## Package 'SIRthresholded'

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Type Package Title Sliced Inverse Regression with Thresholding Version 1.0.0 Author Clement Weinreich [aut, cre], Jerome Saracco [aut], Hadrien Lorenzo [aut] Maintainer Clement Weinreich <clement@weinreich.fr> **License** GPL (>= 2.0) Description Implements a thresholded version of the Sliced Inverse Regression method, which allows to do variable selection. **Encoding** UTF-8 RoxygenNote 7.2.0 Imports strucchange Suggests knitr, rmarkdown, mvtnorm VignetteBuilder knitr URL https://clement-w.github.io/SIRthresholded/

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plot.SIR

#### Description

Display the 10 first eigen values and the estimated index versus Y of the SIR model.

#### Usage

```
## S3 method for class 'SIR'
plot(x, choice = "", ...)
```

#### Arguments

х	A SIR object
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "" Plot every graphs (default).
	arguments to be passed to methods, such as graphical parameters (not used here).

#### Value

No return value

## Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply SIR
res = SIR(Y, X, H = 10, graph = FALSE)
# Eigen values
plot(res,choice="eigvals")
# Estimated index versus Y
plot(res,choice="estim_ind")</pre>
```

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#### Description

Display the 10 first eigen values and the estimated index versus Y of the SIRbootstrap model.

#### Usage

```
## S3 method for class 'SIR_bootstrap'
plot(x, choice = "", ...)
```

#### Arguments

х	A SIR_bootstrap object
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "" Plot every graphs (default).
	arguments to be passed to methods, such as graphical parameters (not used here).

#### Value

No return value

#### Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply bootstrap SIR
res = SIR_bootstrap(Y, X, H = 10, B = 10)
# Eigen values
plot(res,choice="eigvals")
# Estimated index versus Y
plot(res,choice="estim_ind")</pre>
```

#### Description

Display the 10 first eigen values and the estimated index versus Y of the thresholded SIR model.

#### Usage

```
## S3 method for class 'SIR_threshold'
plot(x, choice = "", ...)
```

#### Arguments

х	A SIR_threshold object
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "" Plot every graphs (default).
• • •	arguments to be passed to methods, such as graphical parameters (not used here).

#### Value

No return value

#### Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply SIR with hard thresholding
res = SIR_threshold(Y, X, H = 10, lambda = 0.2, thresholding = "hard")
# Eigen values
plot(res,choice="eigvals")
# Estimated index versus Y
plot(res,choice="estim_ind")
```

plot.SIR\_threshold\_bootstrap

Graphical output of SIR\_threshold\_bootstrap

#### Description

Display the estimated index versus Y of the SIR model, the size of the models, the occurrence of variable selection, the distribution of the coefficients of and  $\hat{b}$  and the distribution of  $\lambda_{opt}$  found across the replications.

#### Usage

```
## S3 method for class 'SIR_threshold_bootstrap'
plot(x, choice = "", ...)
```

#### Arguments

х	A SIR_threshold_bootstrap object
choice	the graph to plot:
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "size" Plot the size of the models across the replications.
	• "selec_var" Plot the occurrence of the selected variables across the replica- tions.
	• "coefs_b" Plot the value of $\hat{b}$ across the replications.
	• "lambdas_replic" Plot the distribution of $\lambda_{opt}$ across the replications.
	• "" Plot every graphs (default).
	arguments to be passed to methods, such as graphical parameters (not used here).

#### Value

No return value

#### Examples

```
# Generate Data
set.seed(10)
n <- 200
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps</pre>
```

res = SIR\_threshold\_bootstrap(Y,X,H=10,n\_lambda=300,thresholding="hard", n\_replications=30,k=2)

# Estimated index versus Y
plot(res,choice="estim\_ind")

```
# Model size
plot(res,choice="size")
# Selected variables
plot(res,choice="selec_var")
# Coefficients of b
plot(res,choice="coefs_b")
# Optimal lambdas
plot(res,choice="lambdas_replic")
```

plot.SIR\_threshold\_opt

Graphical output of SIR\_threshold\_opt

## Description

Display the 10 first eigen values, the estimated index versus Y of the SIR model, the evolution of  $cos^2$  and variable selection according to  $\lambda$ , and the regularization path of  $\hat{b}$ .

#### Usage

## S3 method for class 'SIR\_threshold\_opt'
plot(x, choice = "", ...)

#### Arguments

х	A SIR_threshold_opt object
choice	the graph to plot:
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "opt_lambda" Plot the choice of $\lambda_{opt}$ .
	• " $\cos 2$ _selec" Plot the evolution of $\cos^2$ and variable selection according to
	$\lambda.$
	• "regul_path" Plot the regularization path of $\hat{b}$ .
	• "" Plot every graphs (default).
	arguments to be passed to methods, such as graphical parameters (not used here).

#### Value

No return value

#### SIR

#### Examples

