAB

Circle given by a diameter

Description

Return the circle given by a diameter

Usage

CircleAB(A, B)

Arguments

A, B the endpoints of the diameter

Value

A Circle object.

CircleOA	<i>Circle given by its center and a point</i>
----------	---

Description

Return the circle given by its center and a point it passes through.

Usage

CircleOA(O, A)

Arguments

O the center of the circle
 A a point of the circle

Value

A Circle object.

crossRatio	<i>Cross ratio</i>
------------	--------------------

Description

The cross ratio of four points.

Usage

crossRatio(A, B, C, D)

Arguments

A, B, C, D four distinct points

Value

A complex number. It is real if and only if the four points lie on a generalized circle (that is a circle or a line).

Examples

```

c <- Circle$new(c(0, 0), 1)
A <- c$pointFromAngle(0)
B <- c$pointFromAngle(90)
C <- c$pointFromAngle(180)
D <- c$pointFromAngle(270)
crossRatio(A, B, C, D) # should be real
Mob <- Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
MA <- Mob$transform(A)
MB <- Mob$transform(B)
MC <- Mob$transform(C)
MD <- Mob$transform(D)
crossRatio(MA, MB, MC, MD) # should be identical to `crossRatio(A, B, C, D)`

```

draw

Draw a geometric object

Description

Draw a geometric object on the current plot.

Usage

```

draw(x, ...)

## S3 method for class 'Triangle'
draw(x, ...)

## S3 method for class 'Circle'
draw(x, npoints = 100L, ...)

## S3 method for class 'Arc'
draw(x, npoints = 100L, ...)

## S3 method for class 'Ellipse'
draw(x, npoints = 100L, ...)

## S3 method for class 'EllipticalArc'
draw(x, npoints = 100L, ...)

## S3 method for class 'Line'
draw(x, ...)

```

Arguments

x geometric object (Triangle, Circle, Line, Ellipse, Arc, EllipticalArc)

... arguments passed to `lines` for a Triangle object, an Arc object or an EllipticalArc object, to `polypath` for a Circle object or an Ellipse object, general graphical parameters for a Line object, passed to `lines`, `curve`, or `abline`.

npoints integer, the number of points of the path

Examples

```
# open new plot window
plot(0, 0, type="n", asp = 1, xlim = c(0,2.5), ylim = c(0,2.5),
     xlab = NA, ylab = NA)
grid()
# draw a triangle
t <- Triangle$new(c(0,0), c(1,0), c(0.5,sqrt(3)/2))
draw(t, col = "blue", lwd = 2)
draw(t$rotate(90, t$C), col = "green", lwd = 2)
# draw a circle
circ <- t$incircle()
draw(circ, col = "orange", border = "brown", lwd = 2)
# draw an ellipse
S <- Scaling$new(circ$center, direction = c(2,1), scale = 2)
draw(S$scaleCircle(circ), border = "grey", lwd = 2)
# draw a line
l <- Line$new(c(1,1), c(1.5,1.5), FALSE, TRUE)
draw(l, col = "red", lwd = 2)
perp <- l$perpendicular(c(2,1))
draw(perp, col = "yellow", lwd = 2)
```

ellint2

Extended elliptic integral of the second kind

Description

Evaluates the extended incomplete elliptic integral of the second kind. The function is vectorized in `m` but not in `phi`.

Usage

```
ellint2(phi, m)
```

Arguments

`phi` amplitude, a number

`m` values of the parameter, a vector of numbers lower than $1/\sin(\text{phi})^2$ (NaN is returned if this condition is not satisfied)

Details

For $-\pi/2 \leq \text{phi} \leq \pi/2$, this is the integral of $\sqrt{1 - m \sin(t)^2}$ for t between 0 and phi . Then the function is extended to arbitrary phi by the formula $\text{ellint2}(\text{phi} + k\pi, m) = 2k \cdot \text{ellint2}(\pi/2, m) + \text{ellint2}(\text{phi}, m)$ for any integer k .

Value

A numeric vector of the same length as `m`.

Note

This function is used to calculate the length of an elliptical arc (method `length` of `EllipticalArc`).

Examples

```
phi <- pi/4; m <- 0.6
ellint2(phi, m)
gsl::ellint_E(phi, sqrt(m))
curve(ellint2(phi, x), -5, 1/sin(phi)^2)
```

Ellipse

R6 class representing an ellipse

Description

An ellipse is given by a center, two radii (`rmajor` and `rminor`), and the angle (`alpha`) between the major axis and the horizontal direction.

Active bindings

`center` get or set the center
`rmajor` get or set the major radius of the ellipse
`rminor` get or set the minor radius of the ellipse
`alpha` get or set the angle of the ellipse
`degrees` get or set the degrees field

Methods**Public methods:**

- [Ellipse\\$new\(\)](#)
- [Ellipse\\$print\(\)](#)
- [Ellipse\\$isEqual\(\)](#)
- [Ellipse\\$equation\(\)](#)
- [Ellipse\\$includes\(\)](#)
- [Ellipse\\$contains\(\)](#)
- [Ellipse\\$matrix\(\)](#)
- [Ellipse\\$path\(\)](#)
- [Ellipse\\$diameter\(\)](#)
- [Ellipse\\$pointFromAngle\(\)](#)
- [Ellipse\\$pointFromEccentricAngle\(\)](#)

- `Ellipse$semiMajorAxis()`
- `Ellipse$semiMinorAxis()`
- `Ellipse$foci()`
- `Ellipse$tangent()`
- `Ellipse$normal()`
- `Ellipse$theta2t()`
- `Ellipse$regressionLines()`
- `Ellipse$boundingbox()`
- `Ellipse$randomPoints()`
- `Ellipse$clone()`

Method `new()`: Create a new Ellipse object.

Usage:

```
Ellipse$new(center, rmajor, rminor, alpha, degrees = TRUE)
```

Arguments:

`center` a point, the center of the rotation

`rmajor` positive number, the major radius

`rminor` positive number, the minor radius

`alpha` a number, the angle between the major axis and the horizontal direction

`degrees` logical, whether alpha is given in degrees

Returns: A new Ellipse object.

Examples:

```
Ellipse$new(c(1,1), 3, 2, 30)
```

Method `print()`: Show instance of an Ellipse object.

Usage:

```
Ellipse$print(...)
```

Arguments:

`...` ignored

Method `isEqual()`: Check whether the reference ellipse equals an ellipse.

Usage:

```
Ellipse$isEqual(e1)
```

Arguments:

`e1` An Ellipse object.

Method `equation()`: The coefficients of the implicit equation of the ellipse.

Usage:

```
Ellipse$equation()
```

Details: The implicit equation of the ellipse is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. This method returns A, B, C, D, E and F.

Returns: A named numeric vector.

Method `includes()`: Check whether a point lies on the reference ellipse.

Usage:

```
Ellipse$includes(M)
```

Arguments:

M a point

Method `contains()`: Check whether a point is contained in the reference ellipse.

Usage:

```
Ellipse$contains(M)
```

Arguments:

M a point

Method `matrix()`: Returns the 2x2 matrix S associated to the reference ellipse. The equation of the ellipse is $t(M-O) \%*\% S \%*\% (M-O) = 1$.

Usage:

```
Ellipse$matrix()
```

Examples:

```
ell <- Ellipse$new(c(1,1), 5, 1, 30)
S <- ell$matrix()
O <- ell$center
pts <- ell$path(4L) # four points on the ellipse
apply(pts, 1L, function(M) t(M-O) \%*\% S \%*\% (M-O))
```

Method `path()`: Path that forms the reference ellipse.

Usage:

```
Ellipse$path(npoints = 100L)
```

Arguments:

`npoints` number of points of the path

Returns: A matrix with two columns x and y of length npoints.

Method `diameter()`: Diameter and conjugate diameter of the reference ellipse.

Usage:

```
Ellipse$diameter(t, conjugate = FALSE)
```

Arguments:

t a number, the diameter only depends on t modulo pi; the axes correspond to $t=0$ and $t=\pi/2$

conjugate logical, whether to return the conjugate diameter as well

Returns: A Line object or a list of two Line objects if conjugate = TRUE.

Examples:

```
ell <- Ellipse$new(c(1,1), 5, 2, 30)
diameters <- lapply(c(0, pi/3, 2*pi/3), ell$diameter)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell)
invisible(lapply(diameters, draw))
```

Method `pointFromAngle()`: Intersection point of the ellipse with the half-line starting at the ellipse center and forming angle `theta` with the major axis.

Usage:

`Ellipse$pointFromAngle(theta, degrees = TRUE)`

Arguments:

`theta` a number, the angle, or a numeric vector

`degrees` logical, whether `theta` is given in degrees

Returns: A point of the ellipse if `length(theta)==1` or a two-column matrix of points of the ellipse if `length(theta) > 1` (one point per row).

Method `pointFromEccentricAngle()`: Point of the ellipse with given eccentric angle.

Usage:

`Ellipse$pointFromEccentricAngle(t)`

Arguments:

`t` a number, the eccentric angle in radians, or a numeric vector

Returns: A point of the ellipse if `length(t)==1` or a two-column matrix of points of the ellipse if `length(t) > 1` (one point per row).

Method `semiMajorAxis()`: Semi-major axis of the ellipse.

Usage:

`Ellipse$semiMajorAxis()`

Returns: A segment (Line object).

Method `semiMinorAxis()`: Semi-minor axis of the ellipse.

Usage:

`Ellipse$semiMinorAxis()`

Returns: A segment (Line object).

Method `foci()`: Foci of the reference ellipse.

Usage:

`Ellipse$foci()`

Returns: A list with the two foci.

Method `tangent()`: Tangents of the reference ellipse at a point given by its eccentric angle.

Usage:

`Ellipse$tangent(t)`

Arguments:

`t` eccentric angle, there is one tangent for each value of `t` modulo 2π ; for `t = 0, \pi/2, \pi, -\pi/2`, these are the tangents at the vertices of the ellipse

Examples:

```

ell <- Ellipse$new(c(1,1), 5, 2, 30)
tangents <- lapply(c(0, pi/3, 2*pi/3, pi, 4*pi/3, 5*pi/3), ell$tangent)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell, col = "yellow")
invisible(lapply(tangents, draw, col = "blue"))

```

Method `normal()`: Normal unit vector to the ellipse.

Usage:

```
Ellipse$new(normal(t))
```

Arguments:

`t` a number, the eccentric angle in radians of the point of the ellipse at which we want the normal unit vector

Returns: The normal unit vector to the ellipse at the point given by eccentric angle `t`.

Examples:

```

ell <- Ellipse$new(c(1,1), 5, 2, 30)
t_ <- seq(0, 2*pi, length.out = 13)[-1]
plot(NULL, asp = 1, xlim = c(-5,7), ylim = c(-3,5),
      xlab = NA, ylab = NA)
draw(ell, col = "magenta")
for(i in 1:length(t_)){
  t <- t_[i]
  P <- ell$pointFromEccentricAngle(t)
  v <- ell$normal(t)
  draw(Line$new(P, P+v, FALSE, FALSE))
}

```

Method `theta2t()`: Convert angle to eccentric angle.

Usage:

```
Ellipse$new(theta2t(theta, degrees = TRUE))
```

Arguments:

`theta` angle between the major axis and the half-line starting at the center of the ellipse and passing through the point of interest on the ellipse
`degrees` logical, whether `theta` is given in degrees

Returns: The eccentric angle of the point of interest on the ellipse, in radians.

