# Package 'SPQR'

October 12, 2022

**Description** Methods for flexible estimation of conditional density and quantile func-

tion, as well as model agnostic tools for analyzing quantile covariate effect and variable importance. The estimation method implements the semi-parametric quantile regression model described in Xu and Reich (2021) <doi:10.1111/biom.13576>, and the model agnostic tools extend accumulative local effects (ALE) to quantile regression setting. License GPL (>= 3)**Encoding UTF-8** RoxygenNote 7.1.2 **Depends** R (>= 3.6)**Imports** Rcpp (>= 1.0.8), stats, torch, splines2, ggplot2, loo, progress, progressr, interp, RColorBrewer, yaImpute, coro (>= 1.0.2) LinkingTo Rcpp, RcppArmadillo NeedsCompilation yes SystemRequirements C++11 Author Steven Xu [aut, cre], Reetam Majumder [aut], Brian Reich [ctb] Maintainer Steven Xu <sgxu@ncsu.edu> **Suggests** testthat (>= 3.0.0) Config/testthat/edition 3 URL https://github.com/stevengxu/SPQR BugReports https://github.com/stevengxu/SPQR/issues Repository CRAN **Date/Publication** 2022-05-02 20:02:03 UTC

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2 autoplot.SPQR

# **R** topics documented:

autoplot.SPQR	2
coef.SPQR	3
createFolds.SPQR	4
cv.SPQR	5
load.SPQR	7
plotEstimator	8
plotGOF	9
plotMCMCtrace	C
plotQALE	1
plotQVI	2
predict.SPQR	3
print.SPQR	5
print.summary.SPQR	6
QALE	6
save.SPQR	
SPQR	
summary.SPQR	.1
2	1
	٥

autoplot.SPQR

autoplot method for class SPQR

## **Description**

The function calls one of the following functions: plotEstimator(), plotGOF(), plotMCMCtrace(), plotQALE(), plotQVI()

# Usage

Index

```
## S3 method for class 'SPQR'
autoplot(object, output = c("GOF", "estimator", "trace", "QALE", "QVI"), ...)
```

## **Arguments**

object

An object of class SPQR.

output

A character indicating the type of plot to be returned.

- "GOF": goodness of fit test by comparing the quantiles of probability integral transform (PIT) to that of uniform distribution.
- "estimator": visualization of various estimates, including probability density function (PDF), cumulative density function (CDF) and quantile function (QF).
- "trace": diagnostic trace plots for SPQR fitted with method = "MCMC".
- "QALE": quantile accumulative local effects (ALE) for visualizing covariate effects on predicted quantiles.

coef.SPQR 3

• "QVI": quantile variable importance comparison.

arguments passed into specific plot function, see plotEstimator(), plotGOF(), plotMCMCtrace(), plotQALE() or plotQVI() for required arguments.

#### Value

```
a ggplot object
```

#### **Examples**

coef.SPQR

coef method for class SPQR

## **Description**

Computes the estimated spline coefficients of a SPQR class object

#### Usage

```
## S3 method for class 'SPQR'
coef(object, X, ...)
```

#### **Arguments**

object An object of class SPQR.

X The covariate vector/matrix for which the coefficient is calculated.

Other arguments.

#### Value

A NROW(X) by K matrix containing values of the estimated coefficient, where K is the number of basis functions.

4 createFolds.SPQR

#### **Examples**

createFolds.SPQR

generate cross-validation folds

## **Description**

Helper function to generate cross-validation folds that can be used by cv. SPQR.

# Usage

```
createFolds.SPQR(Y, nfold, stratified = FALSE)
```

## **Arguments**

Y The response vector.

nfold The number of cross-validation folds.

stratified If TRUE, stratified folds based on quantiles of Y are generated.

#### Value

A list of size nfold containing indices of the observations for each fold.

```
set.seed(919)
n <- 1000
X <- rbinom(n, 1, 0.5)
Y <- rnorm(n, X, 0.8)
folds <- createFolds.SPQR(Y, nfold = 5)</pre>
```

cv.SPQR 5

cv.SPQR

cross-validation for SPQR estimator

## **Description**

Fits SPQR using either MLE or MAP method and computes K-fold cross-validation error based on pre-computed folds.