```
# Generate Data
set.seed(10)
n <- 200
beta <- c(1,1,rep(0,8))</pre>
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))</pre>
eps <- rnorm(n)</pre>
Y <- (X%*%beta)**3+eps
# Apply SIR with soft thresholding
res = SIR_threshold_opt(Y,X,H=10,n_lambda=100,thresholding="soft")
# Estimated index versus Y
plot(res,choice="estim_ind")
# Choice of optimal lambda
plot(res,choice="opt_lambda")
# Evolution of cos^2 and var selection according to lambda
plot(res,choice="cos2_selec")
# Regularization path
plot(res,choice="regul_path")
```

SIR

Classic SIR

#### Description

Apply a single-index SIR on (X, Y) with H slices. This function allows to obtain an estimate of a basis of the EDR (Effective Dimension Reduction) space via the eigenvector  $\hat{b}$  associated with the largest nonzero eigenvalue of the matrix of interest  $\hat{\Sigma}_n^{-1}\hat{\Gamma}_n$ . Thus,  $\hat{b}$  is an EDR direction.

#### Usage

SIR(Y, X, H = 10, graph = TRUE, choice = "")

#### Arguments

Y	A numeric vector representing the dependent variable (a response vector).
Х	A matrix representing the quantitative explanatory variables (bind by column).
н	The chosen number of slices (default is 10).
graph	A boolean that must be set to true to display graphics (default is TRUE).
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "" Plot every graphs. (default)

## Value

An object of class SIR, with attributes:

b	This is an estimated EDR direction, which is the principal eigenvector of the interest matrix.
M1	The interest matrix.
eig_val	The eigenvalues of the interest matrix.
n	Sample size.
р	The number of variables in X.
Н	The chosen number of slices.
call	Unevaluated call to the function.
index_pred	The index Xb' estimated by SIR.
Υ	The response vector.

#### Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply SIR
SIR(Y, X, H = 10)</pre>
```

SIR\_bootstrap Bootstrap SIR

#### Description

Apply a single-index SIR on B bootstraped samples of (X, Y) with H slices.

#### Usage

```
SIR_bootstrap(Y, X, H = 10, B = 10, graph = TRUE, choice = "")
```

## Arguments

Y	A numeric vector representing the dependent variable (a response vector).
Х	A matrix representing the quantitative explanatory variables (bind by column).
Н	The chosen number of slices (default is 10).
В	The number of bootstrapped samples to draw (default is 10).

graph	A boolean that must be set to true to display graphics (default is TRUE).
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.

• "" Plot every graphs (default).

#### Value

An object of class SIR\_bootstrap, with attributes:

This is an estimated EDR direction, which is the principal eigenvector of the interest matrix.
A matrix of size p*B that contains an estimation of beta in the columns for each bootstrapped sample.
Sample size.
The number of variables in X.
The chosen number of slices.
Unevaluated call to the function.
The index b'X estimated by SIR.
The response vector.

#### Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply bootstrap SIR
SIR_bootstrap(Y, X, H = 10, B = 10)</pre>
```

SIR\_threshold SIR threshold

## Description

Apply a single-index SIR on (X, Y) with H slices, with a parameter  $\lambda$  which apply a soft/hard thresholding to the interest matrix  $\hat{\Sigma}_n^{-1}\hat{\Gamma}_n$ .

## Usage