Examples:

```

O <- c(1, 1)
ell <- Ellipse$new(O, 5, 2, 30)
theta <- 20
P <- ell$pointFromAngle(theta)
t <- ell$theta2t(theta)
tg <- ell$tangent(t)
OP <- Line$new(O, P, FALSE, FALSE)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,5),
      xlab = NA, ylab = NA)

```

```

draw(ell, col = "antiquewhite")
points(P[1], P[2], pch = 19)
draw(tg, col = "red")
draw(OP)
draw(ell$semiMajorAxis())
text(t(0+c(1,0.9)), expression(theta))

```

Method `regressionLines()`: Regression lines. The regression line of y on x intersects the ellipse at its rightmost point and its leftmost point. The tangents at these points are vertical. The regression line of x on y intersects the ellipse at its topmost point and its bottommost point. The tangents at these points are horizontal.

Usage:

```
Ellipse$regressionLines()
```

Returns: A list with two Line objects: the regression line of y on x and the regression line of x on y .

Examples:

```

ell <- Ellipse$new(c(1,1), 5, 2, 30)
reglines <- ell$regressionLines()
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell, lwd = 2)
draw(reglines$YonX, lwd = 2, col = "blue")
draw(reglines$XonY, lwd = 2, col = "green")

```

Method `boundingbox()`: Return the smallest rectangle parallel to the axes which contains the reference ellipse.

Usage:

```
Ellipse$boundingbox()
```

Returns: A list with two components: the x-limits in x and the y-limits in y .

Examples:

```

ell <- Ellipse$new(c(2,2), 5, 3, 40)
box <- ell$boundingbox()
plot(NULL, asp = 1, xlim = box$x, ylim = box$y, xlab = NA, ylab = NA)
draw(ell, col = "seaShell", border = "blue")
abline(v = box$x, lty = 2); abline(h = box$y, lty = 2)

```

Method `randomPoints()`: Random points on or in the reference ellipse.

Usage:

```
Ellipse$randomPoints(n, where = "in")
```

Arguments:

n an integer, the desired number of points

where "in" to generate inside the ellipse, "on" to generate on the ellipse

Returns: The generated points in a two columns matrix with n rows.

Examples:


```

ell <- Ellipse$new(c(1,1), 5, 2, 30)
pts <- ell$randomPoints(100)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell, lwd = 2)
points(pts, pch = 19, col = "blue")

```

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Ellipse$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```

## -----
## Method `Ellipse$new`
## -----

Ellipse$new(c(1,1), 3, 2, 30)

## -----
## Method `Ellipse$matrix`
## -----

ell <- Ellipse$new(c(1,1), 5, 1, 30)
S <- ell$matrix()
O <- ell$center
pts <- ell$path(4L) # four points on the ellipse
apply(pts, 1L, function(M) t(M-O) %*% S %*% (M-O))

## -----
## Method `Ellipse$diameter`
## -----

ell <- Ellipse$new(c(1,1), 5, 2, 30)
diameters <- lapply(c(0, pi/3, 2*pi/3), ell$diameter)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell)
invisible(lapply(diameters, draw))

## -----
## Method `Ellipse$tangent`
## -----

ell <- Ellipse$new(c(1,1), 5, 2, 30)
tangents <- lapply(c(0, pi/3, 2*pi/3, pi, 4*pi/3, 5*pi/3), ell$tangent)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)

```

```

draw(ell, col = "yellow")
invisible(lapply(tangents, draw, col = "blue"))

## -----
## Method `Ellipse$normal`
## -----

ell <- Ellipse$new(c(1,1), 5, 2, 30)
t_ <- seq(0, 2*pi, length.out = 13)[-1]
plot(NULL, asp = 1, xlim = c(-5,7), ylim = c(-3,5),
      xlab = NA, ylab = NA)
draw(ell, col = "magenta")
for(i in 1:length(t_)){
  t <- t_[i]
  P <- ell$pointFromEccentricAngle(t)
  v <- ell$normal(t)
  draw(Line$new(P, P+v, FALSE, FALSE))
}

## -----
## Method `Ellipse$theta2t`
## -----

O <- c(1, 1)
ell <- Ellipse$new(O, 5, 2, 30)
theta <- 20
P <- ell$pointFromAngle(theta)
t <- ell$theta2t(theta)
tg <- ell$tangent(t)
OP <- Line$new(O, P, FALSE, FALSE)
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,5),
      xlab = NA, ylab = NA)
draw(ell, col = "antiquewhite")
points(P[1], P[2], pch = 19)
draw(tg, col = "red")
draw(OP)
draw(ell$semiMajorAxis())
text(t(O+c(1,0.9)), expression(theta))

## -----
## Method `Ellipse$regressionLines`
## -----

ell <- Ellipse$new(c(1,1), 5, 2, 30)
reglines <- ell$regressionLines()
plot(NULL, asp = 1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell, lwd = 2)
draw(reglines$YonX, lwd = 2, col = "blue")
draw(reglines$XonY, lwd = 2, col = "green")

## -----
## Method `Ellipse$boundingbox`

```

```
## -----
ell <- Ellipse$new(c(2,2), 5, 3, 40)
box <- ell$boundingbox()
plot(NULL, asp = 1, xlim = box$x, ylim = box$y, xlab = NA, ylab = NA)
draw(ell, col = "seaShell", border = "blue")
abline(v = box$x, lty = 2); abline(h = box$y, lty = 2)

## -----
## Method `Ellipse$randomPoints`
## -----

ell <- Ellipse$new(c(1,1), 5, 2, 30)
pts <- ell$randomPoints(100)
plot(NULL, type="n", asp=1, xlim = c(-4,6), ylim = c(-2,4),
      xlab = NA, ylab = NA)
draw(ell, lwd = 2)
points(pts, pch = 19, col = "blue")
```

EllipseEquationFromFivePoints

Ellipse equation from five points

Description

The coefficients of the implicit equation of an ellipse from five points on this ellipse.

Usage

```
EllipseEquationFromFivePoints(P1, P2, P3, P4, P5)
```

Arguments

P1, P2, P3, P4, P5
the five points

Details

The implicit equation of the ellipse is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. This function returns A, B, C, D, E and F.

Value

A named numeric vector.

Examples

```
e11 <- Ellipse$new(c(2,3), 5, 4, 30)
set.seed(666)
pts <- e11$randomPoints(5, "on")
cf1 <- EllipseEquationFromFivePoints(pts[1,],pts[2,],pts[3,],pts[4,],pts[5,])
cf2 <- e11$equation() # should be the same up to a multiplicative factor
all.equal(cf1/cf1["F"], cf2/cf2["F"])
```

EllipseFromCenterAndMatrix

Ellipse from center and matrix

Description

Returns the ellipse of equation $t(X\text{-center}) \%*\% S \%*\% (X\text{-center}) = 1$.

Usage

```
EllipseFromCenterAndMatrix(center, S)
```

Arguments

center	a point, the center of the ellipse
S	a positive symmetric matrix

Value

An Ellipse object.

Examples

```
e11 <- Ellipse$new(c(2,3), 4, 2, 20)
S <- e11$matrix()
EllipseFromCenterAndMatrix(e11$center, S)
```

EllipseFromEquation *Ellipse from its implicit equation*

Description

Return an ellipse from the coefficients of its implicit equation.

Usage

```
EllipseFromEquation(A, B, C, D, E, F)
```

Arguments

A, B, C, D, E, F the coefficients of the equation

Details

The implicit equation of the ellipse is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. This function returns the ellipse given A, B, C, D, E and F.

Value

An Ellipse object.

Examples

```
e11 <- Ellipse$new(c(2,3), 5, 4, 30)
cf <- e11$equation()
e112 <- EllipseFromEquation(cf[1], cf[2], cf[3], cf[4], cf[5], cf[6])
e11$isEqual(e112)
```

EllipseFromFivePoints *Ellipse from five points*

Description

Return an ellipse from five given points on this ellipse.

Usage

```
EllipseFromFivePoints(P1, P2, P3, P4, P5)
```

Arguments

P1, P2, P3, P4, P5
the five points

Value

An Ellipse object.

Examples

```
e11 <- Ellipse$new(c(2,3), 5, 4, 30)
set.seed(666)
pts <- e11$randomPoints(5, "on")
e112 <- EllipseFromFivePoints(pts[1,],pts[2,],pts[3,],pts[4,],pts[5,])
e11$isEqual(e112)
```

EllipseFromFociAndOnePoint

Ellipse from foci and one point

Description

Derive the ellipse with given foci and one point on the boundary.

Usage

```
EllipseFromFociAndOnePoint(F1, F2, P)
```

Arguments

F1, F2	points, the foci
P	a point on the boundary of the ellipse

Value

An Ellipse object.

EllipseFromThreeBoundaryPoints

Smallest ellipse that passes through three boundary points

Description

Returns the smallest area ellipse which passes through three given boundary points.

Usage

```
EllipseFromThreeBoundaryPoints(P1, P2, P3)
```

Arguments

P1, P2, P3	three non-collinear points
------------	----------------------------

Value

An Ellipse object.

Examples

```
P1 <- c(-1,0); P2 <- c(0, 2); P3 <- c(3,0)
ell <- EllipseFromThreeBoundaryPoints(P1, P2, P3)
ell$includes(P1); ell$includes(P2); ell$includes(P3)
```

`EllipticalArc`*R6 class representing an elliptical arc*

Description

An arc is given by an ellipse (Ellipse object), a starting angle and an ending angle. They are respectively named `ell`, `alpha1` and `alpha2`.

Active bindings

`ell` get or set the ellipse

`alpha1` get or set the starting angle

`alpha2` get or set the ending angle

`degrees` get or set the degrees field

Methods**Public methods:**

- `EllipticalArc$new()`
- `EllipticalArc$print()`
- `EllipticalArc$startingPoint()`
- `EllipticalArc$endingPoint()`
- `EllipticalArc$isEqual()`
- `EllipticalArc$complementaryArc()`
- `EllipticalArc$path()`
- `EllipticalArc$length()`
- `EllipticalArc$clone()`

Method `new()`: Create a new `EllipticalArc` object.

Usage:

```
EllipticalArc$new(ell, alpha1, alpha2, degrees = TRUE)
```

Arguments:

`ell` the ellipse

`alpha1` the starting angle

`alpha2` the ending angle

`degrees` logical, whether `alpha1` and `alpha2` are given in degrees

Returns: A new `EllipticalArc` object.

Examples:

```
ell <- Ellipse$new(c(-4,0), 4, 2.5, 140)
EllipticalArc$new(ell, 45, 90)
```

Method `print()`: Show instance of an `EllipticalArc` object.

Usage:

```
EllipticalArc$print(...)
```

Arguments:

... ignored

Method `startingPoint()`: Starting point of the reference elliptical arc.

Usage:

```
EllipticalArc$startingPoint()
```

Method `endingPoint()`: Ending point of the reference elliptical arc.

Usage:

```
EllipticalArc$endingPoint()
```

Method `isEqual()`: Check whether the reference elliptical arc equals another elliptical arc.

Usage:

```
EllipticalArc$isEqual(arc)
```

Arguments:

arc an EllipticalArc object

Method `complementaryArc()`: Complementary elliptical arc of the reference elliptical arc.

Usage:

```
EllipticalArc$complementaryArc()
```

Examples:

```
ell <- Ellipse$new(c(-4,0), 4, 2.5, 140)
arc <- EllipticalArc$new(ell, 30, 60)
plot(NULL, type = "n", asp = 1, xlim = c(-8,0), ylim = c(-3.2,3.2),
      xlab = NA, ylab = NA)
draw(arc, lwd = 3, col = "red")
draw(arc$complementaryArc(), lwd = 3, col = "green")
```

Method `path()`: The reference elliptical arc as a path.

Usage:

```
EllipticalArc$path(npoints = 100L)
```

Arguments:

npoints number of points of the path

Returns: A matrix with two columns x and y of length npoints.

Method `length()`: The length of the elliptical arc.

Usage:

```
EllipticalArc$length()
```

Returns: A number, the arc length.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
EllipticalArc$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

Examples

```
## -----
## Method `EllipticalArc$new`
## -----

ell <- Ellipse$new(c(-4,0), 4, 2.5, 140)
EllipticalArc$new(ell, 45, 90)

## -----
## Method `EllipticalArc$complementaryArc`
## -----

ell <- Ellipse$new(c(-4,0), 4, 2.5, 140)
arc <- EllipticalArc$new(ell, 30, 60)
plot(NULL, type = "n", asp = 1, xlim = c(-8,0), ylim = c(-3.2,3.2),
      xlab = NA, ylab = NA)
draw(arc, lwd = 3, col = "red")
draw(arc$complementaryArc(), lwd = 3, col = "green")
```

fitEllipse

Fit an ellipse

Description

Fit an ellipse to a set of points.

Usage

```
fitEllipse(points)
```

Arguments

points numeric matrix with two columns, one point per row

Value

An Ellipse object representing the fitted ellipse. The residual sum of squares is given in the RSS attribute.

Examples

```
library(PlaneGeometry)
# We add some noise to 30 points on an ellipse:
ell <- Ellipse$new(c(1, 1), 3, 2, 30)
set.seed(666L)
points <- ell$randomPoints(30, "on") + matrix(rnorm(30*2, sd = 0.2), ncol = 2)
# Now we fit an ellipse to these points:
ellFitted <- fitEllipse(points)
```

```
# let's draw all this stuff:
box <- ell$boundingbox()
plot(NULL, asp = 1, xlim = box$x, ylim = box$y, xlab = NA, ylab = NA)
draw(ell, border = "blue", lwd = 2)
points(points, pch = 19)
draw(ellFitted, border = "green", lwd = 2)
```

GaussianEllipse

Gaussian ellipse

Description

Return the ellipse equal to the highest *pdf* region of a bivariate Gaussian distribution with a given probability.

Usage

```
GaussianEllipse(mean, Sigma, p)
```

Arguments

mean	numeric vector of length 2, the mean of the bivariate Gaussian distribution; this is the center of the ellipse
Sigma	covariance matrix of the bivariate Gaussian distribution
p	desired probability level, a number between 0 and 1 (strictly)

Value

An Ellipse object.

Homothety

R6 class representing a homothety

Description

A homothety is given by a center and a scale factor.

Active bindings

center	get or set the center
scale	get or set the scale factor of the homothety

Methods**Public methods:**

- `Homothety$new()`
- `Homothety$print()`
- `Homothety$transform()`
- `Homothety$transformCircle()`
- `Homothety$getMatrix()`
- `Homothety$asAffine()`
- `Homothety$clone()`

Method `new()`: Create a new Homothety object.

Usage:

```
Homothety$new(center, scale)
```

Arguments:

`center` a point, the center of the homothety

`scale` a number, the scale factor of the homothety

Returns: A new Homothety object.

Examples:

```
Homothety$new(c(1,1), 2)
```

Method `print()`: Show instance of a Homothety object.

Usage:

```
Homothety$print(...)
```

Arguments:

`...` ignored

Method `transform()`: Transform a point or several points by the reference homothety.

Usage:

```
Homothety$transform(M)
```

Arguments:

`M` a point or a two-column matrix of points, one point per row

Method `transformCircle()`: Transform a circle by the reference homothety.

Usage:

```
Homothety$transformCircle(circ)
```

Arguments:

`circ` a Circle object

Returns: A Circle object.

Method `getMatrix()`: Augmented matrix of the homothety.

Usage:

```
Homothety$getMatrix()
```

Returns: A 3x3 matrix.

Examples:

```
H <- Homothety$new(c(1,1), 2)
P <- c(1,5)
H$transform(P)
H$getMatrix() %*% c(P,1)
```

Method `asAffine()`: Convert the reference homothety to an Affine object.

Usage:

```
Homothety$asAffine()
```

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Homothety$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## -----
## Method `Homothety$new`
## -----

Homothety$new(c(1,1), 2)

## -----
## Method `Homothety$getMatrix`
## -----

H <- Homothety$new(c(1,1), 2)
P <- c(1,5)
H$transform(P)
H$getMatrix() %*% c(P,1)
```

```
intersectionCircleCircle
```

Intersection of two circles

Description

Return the intersection of two circles.

Usage

```
intersectionCircleCircle(circ1, circ2, epsilon = sqrt(.Machine$double.eps))
```

Arguments

circ1, circ2 two Circle objects
 epsilon a small positive number used for the numerical accuracy

Value

NULL if there is no intersection, a point if the circles touch, a list of two points if the circles meet at two points, a circle if the two circles are identical.

intersectionCircleLine
Intersection of a circle and a line

Description

Return the intersection of a circle and a line.

Usage

```
intersectionCircleLine(circ, line, strict = FALSE)
```

Arguments

circ a Circle object
 line a Line object
 strict logical, whether to take into account line\$extendA and line\$extendB if they are not both TRUE

Value

NULL if there is no intersection; a point if the infinite line is tangent to the circle, or NULL if strict=TRUE and the point is not on the line (segment or half-line); a list of two points if the circle and the infinite line meet at two points, when strict=FALSE; if strict=TRUE and the line is a segment or a half-line, this can return NULL or a single point.

Examples

```
circ <- Circle$new(c(1,1), 2)
line <- Line$new(c(2,-2), c(1,2), FALSE, FALSE)
intersectionCircleLine(circ, line)
intersectionCircleLine(circ, line, strict = TRUE)
```

 intersectionEllipseLine

Intersection of an ellipse and a line

Description

Return the intersection of an ellipse and a line.

Usage

```
intersectionEllipseLine(ell, line, strict = FALSE)
```

Arguments

ell	an Ellipse object or a Circle object
line	a Line object
strict	logical, whether to take into account line\$extendA and line\$extendB if they are not both TRUE

Value

NULL if there is no intersection; a point if the infinite line is tangent to the ellipse, or NULL if strict=TRUE and the point is not on the line (segment or half-line); a list of two points if the ellipse and the infinite line meet at two points, when strict=FALSE; if strict=TRUE and the line is a segment or a half-line, this can return NULL or a single point.

Examples

```
ell <- Ellipse$new(c(1,1), 5, 1, 30)
line <- Line$new(c(2,-2), c(0,4))
( Is <- intersectionEllipseLine(ell, line) )
ell$includes(Is$I1); ell$includes(Is$I2)
```

 intersectionLineLine *Intersection of two lines*

Description

Return the intersection of two lines.

Usage

```
intersectionLineLine(line1, line2, strict = FALSE)
```

Arguments

line1, line2	two Line objects
strict	logical, whether to take into account the extensions of the lines (extendA and extendB)

Value

If `strict = FALSE` this returns either a point, or NULL if the lines are parallel, or a bi-infinite line if the two lines coincide. If `strict = TRUE`, this can also return a half-infinite line or a segment.

Inversion	<i>R6 class representing an inversion</i>
-----------	---

Description

An inversion is given by a pole (a point) and a power (a number, possibly negative, but not zero).

Active bindings

pole	get or set the pole
power	get or set the power

Methods**Public methods:**

- `Inversion$new()`
- `Inversion$print()`
- `Inversion$invert()`
- `Inversion$transform()`
- `Inversion$invertCircle()`
- `Inversion$transformCircle()`
- `Inversion$invertLine()`
- `Inversion$transformLine()`
- `Inversion$invertGcircle()`
- `Inversion$compose()`
- `Inversion$clone()`

Method `new()`: Create a new Inversion object.

Usage:

`Inversion$new(pole, power)`

Arguments:

pole the pole

power the power

Returns: A new Inversion object.

Method print(): Show instance of an inversion object.

Usage:

```
Inversion$print(...)
```

Arguments:

... ignored

Examples:

```
Inversion$new(c(0,0), 2)
```

Method invert(): Inversion of a point.

Usage:

```
Inversion$invert(M)
```

Arguments:

M a point or Inf

Returns: A point or Inf, the image of M.

Method transform(): An alias of invert.

Usage:

```
Inversion$transform(M)
```

Arguments:

M a point or Inf

Returns: A point or Inf, the image of M.

Method invertCircle(): Inversion of a circle.

Usage:

```
Inversion$invertCircle(circ)
```

Arguments:

circ a Circle object

Returns: A Circle object or a Line object.

Examples:

```
# A Pappus chain
# https://www.cut-the-knot.org/Curriculum/Geometry/InversionInArbelos.shtml
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type = "n", asp = 1, xlim = c(0,6), ylim = c(-4,4),
      xlab = NA, ylab = NA, axes = FALSE)
A <- c(0,0); B <- c(6,0)
ABsqr <- c(crossprod(A-B))
iota <- Inversion$new(A, ABsqr)
C <- iota$invert(c(8,0))
Sigma1 <- Circle$new((A+B)/2, sqrt(ABsqr)/2)
Sigma2 <- Circle$new((A+C)/2, sqrt(c(crossprod(A-C)))/2)
draw(Sigma1); draw(Sigma2)
```



```

circ0 <- Circle$new(c(7,0), 1)
iotacirc0 <- iota$invertCircle(circ0)
draw(iotacirc0)
for(i in 1:6){
  circ <- circ0$translate(c(0,2*i))
  iotacirc <- iota$invertCircle(circ)
  draw(iotacirc)
  circ <- circ0$translate(c(0,-2*i))
  iotacirc <- iota$invertCircle(circ)
  draw(iotacirc)
}
par(opar)

```

Method transformCircle(): An alias of invertCircle.

Usage:

```
Inversion$transformCircle(circ)
```

Arguments:

circ a Circle object

Returns: A Circle object or a Line object.

Method invertLine(): Inversion of a line.

Usage:

```
Inversion$invertLine(line)
```

Arguments:

line a Line object

Returns: A Circle object or a Line object.

Method transformLine(): An alias of invertLine.

Usage:

```
Inversion$transformLine(line)
```

Arguments:

line a Line object

Returns: A Circle object or a Line object.

Method invertGcircle(): Inversion of a generalized circle (i.e. a circle or a line).

Usage:

```
Inversion$invertGcircle(gcircle)
```

Arguments:

gcircle a Circle object or a Line object

Returns: A Circle object or a Line object.

Method compose(): Compose the reference inversion with another inversion. The result is a Möbius transformation.

Usage:

```
Inversion$compose(iota1, left = TRUE)
```

Arguments:

`iota1` an Inversion object

`left` logical, whether to compose at left or at right (i.e. returns $iota1 \circ iota0$ or $iota0 \circ iota1$)

Returns: A Mobius object.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Inversion$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

See Also

[inversionSwappingTwoCircles](#), [inversionFixingTwoCircles](#), [inversionFixingThreeCircles](#) to create some inversions.

Examples

```
## -----
## Method `Inversion$print`
## -----

Inversion$new(c(0,0), 2)

## -----
## Method `Inversion$invertCircle`
## -----

# A Pappus chain
# https://www.cut-the-knot.org/Curriculum/Geometry/InversionInArbelos.shtml
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type = "n", asp = 1, xlim = c(0,6), ylim = c(-4,4),
     xlab = NA, ylab = NA, axes = FALSE)
A <- c(0,0); B <- c(6,0)
ABsqr <- c(crossprod(A-B))
iota <- Inversion$new(A, ABsqr)
C <- iota$invert(c(8,0))
Sigma1 <- Circle$new((A+B)/2, sqrt(ABsqr)/2)
Sigma2 <- Circle$new((A+C)/2, sqrt(c(crossprod(A-C)))/2)
draw(Sigma1); draw(Sigma2)
circ0 <- Circle$new(c(7,0), 1)
iotacirc0 <- iota$invertCircle(circ0)
draw(iotacirc0)
for(i in 1:6){
  circ <- circ0$translate(c(0,2*i))
```

```
iotacirc <- iota$invertCircle(circ)
draw(iotacirc)
circ <- circ0$translate(c(0,-2*i))
iotacirc <- iota$invertCircle(circ)
draw(iotacirc)
}
par(opar)
```

inversionFixingThreeCircles

Inversion fixing three circles

Description

Return the inversion which lets invariant three given circles.