```
SIR_threshold(
    Y,
    X,
    H = 10,
    lambda = 0,
    thresholding = "hard",
    graph = TRUE,
    choice = ""
)
```

## Arguments

Υ	A numeric vector representing the dependent variable (a response vector).
Х	A matrix representing the quantitative explanatory variables (bind by column).
Н	The chosen number of slices (default is 10).
lambda	The thresholding parameter (default is 0).
thresholding	The thresholding method to choose between hard and soft (default is hard).
graph	A boolean that must be set to true to display graphics (default is TRUE).
choice	the graph to plot:
	• "eigvals" Plot the eigen values of the matrix of interest.
	• "estim_ind" Plot the estimated index by the SIR model versus Y.

• "" Plot every graphs (default).

#### Value

An object of class SIR\_threshold, with attributes:

b	This is an estimated EDR direction, which is the principal eigenvector of the interest matrix.		
M1	The interest matrix thresholded.		
eig_val	The eigenvalues of the interest matrix thresholded.		
eig_vect	A matrix corresponding to the eigenvectors of the interest matrix.		
Υ	The response vector.		
n	Sample size.		
р	The number of variables in X.		
Н	The chosen number of slices.		
nb.zeros	The number of 0 in the estimation of the vector beta.		
index_pred	The index Xb' estimated by SIR.		
list.relevant.variables			
	A list that contains the variables selected by the model.		
cos_squared	The cosine squared between vanilla SIR and SIR thresholded.		
lambda	The thresholding parameter used.		

thresholding	The thresholding method used.
call	Unevaluated call to the function.
X_reduced	The X data restricted to the variables selected by the model. It can be used to estimate a new SIR model on the relevant variables to improve the estimation of b.

#### Examples

```
# Generate Data
set.seed(10)
n <- 500
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps
# Apply SIR with hard thresholding
SIR_threshold(Y, X, H = 10, lambda = 0.2, thresholding = "hard")</pre>
```

SIR\_threshold\_bootstrap

SIR optimally thresholded on bootstraped replications

#### Description

Apply a single-index optimally soft/hard thresholded SIR with H slices on 'n\_replications' bootstraped replications of (X, Y). The optimal number of selected variables is the number of selected variables that came back most often among the replications performed. From this, we can get the corresponding  $\hat{b}$  and  $\lambda_{opt}$  that produce the same number of selected variables in the result of 'SIR\_threshold\_opt'.

#### Usage

```
SIR_threshold_bootstrap(
   Y,
   X,
   H = 10,
   thresholding = "hard",
   n_replications = 50,
   graph = TRUE,
   output = TRUE,
   n_lambda = 100,
   k = 2,
   choice = ""
)
```

## Arguments

Υ	A numeric vector representing the dependent variable (a response vector).
Х	A matrix representing the quantitative explanatory variables (bind by column).
Н	The chosen number of slices (default is 10).
thresholding	The thresholding method to choose between hard and soft (default is hard).
n_replications	The number of bootstraped replications of $(X,Y)$ done to estimate the model (default is 50).
graph	A boolean, set to TRUE to plot graphs (default is TRUE).
output	A boolean, set to TRUE to print information (default is TRUE).
n_lambda	The number of lambda to test. The n_lambda tested lambdas are uniformally distributed between 0 and the maximum value of the interest matrix (default is 100).
k	Multiplication factor of the bootstrapped sample size (default is 1 = keep the same size as original data).
choice	the graph to plot:
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	• "size" Plot the size of the models across the replications.
	• "selec_var" Plot the occurrence of the selected variables across the replica- tions.
	<ul> <li>"coefs_b" Plot the value of b across the replications.</li> </ul>
	<ul> <li>"lambdas_replic" Plot the optimal lambdas across the replications.</li> </ul>
	• "" Plot every graphs (default).

## Value

An object of class SIR\_threshold\_bootstrap, with attributes:

b	This is the optimal estimated EDR direction, which is the principal eigenvector of the interest matrix.		
lambda_opt	The optimal lambda.		
<pre>vec_nb_var_sele</pre>	c		
	Vector that contains the number of selected variables for each replications.		
occurrences_var			
	Vector that contains at index $i$ the number of times the $i\_th$ variable has been selected in a replication.		
call nb_var_selec_op	Unevaluated call to the function.		
	Optimal number of selected variables which is the number of selected variables that came back most often among the replications performed.		
list_relevant_variables			
	A list that contains the variables selected by the model.		
n	Sample size.		
р	The number of variables in X.		

Н	The chosen number of slices.			
n_replications	The number of bootstraped replications of (X,Y) done to estimate the model.			
thresholding	The thresholding method used.			
X_reduced	The X data restricted to the variables selected by the model. It can be used estimate a new SIR model on the relevant variables to improve the estimation b.			
mat_b	Contains the estimation b at each bootstraped replications.			
lambdas_opt_boo	t			
	Contains the optimal lambda found by SIR_threshold_opt at each replication.			
index_pred	The index Xb' estimated by SIR.			
Y	The response vector.			
M1	The interest matrix thresholded with the optimal lambda.			

#### Examples