Usage

```
inversionFixingThreeCircles(circ1, circ2, circ3)
```

Arguments

```
circ1, circ2, circ3
      Circle objects
```

Value

An Inversion object, which lets each of circ1, circ2 and circ3 invariant.

inversionFixingTwoCircles

Inversion fixing two circles

Description

Return the inversion which lets invariant two given circles.

Usage

```
inversionFixingTwoCircles(circ1, circ2)
```

Arguments

```
circ1, circ2    Circle objects
```

Value

An Inversion object, which maps circ1 to circ2 and circ2 to circ1.

inversionFromCircle *Inversion on a circle*

Description

Return the inversion on a given circle.

Usage

```
inversionFromCircle(circ)
```

Arguments

circ a Circle object

Value

An Inversion object

inversionKeepingCircle
 Inversion keeping a circle unchanged

Description

Return an inversion with a given pole which keeps a given circle unchanged.

Usage

```
inversionKeepingCircle(pole, circ)
```

Arguments

pole inversion pole, a point
circ a Circle object

Value

An Inversion object.

Examples

```
circ <- Circle$new(c(4,3), 2)  
iota <- inversionKeepingCircle(c(1,2), circ)  
iota$transformCircle(circ)
```

 inversionSwappingTwoCircles

Inversion swapping two circles

Description

Return the inversion which swaps two given circles.

Usage

```
inversionSwappingTwoCircles(circ1, circ2, positive = TRUE)
```

Arguments

circ1, circ2	Circle objects
positive	logical, whether the sign of the desired inversion power must be positive or negative

Value

An Inversion object, which maps circ1 to circ2 and circ2 to circ1, except in the case when circ1 and circ2 are congruent and tangent: in this case a Reflection object is returned (a reflection is an inversion on a line).

 Line

R6 class representing a line

Description

A line is given by two distinct points, named A and B, and two logical values extendA and extendB, indicating whether the line must be extended beyond A and B respectively. Depending on extendA and extendB, the line is an infinite line, a half-line, or a segment.

Active bindings

A	get or set the point A
B	get or set the point B
extendA	get or set extendA
extendB	get or set extendB

Methods**Public methods:**

- `Line$new()`
- `Line$print()`
- `Line$length()`
- `Line$directionAndOffset()`
- `Line$isEqual()`
- `Line$isParallel()`
- `Line$isPerpendicular()`
- `Line$includes()`
- `Line$perpendicular()`
- `Line$parallel()`
- `Line$projection()`
- `Line$distance()`
- `Line$reflection()`
- `Line$rotate()`
- `Line$translate()`
- `Line$invert()`
- `Line$clone()`

Method `new()`: Create a new Line object.

Usage:

```
Line$new(A, B, extendA = TRUE, extendB = TRUE)
```

Arguments:

A, B points

extendA, extendB logical values

Returns: A new Line object.

Examples:

```
l <- Line$new(c(1,1), c(1.5,1.5), FALSE, TRUE)
l
l$A
l$A <- c(0,0)
l
```

Method `print()`: Show instance of a line object.

Usage:

```
Line$print(...)
```

Arguments:

... ignored

Examples:

```
Line$new(c(0,0), c(1,0), FALSE, TRUE)
```

Method length(): Segment length, returns the length of the segment joining the two point defining the line.

Usage:

Line\$length()

Method directionAndOffset(): Direction (angle between 0 and 2pi) and offset (positive number) of the reference line.

Usage:

Line\$directionAndOffset()

Details: The equation of the line is $\cos(\theta)x + \sin(\theta)y = d$ where θ is the direction and d is the offset.

Method isEqual(): Check whether the reference line equals a given line, without taking into account extendA and extendB.

Usage:

Line\$isEqual(line)

Arguments:

line a Line object

Returns: TRUE or FALSE.

Method isParallel(): Check whether the reference line is parallel to a given line.

Usage:

Line\$isParallel(line)

Arguments:

line a Line object

Returns: TRUE or FALSE.

Method isPerpendicular(): Check whether the reference line is perpendicular to a given line.

Usage:

Line\$isPerpendicular(line)

Arguments:

line a Line object

Returns: TRUE or FALSE.

Method includes(): Whether a point belongs to the reference line.

Usage:

Line\$includes(M, strict = FALSE, checkCollinear = TRUE)

Arguments:

M the point for which we want to test whether it belongs to the line

strict logical, whether to take into account extendA and extendB

checkCollinear logical, whether to check the collinearity of A, B, M; set to FALSE only if you are sure that M is on the line (AB) (if you use strict=TRUE)

Returns: TRUE or FALSE.

Examples:

```
A <- c(0,0); B <- c(1,2); M <- c(3,6)
l <- Line$new(A, B, FALSE, FALSE)
l$includes(M, strict = TRUE)
```

Method `perpendicular()`: Perpendicular line passing through a given point.

Usage:

```
Line$perpendicular(M, extendH = FALSE, extendM = TRUE)
```

Arguments:

M the point through which the perpendicular passes.

extendH logical, whether to extend the perpendicular line beyond the meeting point

extendM logical, whether to extend the perpendicular line beyond the point M

Returns: A Line object; its two points are the meeting point and the point M.

Method `parallel()`: Parallel to the reference line passing through a given point.

Usage:

```
Line$parallel(M)
```

Arguments:

M a point

Returns: A Line object.

Method `projection()`: Orthogonal projection of a point to the reference line.

Usage:

```
Line$projection(M)
```

Arguments:

M a point

Returns: A point.

Method `distance()`: Distance from a point to the reference line.

Usage:

```
Line$distance(M)
```

Arguments:

M a point

Returns: A positive number.

Method `reflection()`: Reflection of a point with respect to the reference line.

Usage:

```
Line$reflection(M)
```

Arguments:

M a point

Returns: A point.

Method rotate(): Rotate the reference line.

Usage:

```
Line$rotate(alpha, 0, degrees = TRUE)
```

Arguments:

alpha angle of rotation

0 center of rotation

degrees logical, whether alpha is given in degrees

Returns: A Line object.

Method translate(): Translate the reference line.

Usage:

```
Line$translate(v)
```

Arguments:

v the vector of translation

Returns: A Line object.

Method invert(): Invert the reference line.

Usage:

```
Line$invert(inversion)
```

Arguments:

inversion an Inversion object

Returns: A Circle object or a Line object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

```
Line$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

Examples

```
## -----
## Method `Line$new`
## -----

l <- Line$new(c(1,1), c(1.5,1.5), FALSE, TRUE)
l
l$a
l$a <- c(0,0)
l
## -----
```

```
## Method `Line$print`
## -----

Line$new(c(0,0), c(1,0), FALSE, TRUE)

## -----
## Method `Line$includes`
## -----

A <- c(0,0); B <- c(1,2); M <- c(3,6)
l <- Line$new(A, B, FALSE, FALSE)
l$includes(M, strict = TRUE)
```

LownerJohnEllipse *Löwner-John ellipse (ellipse hull)*

Description

Minimum area ellipse containing a set of points.

Usage

```
LownerJohnEllipse(pts)
```

Arguments

pts	the points in a two-columns matrix (one point per row); at least three distinct points
-----	--

Value

An Ellipse object.

Examples

```
pts <- cbind(rnorm(30, sd=2), rnorm(30))
ell <- LownerJohnEllipse(pts)
box <- ell$boundingbox()
plot(NULL, asp = 1, xlim = box$x, ylim = box$y, xlab = NA, ylab = NA)
draw(ell, col = "seaShell")
points(pts, pch = 19)
all(apply(pts, 1, ell$contains)) # should be TRUE
```

midCircles	<i>Mid-circle(s)</i>
------------	----------------------

Description

Return the mid-circle(s) of two circles.

Usage

```
midCircles(circ1, circ2)
```

Arguments

circ1, circ2 Circle objects

Details

A mid-circle of two circles is a generalized circle (i.e. a circle or a line) such that the inversion on this circle swaps the two circles. The case of a line appears only when the two circles have equal radii.

Value

A Circle object, or a Line object, or a list of two such objects.

See Also

[inversionSwappingTwoCircles](#)

Examples

```
circ1 <- Circle$new(c(5,4),2)
circ2 <- Circle$new(c(6,4),1)
midcircle <- midCircles(circ1, circ2)
inversionFromCircle(midcircle)
inversionSwappingTwoCircles(circ1, circ2)
```

Mobius

R6 class representing a Möbius transformation.

Description

A Möbius transformation is given by a matrix of complex numbers with non-null determinant.

Active bindings

- a get or set a
- b get or set b
- c get or set c
- d get or set d

Methods

Public methods:

- `Mobius$new()`
- `Mobius$print()`
- `Mobius$getM()`
- `Mobius$compose()`
- `Mobius$inverse()`
- `Mobius$power()`
- `Mobius$gpowers()`
- `Mobius$transform()`
- `Mobius$fixedPoints()`
- `Mobius$transformCircle()`
- `Mobius$transformLine()`
- `Mobius$transformGcircle()`
- `Mobius$clone()`

Method `new()`: Create a new Mobius object.

Usage:

`Mobius$new(M)`

Arguments:

M the matrix corresponding to the Möbius transformation

Returns: A new Mobius object.

Method `print()`: Show instance of a Mobius object.

Usage:

`Mobius$print(...)`

Arguments:

... ignored

Examples:

```
Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
```

Method `getM()`: Get the matrix corresponding to the Möbius transformation.

Usage:

```
Mobius$getM()
```

Method `compose()`: Compose the reference Möbius transformation with another Möbius transformation

Usage:

```
Mobius$compose(M1, left = TRUE)
```

Arguments:

M1 a Mobius object

left logical, whether to compose at left or at right (i.e. returns $M1 \circ M0$ or $M0 \circ M1$)

Returns: A Mobius object.

Method `inverse()`: Inverse of the reference Möbius transformation.

Usage:

```
Mobius$inverse()
```

Returns: A Mobius object.

Method `power()`: Power of the reference Möbius transformation.

Usage:

```
Mobius$power(k)
```

Arguments:

k an integer, possibly negative

Returns: The Möbius transformation M^k , where M is the reference Möbius transformation.

Method `gpower()`: Generalized power of the reference Möbius transformation.

Usage:

```
Mobius$gpower(k)
```

Arguments:

k a real number, possibly negative

Returns: A Mobius object, the generalized k-th power of the reference Möbius transformation.

Examples:

```
M <- Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
```

```
Mroot <- M$gpower(1/2)
```

```
Mroot$compose(Mroot) # should be M
```

Method `transform()`: Transformation of a point by the reference Möbius transformation.

Usage:

```
Mobius$transform(M)
```

Arguments:

M a point or Inf

Returns: A point or Inf, the image of M.

Examples:

```
Mob <- Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
```

```
Mob$transform(c(1,1))
```

```
Mob$transform(Inf)
```

Method `fixedPoints()`: Returns the fixed points of the reference Möbius transformation.

Usage:

```
Mobius$fixedPoints()
```

Returns: One point, or a list of two points, or a message in the case when the transformation is the identity map.

Method `transformCircle()`: Transformation of a circle by the reference Möbius transformation.

Usage:

```
Mobius$transformCircle(circ)
```

Arguments:

circ a Circle object

Returns: A Circle object or a Line object.

Method `transformLine()`: Transformation of a line by the reference Möbius transformation.

Usage:

```
Mobius$transformLine(line)
```

Arguments:

line a Line object

Returns: A Circle object or a Line object.

Method `transformGcircle()`: Transformation of a generalized circle (i.e. a circle or a line) by the reference Möbius transformation.

Usage:

```
Mobius$transformGcircle(gcirc)
```

Arguments:

gcirc a Circle object or a Line object

Returns: A Circle object or a Line object.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Mobius$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

See Also

[MobiusMappingThreePoints](#) to create a Möbius transformation, and also the `compose` method of the [Inversion R6](#) class.

Examples

```
## -----
## Method `Mobius$print`
## -----

Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))

## -----
## Method `Mobius$gpower`
## -----

M <- Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
Mroot <- M$gpower(1/2)
Mroot$compose(Mroot) # should be M

## -----
## Method `Mobius$transform`
## -----

Mob <- Mobius$new(rbind(c(1+1i,2),c(0,3-2i)))
Mob$transform(c(1,1))
Mob$transform(Inf)
```

MobiusMappingCircle *Möbius transformation mapping a given circle to a given circle*

Description

Returns a Möbius transformation mapping a given circle to another given circle.

Usage

```
MobiusMappingCircle(circ1, circ2)
```

Arguments

circ1, circ2 Circle objects

Value

A Möbius transformation which maps circ1 to circ2.

Examples

```
library(PlaneGeometry)
C1 <- Circle$new(c(0, 0), 1)
C2 <- Circle$new(c(1, 2), 3)
M <- MobiusMappingCircle(C1, C2)
C3 <- M$transformCircle(C1)
C3$isEqual(C2)
```

MobiusMappingThreePoints

Möbius transformation mapping three given points to three given points

Description

Return a Möbius transformation which sends P1 to Q1, P2 to Q2 and P3 to Q3.

Usage

```
MobiusMappingThreePoints(P1, P2, P3, Q1, Q2, Q3)
```

Arguments

P1, P2, P3	three distinct points, Inf allowed
Q1, Q2, Q3	three distinct points, Inf allowed

Value

A Mobius object.

MobiusSwappingTwoPoints

Möbius transformation swapping two given points

Description

Return a Möbius transformation which sends A to B and B to A.

Usage

```
MobiusSwappingTwoPoints(A, B)
```

Arguments

A, B	two distinct points, Inf not allowed
------	--------------------------------------

Value

A Mobius object.

Projection	<i>R6 class representing a projection</i>
------------	---

Description

A projection on a line D parallel to another line Delta is given by the line of projection (D) and the directrix line (Delta).

Active bindings

D get or set the projection line
 Delta get or set the directrix line

Methods**Public methods:**

- [Projection\\$new\(\)](#)
- [Projection\\$print\(\)](#)
- [Projection\\$project\(\)](#)
- [Projection\\$transform\(\)](#)
- [Projection\\$getMatrix\(\)](#)
- [Projection\\$asAffine\(\)](#)
- [Projection\\$clone\(\)](#)

Method `new()`: Create a new Projection object.

Usage:

```
Projection$new(D, Delta)
```

Arguments:

D, Delta two Line objects such that the two lines meet (not parallel); or Delta = NULL for orthogonal projection onto D

Returns: A new Projection object.

Examples:

```
D <- Line$new(c(1,1), c(5,5))
Delta <- Line$new(c(0,0), c(3,4))
Projection$new(D, Delta)
```

Method `print()`: Show instance of a projection object.

Usage:

```
Projection$print(...)
```

Arguments:

... ignored

Method `project()`: Project a point.

Usage:

```
Projection$project(M)
```

Arguments:

M a point

Examples:

```
D <- Line$new(c(1,1), c(5,5))
Delta <- Line$new(c(0,0), c(3,4))
P <- Projection$new(D, Delta)
M <- c(1,3)
Mprime <- P$project(M)
D$includes(Mprime) # should be TRUE
Delta$isParallel(Line$new(M, Mprime)) # should be TRUE
```

Method `transform()`: An alias of `project`.

Usage:

```
Projection$transform(M)
```

Arguments:

M a point

Method `getMatrix()`: Augmented matrix of the projection.

Usage:

```
Projection$getMatrix()
```

Returns: A 3x3 matrix.

Examples:

```
P <- Projection$new(Line$new(c(2,2), c(4,5)), Line$new(c(0,0), c(1,1)))
M <- c(1,5)
P$project(M)
P$getMatrix() %*% c(M,1)
```

Method `asAffine()`: Convert the reference projection to an Affine object.

Usage:

```
Projection$asAffine()
```

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Projection$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

Note

For an orthogonal projection, you can use the projection method of the [Line](#) R6 class.

Examples

```

## -----
## Method `Projection$new`
## -----

D <- Line$new(c(1,1), c(5,5))
Delta <- Line$new(c(0,0), c(3,4))
Projection$new(D, Delta)

## -----
## Method `Projection$project`
## -----

D <- Line$new(c(1,1), c(5,5))
Delta <- Line$new(c(0,0), c(3,4))
P <- Projection$new(D, Delta)
M <- c(1,3)
Mprime <- P$project(M)
D$includes(Mprime) # should be TRUE
Delta$isParallel(Line$new(M, Mprime)) # should be TRUE

## -----
## Method `Projection$getMatrix`
## -----

P <- Projection$new(Line$new(c(2,2), c(4,5)), Line$new(c(0,0), c(1,1)))
M <- c(1,5)
P$project(M)
P$getMatrix() %*% c(M,1)

```

radicalCenter

Radical center

Description

Returns the radical center of three circles.

Usage

```
radicalCenter(circ1, circ2, circ3)
```

Arguments

```
circ1, circ2, circ3
      Circle objects
```

Value

A point.

Reflection

R6 class representing a reflection

Description

A reflection is given by a line.

Active bindings

`line` get or set the line of the reflection

Methods

Public methods:

- `Reflection$new()`
- `Reflection$print()`
- `Reflection$reflect()`
- `Reflection$transform()`
- `Reflection$reflectCircle()`
- `Reflection$transformCircle()`
- `Reflection$reflectLine()`
- `Reflection$transformLine()`
- `Reflection$getMatrix()`
- `Reflection$asAffine()`
- `Reflection$clone()`

Method `new()`: Create a new Reflection object.

Usage:

```
Reflection$new(line)
```

Arguments:

`line` a Line object

Returns: A new Reflection object.

Examples:

```
l <- Line$new(c(1,1), c(1.5,1.5), FALSE, TRUE)
Reflection$new(l)
```

Method `print()`: Show instance of a reflection object.

Usage:

```
Reflection$print(...)
```

Arguments:

`...` ignored

Method `reflect()`: Reflect a point.

Usage:

Reflection\$reflect(M)

Arguments:

M a point, Inf allowed

Method transform(): An alias of reflect.

Usage:

Reflection\$transform(M)

Arguments:

M a point, Inf allowed

Method reflectCircle(): Reflect a circle.

Usage:

Reflection\$reflectCircle(circ)

Arguments:

circ a Circle object

Returns: A Circle object.

Method transformCircle(): An alias of reflectCircle.

Usage:

Reflection\$transformCircle(circ)

Arguments:

circ a Circle object

Returns: A Circle object.

Method reflectLine(): Reflect a line.

Usage:

Reflection\$reflectLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method transformLine(): An alias of reflectLine.

Usage:

Reflection\$transformLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method getMatrix(): Augmented matrix of the reflection.

Usage:

Reflection\$getMatrix()

Returns: A 3x3 matrix.

Examples:

```
R <- Reflection$new(Line$new(c(2,2), c(4,5)))
P <- c(1,5)
R$reflect(P)
R$getMatrix() %*% c(P,1)
```

Method `asAffine()`: Convert the reference reflection to an Affine object.

Usage:

```
Reflection$asAffine()
```

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Reflection$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## -----
## Method `Reflection$new`
## -----

l <- Line$new(c(1,1), c(1.5,1.5), FALSE, TRUE)
Reflection$new(l)

## -----
## Method `Reflection$getMatrix`
## -----

R <- Reflection$new(Line$new(c(2,2), c(4,5)))
P <- c(1,5)
R$reflect(P)
R$getMatrix() %*% c(P,1)
```

Rotation

R6 class representing a rotation

Description

A rotation is given by an angle (`theta`) and a center.

Active bindings

`theta` get or set the angle of the rotation

`center` get or set the center

`degrees` get or set the degrees field

Methods**Public methods:**

- `Rotation$new()`
- `Rotation$print()`
- `Rotation$rotate()`
- `Rotation$transform()`
- `Rotation$rotateCircle()`
- `Rotation$transformCircle()`
- `Rotation$rotateEllipse()`
- `Rotation$transformEllipse()`
- `Rotation$rotateLine()`
- `Rotation$transformLine()`
- `Rotation$getMatrix()`
- `Rotation$asAffine()`
- `Rotation$clone()`

Method `new()`: Create a new Rotation object.

Usage:

```
Rotation$new(theta, center, degrees = TRUE)
```

Arguments:

`theta` a number, the angle of the rotation

`center` a point, the center of the rotation

`degrees` logical, whether theta is given in degrees

Returns: A new Rotation object.

Examples:

```
Rotation$new(60, c(1,1))
```

Method `print()`: Show instance of a Rotation object.

Usage:

```
Rotation$print(...)
```

Arguments:

... ignored

Method `rotate()`: Rotate a point or several points.

Usage:

```
Rotation$rotate(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Method `transform()`: An alias of rotate.

Usage:

```
Rotation$transform(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Method rotateCircle(): Rotate a circle.

Usage:

Rotation\$rotateCircle(circ)

Arguments:

circ a Circle object

Returns: A Circle object.

Method transformCircle(): An alias of rotateCircle.

Usage:

Rotation\$transformCircle(circ)

Arguments:

circ a Circle object

Returns: A Circle object.

Method rotateEllipse(): Rotate an ellipse.

Usage:

Rotation\$rotateEllipse(ell)

Arguments:

ell an Ellipse object

Returns: An Ellipse object.

Method transformEllipse(): An alias of rotateEllipse.

Usage:

Rotation\$transformEllipse(ell)

Arguments:

ell an Ellipse object

Returns: An Ellipse object.

Method rotateLine(): Rotate a line.

Usage:

Rotation\$rotateLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method transformLine(): An alias of rotateLine.

Usage:

Rotation\$transformLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method `getMatrix()`: Augmented matrix of the rotation.

Usage:

`Rotation$getMatrix()`

Returns: A 3x3 matrix.

Examples:

```
R <- Rotation$new(60, c(1,1))
P <- c(1,5)
R$rotate(P)
R$getMatrix() %*% c(P,1)
```

Method `asAffine()`: Convert the reference rotation to an Affine object.

Usage:

`Rotation$asAffine()`

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

`Rotation$clone(deep = FALSE)`

Arguments:

deep Whether to make a deep clone.

Examples

```
## -----
## Method `Rotation$new`
## -----

Rotation$new(60, c(1,1))

## -----
## Method `Rotation$getMatrix`
## -----

R <- Rotation$new(60, c(1,1))
P <- c(1,5)
R$rotate(P)
R$getMatrix() %*% c(P,1)
```

Scaling

R6 class representing a (non-uniform) scaling

Description

A (non-uniform) scaling is given by a center, a direction vector, and a scale factor.

Active bindings

`center` get or set the center

`direction` get or set the direction

`scale` get or set the scale factor

Methods

Public methods:

- `Scaling$new()`
- `Scaling$print()`
- `Scaling$transform()`
- `Scaling$getMatrix()`
- `Scaling$asAffine()`
- `Scaling$scaleCircle()`
- `Scaling$clone()`

Method `new()`: Create a new `Scaling` object.