```
# Generate Data
set.seed(8)
n <- 170
beta <- c(1,1,1,1,1,rep(0,15))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,20))
eps <- rnorm(n,sd=8)
Y <- (X%*%beta)**3+eps
# Apply SIR with hard thresholding
SIR_threshold_bootstrap(Y,X,H=10,n_lambda=300,thresholding="hard", n_replications=30,k=2)</pre>
```

SIR\_threshold\_opt SIR optimally thresholded

#### Description

Apply a single-index SIR on (X, Y) with H slices, with a soft/hard thresholding of the interest matrix  $\hat{\Sigma}_n^{-1}\hat{\Gamma}_n$  by an optimal parameter  $\lambda_{opt}$ . The  $\lambda_{opt}$  is found automatically among a vector of n\_lambda  $\lambda$ , starting from 0 to the maximum value of  $\hat{\Sigma}_n^{-1}\hat{\Gamma}_n$ . For each feature of X, the number of  $\lambda$  associated with a selection of this feature is stored (in a vector of size p). This vector is sorted in a decreasing way. Then, thanks to strucchange::breakpoints, a breakpoint is found in this sorted vector. The coefficients of the variables at the left of the breakpoint, tend to be automatically toggled to 0 due to the thresholding operation based on  $\lambda_{opt}$ , and so should be removed (useless variables). Finally,  $\lambda_{opt}$  corresponds to the first  $\lambda$  such that the associated  $\hat{b}$  provides the same number of zeros as the breakpoint's value.

For example, for  $X \in \mathbb{R}^{10}$  and n\_lambda=100, this sorted vector can look like this :

X10	X3	X8	X5	X7	X9	X4	X6	X2	X1
2	3	3	4	4	4	6	10	95	100

Here, the breakpoint would be 8.

## Usage

```
SIR_threshold_opt(
   Y,
   X,
   H = 10,
   n_lambda = 100,
   thresholding = "hard",
   graph = TRUE,
   output = TRUE,
   choice = ""
)
```

## Arguments

Υ	A numeric vector representing the dependent variable (a response vector).
Х	A matrix representing the quantitative explanatory variables (bind by column).
Н	The chosen number of slices (default is 10).
n_lambda	The number of lambda to test. The n_lambda tested lambdas are uniformally distributed between 0 and the maximum value of the interest matrix. (default is 100).
thresholding	The thresholding method to choose between hard and soft (default is hard).
graph	A boolean, set to TRUE to plot graphs (default is TRUE).
output	A boolean, set to TRUE to print informations (default is TRUE).
choice	the graph to plot:
	• "estim_ind" Plot the estimated index by the SIR model versus Y.
	<ul> <li>"opt_lambda" Plot the choice of the optimal lambda.</li> </ul>
	• "cos2_selec" Plot the evolution of cos^2 and variable selection according to lambda.
	<ul> <li>"regul_path" Plot the regularization path of b.</li> </ul>
	• "" Plot every graphs (default).

## Value

An object of class SIR\_threshold\_opt, with attributes:

b	This is the optimal estimated EDR direction, which is the principal eigenvector of the interest matrix.
lambdas	A vector that contains the tested lambdas.
lambda_opt	The optimal lambda.
mat_b	A matrix of size p*n_lambda that contains an estimation of beta in the columns for each lambda.
n_lambda	The number of lambda tested.

vect_nb_zeros	The number of 0 in b for each lambda.			
list_relevant_variables				
	A list that contains the variables selected by the model.			
fit_bp	An object of class breakpoints from the strucchange package, that contains in- formations about the breakpoint which allows to deduce the optimal lambda.			
indices_useless	_var			
	A vector that contains p items: each variable is associated with the number of lambda that selects this variable.			
vect_cos_square	d			
	A vector that contains for each lambda, the cosine squared between vanilla SIR and SIR thresholded.			
Y	The response vector.			
n	Sample size.			
р	The number of variables in X.			
Н	The chosen number of slices.			
M1	The interest matrix thresholded with the optimal lambda.			
thresholding	The thresholding method used.			
call	Unevaluated call to the function.			
X_reduced	The X data restricted to the variables selected by the model. It can be used to estimate a new SIR model on the relevant variables to improve the estimation of b.			
index_pred	The index Xb' estimated by SIR.			

## Examples

```
# Generate Data
set.seed(2)
n <- 200
beta <- c(1,1,rep(0,8))
X <- mvtnorm::rmvnorm(n,sigma=diag(1,10))
eps <- rnorm(n)
Y <- (X%*%beta)**3+eps</pre>
```

```
# Apply SIR with soft thresholding
SIR_threshold_opt(Y,X,H=10,n_lambda=300,thresholding="soft")
```

# Index

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