Usage:

```
Scaling$new(center, direction, scale)
```

Arguments:

`center` a point, the center of the scaling

`direction` a vector, the direction of the scaling

`scale` a number, the scale factor

Returns: A new `Scaling` object.

Examples:

```
Scaling$new(c(1,1), c(1,3), 2)
```

Method `print()`: Show instance of a `Scaling` object.

Usage:

```
Scaling$print(...)
```

Arguments:

... ignored

Method `transform()`: Transform a point or several points by the reference scaling.

Usage:

```
Scaling$transform(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Method `getMatrix()`: Augmented matrix of the scaling.

Usage:

```
Scaling$getMatrix()
```

Returns: A 3x3 matrix.

Examples:

```
S <- Scaling$new(c(1,1), c(2,3), 2)
P <- c(1,5)
S$transform(P)
S$getMatrix() %*% c(P,1)
```

Method `asAffine()`: Convert the reference scaling to an Affine object.

Usage:

```
Scaling$asAffine()
```

Method `scaleCircle()`: Scale a circle. The result is an ellipse.

Usage:

```
Scaling$scaleCircle(circ)
```

Arguments:

circ a Circle object

Returns: An Ellipse object.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Scaling$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

References

R. Goldman, *An Integrated Introduction to Computer Graphics and Geometric Modeling*. CRC Press, 2009.

Examples

```
Q <- c(1,1); w <- c(1,3); s <- 2
S <- Scaling$new(Q, w, s)
# the center is mapped to itself:
S$transform(Q)
# any vector \code{u} parallel to the direction vector is mapped to \code{s*u}:
u <- 3*w
```

```

all.equal(s*u, S$transform(u) - S$transform(c(0,0)))
# any vector perpendicular to the direction vector is mapped to itself
wt <- 3*c(-w[2], w[1])
all.equal(wt, S$transform(wt) - S$transform(c(0,0)))

## -----
## Method `Scaling$new`
## -----

Scaling$new(c(1,1), c(1,3), 2)

## -----
## Method `Scaling$getMatrix`
## -----

S <- Scaling$new(c(1,1), c(2,3), 2)
P <- c(1,5)
S$transform(P)
S$getMatrix() %%% c(P,1)

```

ScalingXY

R6 class representing an axis-scaling

Description

An axis-scaling is given by a center, and two scale factors s_x and s_y , one for the x-axis and one for the y-axis.

Active bindings

center get or set the center
 s_x get or set the scale factor of the x-axis
 s_y get or set the scale factor of the y-axis

Methods

Public methods:

- `ScalingXY$new()`
- `ScalingXY$print()`
- `ScalingXY$transform()`
- `ScalingXY$getMatrix()`
- `ScalingXY$asAffine()`
- `ScalingXY$clone()`

Method `new()`: Create a new `ScalingXY` object.

Usage:

```
ScalingXY$new(center, sx, sy)
```

Arguments:

center a point, the center of the scaling

sx a number, the scale factor of the x-axis

sy a number, the scale factor of the y-axis

Returns: A new ScalingXY object.

Examples:

```
ScalingXY$new(c(1,1), 4, 2)
```

Method print(): Show instance of a ScalingXY object.

Usage:

```
ScalingXY$print(...)
```

Arguments:

... ignored

Method transform(): Transform a point or several points by the reference axis-scaling.

Usage:

```
ScalingXY$transform(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Returns: A point or a two-column matrix of points.

Method getMatrix(): Augmented matrix of the axis-scaling.

Usage:

```
ScalingXY$getMatrix()
```

Returns: A 3x3 matrix.

Examples:

```
S <- ScalingXY$new(c(1,1), 4, 2)
```

```
P <- c(1,5)
```

```
S$transform(P)
```

```
S$getMatrix() %*% c(P,1)
```

Method asAffine(): Convert the reference axis-scaling to an Affine object.

Usage:

```
ScalingXY$asAffine()
```

Method clone(): The objects of this class are cloneable with this method.

Usage:

```
ScalingXY$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

Examples

```
## -----
## Method `ScalingXY$new`
## -----

ScalingXY$new(c(1,1), 4, 2)

## -----
## Method `ScalingXY$getMatrix`
## -----

S <- ScalingXY$new(c(1,1), 4, 2)
P <- c(1,5)
S$transform(P)
S$getMatrix() %*% c(P,1)
```

Shear

R6 class representing a shear transformation

Description

A shear is given by a vertex, two perpendicular vectors, and an angle.

Active bindings

vertex get or set the vertex

vector get or set the first vector

ratio get or set the ratio between the length of vector and the length of the second vector, perpendicular to the first one

angle get or set the angle

degrees get or set the degrees field

Methods**Public methods:**

- [Shear\\$new\(\)](#)
- [Shear\\$print\(\)](#)
- [Shear\\$transform\(\)](#)
- [Shear\\$getMatrix\(\)](#)
- [Shear\\$asAffine\(\)](#)
- [Shear\\$clone\(\)](#)

Method `new()`: Create a new Shear object.

Usage:

```
Shear$new(vertex, vector, ratio, angle, degrees = TRUE)
```

Arguments:

vertex a point

vector a vector

ratio a positive number, the ratio between the length of vector and the length of the second vector, perpendicular to the first one

angle an angle strictly between -90 degrees and 90 degrees

degrees logical, whether angle is given in degrees

Returns: A new Shear object.

Examples:

```
Shear$new(c(1,1), c(1,3), 0.5, 30)
```

Method print(): Show instance of a Shear object.

Usage:

```
Shear$print(...)
```

Arguments:

... ignored

Method transform(): Transform a point or several points by the reference shear.

Usage:

```
Shear$transform(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Method getMatrix(): Augmented matrix of the shear.

Usage:

```
Shear$getMatrix()
```

Returns: A 3x3 matrix.

Examples:

```
S <- Shear$new(c(1,1), c(1,3), 0.5, 30)
```

```
S$getMatrix()
```

Method asAffine(): Convert the reference shear to an Affine object.

Usage:

```
Shear$asAffine()
```

Examples:

```
Shear$new(c(0,0), c(1,0), 1, atan(30), FALSE)$asAffine()
```

Method clone(): The objects of this class are cloneable with this method.

Usage:

```
Shear$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

References

R. Goldman, *An Integrated Introduction to Computer Graphics and Geometric Modeling*. CRC Press, 2009.

Examples

```
P <- c(0,0); w <- c(1,0); ratio <- 1; angle <- 45
shear <- Shear$new(P, w, ratio, angle)
wt <- ratio * c(-w[2], w[1])
Q <- P + w; R <- Q + wt; S <- P + wt
A <- shear$transform(P)
B <- shear$transform(Q)
C <- shear$transform(R)
D <- shear$transform(S)
plot(0, 0, type = "n", asp = 1, xlim = c(0,1), ylim = c(0,2))
lines(rbind(P,Q,R,S,P), lwd = 2) # unit square
lines(rbind(A,B,C,D,A), lwd = 2, col = "blue") # image by the shear
```

```
## -----
## Method `Shear$new`
## -----

Shear$new(c(1,1), c(1,3), 0.5, 30)

## -----
## Method `Shear$getMatrix`
## -----

S <- Shear$new(c(1,1), c(1,3), 0.5, 30)
S$getMatrix()

## -----
## Method `Shear$asAffine`
## -----

Shear$new(c(0,0), c(1,0), 1, atan(30), FALSE)$asAffine()
```

soddyCircle

Inner Soddy circle

Description

Inner Soddy circles associated to three circles.

Usage

```
soddyCircle(circ1, circ2, circ3)
```


Arguments

circ1, circ2, circ3
distinct circles

Value

A Circle object.

SteinerChain	<i>Steiner chain</i>
--------------	----------------------

Description

Return a Steiner chain of circles.

Usage

```
SteinerChain(c0, n, phi, shift, ellipse = FALSE)
```

Arguments

c0	exterior circle, a Circle object
n	number of circles, not including the inner circle; at least 3
phi	$-1 < \phi < 1$ controls the radii of the circles
shift	any number; it produces a kind of rotation around the inner circle; values between 0 and n cover all possibilities
ellipse	logical; the centers of the circles of the Steiner chain lie on an ellipse, and this ellipse is returned as an attribute if you set this argument to TRUE

Value

A list of n+1 Circle objects. The inner circle is stored at the last position.

Examples

```
c0 <- Circle$new(c(1,1), 3)
chain <- SteinerChain(c0, 5, 0.3, 0.5, ellipse = TRUE)
plot(0, 0, type = "n", asp = 1, xlim = c(-4,4), ylim = c(-4,4))
invisible(lapply(chain, draw, lwd = 2, border = "blue"))
draw(c0, lwd = 2)
draw(attr(chain, "ellipse"), lwd = 2, border = "red")
```

Translation

R6 class representing a translation

Description

A translation is given by a vector v .

Active bindings

v get or set the vector of translation

Methods

Public methods:

- [Translation\\$new\(\)](#)
- [Translation\\$print\(\)](#)
- [Translation\\$project\(\)](#)
- [Translation\\$transform\(\)](#)
- [Translation\\$translateLine\(\)](#)
- [Translation\\$transformLine\(\)](#)
- [Translation\\$translateEllipse\(\)](#)
- [Translation\\$transformEllipse\(\)](#)
- [Translation\\$getMatrix\(\)](#)
- [Translation\\$asAffine\(\)](#)
- [Translation\\$clone\(\)](#)

Method `new()`: Create a new Translation object.

Usage:

```
Translation$new(v)
```

Arguments:

v a numeric vector of length two, the vector of translation

Returns: A new Translation object.

Method `print()`: Show instance of a translation object.

Usage:

```
Translation$print(...)
```

Arguments:

... ignored

Method `project()`: Transform a point or several points by the reference translation.

Usage:

```
Translation$project(M)
```

Arguments:

M a point or a two-column matrix of points, one point per row

Method transform(): An alias of translate.

Usage:

Translation\$transform(M)

Arguments:

M a point or a two-column matrix of points, one point per row

Method translateLine(): Translate a line.

Usage:

Translation\$translateLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method transformLine(): An alias of translateLine.

Usage:

Translation\$transformLine(line)

Arguments:

line a Line object

Returns: A Line object.

Method translateEllipse(): Translate a circle or an ellipse.

Usage:

Translation\$translateEllipse(ell)

Arguments:

ell an Ellipse object or a Circle object

Returns: An Ellipse object or a Circle object.

Method transformEllipse(): An alias of translateEllipse.

Usage:

Translation\$transformEllipse(ell)

Arguments:

ell an Ellipse object or a Circle object

Returns: An Ellipse object or a Circle object.

Method getMatrix(): Augmented matrix of the translation.

Usage:

Translation\$getMatrix()

Returns: A 3x3 matrix.

Method `asAffine()`: Convert the reference translation to an Affine object.

Usage:

`Translation$asAffine()`

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

`Translation$clone(deep = FALSE)`

Arguments:

`deep` Whether to make a deep clone.

Triangle

R6 class representing a triangle

Description

A triangle has three vertices. They are named A, B, C.

Active bindings

A get or set the vertex A

B get or set the vertex B

C get or set the vertex C

Methods

Public methods:

- `Triangle$new()`
- `Triangle$print()`
- `Triangle$flatness()`
- `Triangle$a()`
- `Triangle$b()`
- `Triangle$c()`
- `Triangle$edges()`
- `Triangle$orientation()`
- `Triangle$contains()`
- `Triangle$isAcute()`
- `Triangle$angleA()`
- `Triangle$angleB()`
- `Triangle$angleC()`
- `Triangle$angles()`
- `Triangle$X175()`
- `Triangle$VeldkampIsoperimetricPoint()`

- Triangle\$centroid()
- Triangle\$orthocenter()
- Triangle\$area()
- Triangle\$incircle()
- Triangle\$inradius()
- Triangle\$incenter()
- Triangle\$excircles()
- Triangle\$excentralTriangle()
- Triangle\$BevanPoint()
- Triangle\$medialTriangle()
- Triangle\$orthicTriangle()
- Triangle\$incentralTriangle()
- Triangle\$NagelTriangle()
- Triangle\$NagelPoint()
- Triangle\$GergonneTriangle()
- Triangle\$GergonnePoint()
- Triangle\$tangentialTriangle()
- Triangle\$symmedialTriangle()
- Triangle\$symmedianPoint()
- Triangle\$circumcircle()
- Triangle\$circumcenter()
- Triangle\$circumradius()
- Triangle\$BrocardCircle()
- Triangle\$BrocardPoints()
- Triangle\$LemoineCircleI()
- Triangle\$LemoineCircleII()
- Triangle\$LemoineTriangle()
- Triangle\$LemoineCircleIII()
- Triangle\$ParryCircle()
- Triangle\$pedalTriangle()
- Triangle\$CevianTriangle()
- Triangle\$MalfattiCircles()
- Triangle\$AjimaMalfatti1()
- Triangle\$AjimaMalfatti2()
- Triangle\$equalDetourPoint()
- Triangle\$trilinearToPoint()
- Triangle\$pointToTrilinear()
- Triangle\$isogonalConjugate()
- Triangle\$rotate()
- Triangle\$translate()
- Triangle\$SteinerEllipse()
- Triangle\$SteinerInellipse()

- `Triangle$MandartInellipse()`
- `Triangle$randomPoints()`
- `Triangle$hexylTriangle()`
- `Triangle$clone()`

Method `new()`: Create a new Triangle object.

Usage:

```
Triangle$new(A, B, C)
```

Arguments:

A, B, C vertices

Returns: A new Triangle object.

Examples:

```
t <- Triangle$new(c(0,0), c(1,0), c(1,1))
t
t$C
t$C <- c(2,2)
t
```

Method `print()`: Show instance of a triangle object

Usage:

```
Triangle$print(...)
```

Arguments:

... ignored

Examples:

```
Triangle$new(c(0,0), c(1,0), c(1,1))
```

Method `flatness()`: Flatness of the triangle.

Usage:

```
Triangle$flatness()
```

Returns: A number between 0 and 1. A triangle is flat when its flatness is 1.

Method `a()`: Length of the side BC.

Usage:

```
Triangle$a()
```

Method `b()`: Length of the side AC.

Usage:

```
Triangle$b()
```

Method `c()`: Length of the side AB.

Usage:

```
Triangle$c()
```

Method `edges()`: The lengths of the sides of the triangle.

Usage:

Triangle\$edges()

Returns: A named numeric vector.

Method orientation(): Determine the orientation of the triangle.

Usage:

Triangle\$orientation()

Returns: An integer: 1 for counterclockwise, -1 for clockwise, 0 for collinear.

Method contains(): Determine whether a point lies inside the reference triangle.

Usage:

Triangle\$contains(M)

Arguments:

M a point

Method isAcute(): Determines whether the reference triangle is acute.

Usage:

Triangle\$isAcute()

Returns: 'TRUE' if the triangle is acute (or right), 'FALSE' otherwise.

Method angleA(): Angle at the vertex A.

Usage:

Triangle\$angleA()

Returns: The angle at the vertex A in radians.

Method angleB(): Angle at the vertex B.

Usage:

Triangle\$angleB()

Returns: The angle at the vertex B in radians.

Method angleC(): Angle at the vertex C.

Usage:

Triangle\$angleC()

Returns: The angle at the vertex C in radians.

Method angles(): The three angles of the triangle.

Usage:

Triangle\$angles()

Returns: A named vector containing the values of the angles in radians.

Method X175(): Isoperimetric point, also known as the X(175) triangle center; this is the center of the outer Soddy circle.

Usage:

Triangle\$X175()

Method VeldkampIsoperimetricPoint(): Isoperimetric point in the sense of Veldkamp.

Usage:

Triangle\$VeldkampIsoperimetricPoint()

Returns: The isoperimetric point in the sense of Veldkamp, if it exists. Otherwise, returns 'NULL'.

Method centroid(): Centroid.

Usage:

Triangle\$centroid()

Method orthocenter(): Orthocenter.

Usage:

Triangle\$orthocenter()

Method area(): Area of the triangle.

Usage:

Triangle\$area()

Method incircle(): Incircle of the triangle.

Usage:

Triangle\$incircle()

Returns: A Circle object.

Method inradius(): Inradius of the reference triangle.

Usage:

Triangle\$inradius()

Method incenter(): Incenter of the reference triangle.

Usage:

Triangle\$incenter()

Method excircles(): Excircles of the triangle.

Usage:

Triangle\$excircles()

Returns: A list with the three excircles, Circle objects.

Method excentralTriangle(): Excentral triangle of the reference triangle.

Usage:

Triangle\$excentralTriangle()

Returns: A Triangle object.

Method BevanPoint(): Bevan point. This is the circumcenter of the excentral triangle.

Usage:

Triangle\$BevanPoint()

Method medialTriangle(): Medial triangle. Its vertices are the mid-points of the sides of the reference triangle.

Usage:

Triangle\$medialTriangle()

Method orthicTriangle(): Orthic triangle. Its vertices are the feet of the altitudes of the reference triangle.

Usage:

Triangle\$orthicTriangle()

Method incentralTriangle(): Incentral triangle.

Usage:

Triangle\$incentralTriangle()

Details: It is the triangle whose vertices are the intersections of the reference triangle's angle bisectors with the respective opposite sides.

Returns: A Triangle object.

Method NagelTriangle(): Nagel triangle (or extouch triangle) of the reference triangle.

Usage:

Triangle\$NagelTriangle(NagelPoint = FALSE)

Arguments:

NagelPoint logical, whether to return the Nagel point as attribute

Returns: A Triangle object.

Examples:

```
t <- Triangle$new(c(0,-2), c(0.5,1), c(3,0.6))
lineAB <- Line$new(t$A, t$B)
lineAC <- Line$new(t$A, t$C)
lineBC <- Line$new(t$B, t$C)
NagelTriangle <- t$NagelTriangle(NagelPoint = TRUE)
NagelPoint <- attr(NagelTriangle, "Nagel point")
excircles <- t$excircles()
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type="n", asp = 1, xlim = c(-1,5), ylim = c(-3,3),
      xlab = NA, ylab = NA, axes = FALSE)
draw(t, lwd = 2)
draw(lineAB); draw(lineAC); draw(lineBC)
draw(excircles$A, border = "orange")
draw(excircles$B, border = "orange")
draw(excircles$C, border = "orange")
draw(NagelTriangle, lwd = 2, col = "red")
draw(Line$new(t$A, NagelTriangle$A, FALSE, FALSE), col = "blue")
draw(Line$new(t$B, NagelTriangle$B, FALSE, FALSE), col = "blue")
draw(Line$new(t$C, NagelTriangle$C, FALSE, FALSE), col = "blue")
points(rbind(NagelPoint), pch = 19)
par(opar)
```

Method NagelPoint(): Nagel point of the triangle.

Usage:

```
Triangle$NagelPoint()
```

Method GergonneTriangle(): Gergonne triangle of the reference triangle.

Usage:

```
Triangle$GergonneTriangle(GergonnePoint = FALSE)
```

Arguments:

GergonnePoint logical, whether to return the Gergonne point as an attribute

Details: The Gergonne triangle is also known as the *intouch triangle* or the *contact triangle*. This is the triangle made of the three tangency points of the incircle.

Returns: A Triangle object.

Method GergonnePoint(): Gergonne point of the reference triangle.

Usage:

```
Triangle$GergonnePoint()
```

Method tangentialTriangle(): Tangential triangle of the reference triangle. This is the triangle formed by the lines tangent to the circumcircle of the reference triangle at its vertices. It does not exist for a right triangle.

Usage:

```
Triangle$tangentialTriangle()
```

Returns: A Triangle object.

Method symmedialTriangle(): Symmedial triangle of the reference triangle.

Usage:

```
Triangle$symmedialTriangle()
```

Returns: A Triangle object.

Examples:

```
t <- Triangle$new(c(0,-2), c(0.5,1), c(3,0.6))
symt <- t$symmedialTriangle()
symmedianA <- Line$new(t$A, symt$A, FALSE, FALSE)
symmedianB <- Line$new(t$B, symt$B, FALSE, FALSE)
symmedianC <- Line$new(t$C, symt$C, FALSE, FALSE)
K <- t$symmedianPoint()
opar <- par(mar = c(0,0,0,0))
plot(NULL, asp = 1, xlim = c(-1,5), ylim = c(-3,3),
      xlab = NA, ylab = NA, axes = FALSE)
draw(t, lwd = 2)
draw(symmedianA, lwd = 2, col = "blue")
draw(symmedianB, lwd = 2, col = "blue")
draw(symmedianC, lwd = 2, col = "blue")
points(rbind(K), pch = 19, col = "red")
par(opar)
```

Method `symmedianPoint()`: Symmedian point of the reference triangle.

Usage:

`Triangle$symmedianPoint()`

Returns: A point.

Method `circumcircle()`: Circumcircle of the reference triangle.

Usage:

`Triangle$circumcircle()`

Returns: A Circle object.

Method `circumcenter()`: Circumcenter of the reference triangle.

Usage:

`Triangle$circumcenter()`

Method `circumradius()`: Circumradius of the reference triangle.

Usage:

`Triangle$circumradius()`

Method `BrocardCircle()`: The Brocard circle of the reference triangle (also known as the seven-point circle).

Usage:

`Triangle$BrocardCircle()`

Returns: A Circle object.

Method `BrocardPoints()`: Brocard points of the reference triangle.

Usage:

`Triangle$BrocardPoints()`

Returns: A list of two points, the first Brocard point and the second Brocard point.

Method `LemoineCircleI()`: The first Lemoine circle of the reference triangle.

Usage:

`Triangle$LemoineCircleI()`

Returns: A Circle object.

Method `LemoineCircleII()`: The second Lemoine circle of the reference triangle (also known as the cosine circle)

Usage:

`Triangle$LemoineCircleII()`

Returns: A Circle object.

Method `LemoineTriangle()`: The Lemoine triangle of the reference triangle.

Usage:

`Triangle$LemoineTriangle()`

Returns: A Triangle object.

Method `LemoineCircleIII()`: The third Lemoine circle of the reference triangle.

Usage:

```
Triangle$LemoineCircleIII()
```

Returns: A Circle object.

Method `ParryCircle()`: Parry circle of the reference triangle.

Usage:

```
Triangle$ParryCircle()
```

Returns: A Circle object.

Method `pedalTriangle()`: Pedal triangle of a point with respect to the reference triangle. The pedal triangle of a point P is the triangle whose vertices are the feet of the perpendiculars from P to the sides of the reference triangle.

Usage:

```
Triangle$pedalTriangle(P)
```

Arguments:

P a point

Returns: A Triangle object.

Method `CevianTriangle()`: Cevian triangle of a point with respect to the reference triangle.

Usage:

```
Triangle$CevianTriangle(P)
```

Arguments:

P a point

Returns: A Triangle object.

Method `MalfattiCircles()`: Malfatti circles of the triangle.

Usage:

```
Triangle$MalfattiCircles(tangencyPoints = FALSE)
```

Arguments:

tangencyPoints logical, whether to return the tangency points of the Malfatti circles as an attribute.

Returns: A list with the three Malfatti circles, Circle objects.

Examples:

```
t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
Mcircles <- t$MalfattiCircles(TRUE)
plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(0,2.5),
      xlab = NA, ylab = NA)
grid()
draw(t, col = "blue", lwd = 2)
invisible(lapply(Mcircles, draw, col = "green", border = "red"))
invisible(lapply(attr(Mcircles, "tangencyPoints"), function(P){
  points(P[1], P[2], pch = 19)
}))
```

Method `AjimaMalfatti1()`: First Ajima-Malfatti point of the triangle.

Usage:

```
Triangle$AjimaMalfatti1()
```

Method `AjimaMalfatti2()`: Second Ajima-Malfatti point of the triangle.

Usage:

```
Triangle$AjimaMalfatti2()
```

Method `equalDetourPoint()`: Equal detour point of the triangle.

Usage:

```
Triangle$equalDetourPoint(detour = FALSE)
```

Arguments:

`detour` logical, whether to return the detour as an attribute

Details: Also known as the X(176) triangle center.

Method `trilinearToPoint()`: Point given by trilinear coordinates.

Usage:

```
Triangle$trilinearToPoint(x, y, z)
```

Arguments:

`x, y, z` trilinear coordinates

Returns: The point with trilinear coordinates $x:y:z$ with respect to the reference triangle.

Examples:

```
t <- Triangle$new(c(0,0), c(2,1), c(5,7))
incircle <- t$incircle()
t$trilinearToPoint(1, 1, 1)
incircle$center
```

Method `pointToTrilinear()`: Give the trilinear coordinates of a point with respect to the reference triangle.

Usage:

```
Triangle$pointToTrilinear(P)
```

Arguments:

`P` a point

Returns: The trilinear coordinates, a numeric vector of length 3.

Method `isogonalConjugate()`: Isogonal conjugate of a point with respect to the reference triangle.

Usage:

```
Triangle$isogonalConjugate(P)
```

Arguments:

`P` a point

Returns: A point, the isogonal conjugate of `P`.

Method rotate(): Rotate the triangle.

Usage:

```
Triangle$rotate(alpha, 0, degrees = TRUE)
```

Arguments:

alpha angle of rotation

0 center of rotation

degrees logical, whether alpha is given in degrees

Returns: A Triangle object.

Method translate(): Translate the triangle.

Usage:

```
Triangle$translate(v)
```

Arguments:

v the vector of translation

Returns: A Triangle object.

Method SteinerEllipse(): The Steiner ellipse (or circumellipse) of the reference triangle. This is the ellipse passing through the three vertices of the triangle and centered at the centroid of the triangle.

Usage:

```
Triangle$SteinerEllipse()
```

Returns: An Ellipse object.

Examples:

```
t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
ell <- t$SteinerEllipse()
plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(-0.7,2.4),
      xlab = NA, ylab = NA)
draw(t, col = "blue", lwd = 2)
draw(ell, border = "red", lwd = 2)
```

Method SteinerInellipse(): The Steiner inellipse (or midpoint ellipse) of the reference triangle. This is the ellipse tangent to the sides of the triangle at their midpoints, and centered at the centroid of the triangle.

Usage:

```
Triangle$SteinerInellipse()
```

Returns: An Ellipse object.

Examples:

```
t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
ell <- t$SteinerInellipse()
plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(-0.1,2.4),
      xlab = NA, ylab = NA)
draw(t, col = "blue", lwd = 2)
draw(ell, border = "red", lwd = 2)
```

Method `MandartInellipse()`: The Mandart inellipse of the reference triangle. This is the unique ellipse tangent to the triangle's sides at the contact points of its excircles

Usage:

```
Triangle$MandartInellipse()
```

Returns: An Ellipse object.

Method `randomPoints()`: Random points on or in the reference triangle.

Usage:

```
Triangle$randomPoints(n, where = "in")
```

Arguments:

n an integer, the desired number of points

where "in" to generate inside the triangle, "on" to generate on the triangle

Returns: The generated points in a two columns matrix with n rows.

Method `hexylTriangle()`: Hexyl triangle.

Usage:

```
Triangle$hexylTriangle()
```

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Triangle$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

Note

The Steiner ellipse is also the smallest area ellipse which passes through the vertices of the triangle, and thus can be obtained with the function [EllipseFromThreeBoundaryPoints](#). We can also note that the major axis of the Steiner ellipse is the Deming least squares line of the three triangle vertices.

See Also

[TriangleThreeLines](#) to define a triangle by three lines.

Examples

```
# incircle and excircles
A <- c(0,0); B <- c(1,2); C <- c(3.5,1)
t <- Triangle$new(A, B, C)
incircle <- t$incircle()
excircles <- t$excircles()
JA <- excircles$A$center
JB <- excircles$B$center
JC <- excircles$C$center
JAJBJC <- Triangle$new(JA, JB, JC)
A_JA <- Line$new(A, JA, FALSE, FALSE)
B_JB <- Line$new(B, JB, FALSE, FALSE)
```

```

C_JC <- Line$new(C, JC, FALSE, FALSE)
opar <- par(mar = c(0,0,0,0))
plot(NULL, asp = 1, xlim = c(0,6), ylim = c(-4,4),
      xlab = NA, ylab = NA, axes = FALSE)
draw(t, lwd = 2)
draw(incircle, border = "orange")
draw(excircles$A); draw(excircles$B); draw(excircles$C)
draw(JAJBJC, col = "blue")
draw(A_JA, col = "green")
draw(B_JB, col = "green")
draw(C_JC, col = "green")
par(opar)

## -----
## Method `Triangle$new`
## -----

t <- Triangle$new(c(0,0), c(1,0), c(1,1))
t
t$C
t$C <- c(2,2)
t

## -----
## Method `Triangle$print`
## -----

Triangle$new(c(0,0), c(1,0), c(1,1))

## -----
## Method `Triangle$NagelTriangle`
## -----

t <- Triangle$new(c(0,-2), c(0.5,1), c(3,0.6))
lineAB <- Line$new(t$A, t$B)
lineAC <- Line$new(t$A, t$C)
lineBC <- Line$new(t$B, t$C)
NagelTriangle <- t$NagelTriangle(NagelPoint = TRUE)
NagelPoint <- attr(NagelTriangle, "Nagel point")
excircles <- t$excircles()
opar <- par(mar = c(0,0,0,0))
plot(0, 0, type="n", asp = 1, xlim = c(-1,5), ylim = c(-3,3),
      xlab = NA, ylab = NA, axes = FALSE)
draw(t, lwd = 2)
draw(lineAB); draw(lineAC); draw(lineBC)
draw(excircles$A, border = "orange")
draw(excircles$B, border = "orange")
draw(excircles$C, border = "orange")
draw(NagelTriangle, lwd = 2, col = "red")
draw(Line$new(t$A, NagelTriangle$A, FALSE, FALSE), col = "blue")
draw(Line$new(t$B, NagelTriangle$B, FALSE, FALSE), col = "blue")
draw(Line$new(t$C, NagelTriangle$C, FALSE, FALSE), col = "blue")

```



```

points(rbind(NagelPoint), pch = 19)
par(opar)

## -----
## Method `Triangle$symmedialTriangle`
## -----

t <- Triangle$new(c(0,-2), c(0.5,1), c(3,0.6))
symt <- t$symmedialTriangle()
symmedianA <- Line$new(t$A, symt$A, FALSE, FALSE)
symmedianB <- Line$new(t$B, symt$B, FALSE, FALSE)
symmedianC <- Line$new(t$C, symt$C, FALSE, FALSE)
K <- t$symmedianPoint()
opar <- par(mar = c(0,0,0,0))
plot(NULL, asp = 1, xlim = c(-1,5), ylim = c(-3,3),
      xlab = NA, ylab = NA, axes = FALSE)
draw(t, lwd = 2)
draw(symmedianA, lwd = 2, col = "blue")
draw(symmedianB, lwd = 2, col = "blue")
draw(symmedianC, lwd = 2, col = "blue")
points(rbind(K), pch = 19, col = "red")
par(opar)

## -----
## Method `Triangle$MalfattiCircles`
## -----

t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
Mcircles <- t$MalfattiCircles(TRUE)
plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(0,2.5),
      xlab = NA, ylab = NA)
grid()
draw(t, col = "blue", lwd = 2)
invisible(lapply(Mcircles, draw, col = "green", border = "red"))
invisible(lapply(attr(Mcircles, "tangencyPoints"), function(P){
  points(P[1], P[2], pch = 19)
}))

## -----
## Method `Triangle$trilinearToPoint`
## -----

t <- Triangle$new(c(0,0), c(2,1), c(5,7))
incircle <- t$incircle()
t$trilinearToPoint(1, 1, 1)
incircle$center

## -----
## Method `Triangle$SteinerEllipse`
## -----

t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
ell <- t$SteinerEllipse()

```

```

plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(-0.7,2.4),
      xlab = NA, ylab = NA)
draw(t, col = "blue", lwd = 2)
draw(ell, border = "red", lwd =2)

## -----
## Method `Triangle$SteinerInellipse`
## -----

t <- Triangle$new(c(0,0), c(2,0.5), c(1.5,2))
ell <- t$SteinerInellipse()
plot(NULL, asp = 1, xlim = c(0,2.5), ylim = c(-0.1,2.4),
      xlab = NA, ylab = NA)
draw(t, col = "blue", lwd = 2)
draw(ell, border = "red", lwd =2)

```

TriangleThreeLines *Triangle defined by three lines*

Description

Return the triangle formed by three lines.

Usage

```
TriangleThreeLines(line1, line2, line3)
```

Arguments

```
line1, line2, line3
      Line objects
```

Value

A Triangle object.

unitCircle *Unit circle*

Description

Circle centered at the origin with radius 1.

Usage

```
unitCircle
```

Format

An object of class Circle (inherits from R6) of length 25.

Index

- * **datasets**
 - unitCircle, 90
- abline, 18
- Affine, 3
- AffineMappingEllipse2Ellipse, 5
- AffineMappingThreePoints, 5
- Arc, 6
- Circle, 8
- CircleAB, 15
- CircleOA, 16
- crossRatio, 16
- curve, 18
- draw, 17
- ellint2, 18
- Ellipse, 19
- EllipseEquationFromFivePoints, 27
- EllipseFromCenterAndMatrix, 28
- EllipseFromEquation, 28
- EllipseFromFivePoints, 29
- EllipseFromFociAndOnePoint, 30
- EllipseFromThreeBoundaryPoints, 30, 87
- EllipticalArc, 31
- fitEllipse, 33
- GaussianEllipse, 34
- Homothety, 34
- intersectionCircleCircle, 36
- intersectionCircleLine, 37
- intersectionEllipseLine, 38
- intersectionLineLine, 38
- Inversion, 39, 55
- inversionFixingThreeCircles, 42, 43
- inversionFixingTwoCircles, 42, 43
- inversionFromCircle, 44
- inversionKeepingCircle, 44
- inversionSwappingTwoCircles, 42, 45, 51
- Line, 45, 58
- lines, 18
- LownerJohnEllipse, 50
- midCircles, 51
- Mobius, 52
- MobiusMappingCircle, 55
- MobiusMappingThreePoints, 55, 56
- MobiusSwappingTwoPoints, 56
- polypath, 18
- Projection, 57
- radicalCenter, 14, 59
- Reflection, 60
- Rotation, 62
- Scaling, 66
- ScalingXY, 68
- Shear, 70
- soddyCircle, 72
- SteinerChain, 73
- Translation, 74
- Triangle, 76
- TriangleThreeLines, 87, 90
- unitCircle, 90