# Usage

```
cv.SPQR(
  folds,
  X,
  Y,
  n.knots = 10,
  n.hidden = 10,
  activation = c("tanh", "relu", "sigmoid"),
  method = c("MLE", "MAP", "MCMC"),
  prior = c("ARD", "GP", "GSM"),
  hyperpar = list(),
  control = list(),
  normalize = FALSE,
  verbose = TRUE,
  seed = NULL,
  ...
)
```

#### **Arguments**

folds	A list of CV folds, possibly that generated from createFolds.SPQR().
Χ	The covariate matrix (without intercept column)
Υ	The response vector.
n.knots	The number of basis functions. Default: 10.
n.hidden	A vector specifying the number of hidden neurons in each hidden layer. Default: 10.
activation	The hidden layer activation. Either "tanh" (default) or "relu".
method	Method for estimating SPQR. One of "MLE", "MAP" (default) or "MCMC".
prior	The prior model for variance hyperparameters. One of "GP", "ARD" (default) or "GSM".
hyperpar	A list of named hyper-prior hyperparameters to use instead of the default values, including a_lambda, b_lambda, a_sigma and b_sigma. The default value is 0.001 for all four hyperparameters.
control	A list of named and method-dependent parameters that allows finer control of the behavior of the computational approaches.

1. Parameters for MLE and MAP methods

6 cv.SPQR

> • use.GPU If TRUE GPU computing will be used if torch::cuda\_is\_available() returns TRUE. Default: FALSE.

- 1r The learning rate used by the Adam optimizer, i.e., torch::optim\_adam().
- dropout A length two vector specifying the dropout probabilities in the input and hidden layers respectively. The default is c(0,0) indicating no dropout.
- batchnorm If TRUE batch normalization will be used after each hidden activation
- epochs The number of passes of the entire training dataset in gradient descent optimization. If early.stopping.epochs is used then this is the maximum number of passes. Default: 200.
- batch.size The size of mini batches for gradient calculation. Default: 128.
- valid.pct The fraction of data used as validation set. Default: 0.2.
- early. stopping. epochs The number of epochs before stopping if the validation loss does not decrease. Default: 10.
- print.every.epochs The number of epochs before next training progress in printed. Default: 10.
- save.path The path to save the fitted torch model. By default a folder named "SPQR\_model" is created in the current working directory to store the model.
- save.name The name of the file to save the fitted torch model. Default is "SPQR.model.pt".

#### 2. Parameters for MCMC method

These parameters are similar to those in rstan::stan(). Detailed explanations can be found in the Stan reference manual.

- algorithm The sampling algorithm; "HMC": Hamiltonian Monte Carlo with dual-averaging, "NUTS": No-U-Turn sampler (default).
- iter The number of MCMC iterations (including warmup). Default: 2000.
- warmup The number of warm-up/burn-in iterations for step-size and mass matrix adaptation. Default: 500.
- thin The number of iterations before saving next post-warmup samples. Default: 1.
- stepsize The discretization interval/step-size  $\epsilon$  of leap-frog integrator. Default is NULL which indicates that it will be adaptively selected during warmup iterations.
- metric The type of mass matrix; "unit": diagonal matrix of ones, "diag": diagonal matrix with positive diagonal entries estimated during warmup iterations (default), "dense": a dense, symmetric positive definite matrix with entries estimated during warm-up iterations.
- delta The target Metropolis acceptance rate. Default: 0.9.
- max. treedepth The maximum tree depth in NUTS. Default: 6.
- int.time The integration time in HMC. The number of leap-frog steps is calculated as  $L_{\epsilon} = |t/\epsilon|$ . Default: 0.3.

If TRUE, all covariates will be normalized to take values between [0,1].

Ioad.SPQR 7

verbose If TRUE (default), training progress will be printed.

seed Random number generation seed.
... other parameters to pass to control.

#### Value

control the list of all control parameters.

cve the cross-validation error.

folds the CV folds.

#### See Also

```
createFolds.SPQR()
```

## **Examples**

load.SPQR

load saved SPQR model

# Description

Load saved SPQR model from a designated path. The function first loads the .SPQR file that stores the SPQR object. It then checks whether the SPQR model is fitted with method = "MCMC". If not, it also loads the .pt file storing the torch model with the same name and attach it to the SPQR object.

## Usage

```
load.SPQR(name = stop("`name` must be specified"), path = NULL)
```

#### **Arguments**

name The name of the saved object excluding extension.

path The path to look for the saved object. Default is the current working directory.

#### Value

An object of class SPQR.

8 plotEstimator

#### **Examples**

```
set.seed(919)
n <- 200
X <- rbinom(n, 1, 0.5)
Y <- rnorm(n, X, 0.8)
fit <- SPQR(X = X, Y = Y, method = "MCMC", normalize = TRUE, verbose = FALSE)
# save.SPQR(fit, name = "mcmc_fit")
# fit <- load.SPQR("mcmc_fit")</pre>
```

plotEstimator

plot SPQR estimators

## Description

Computes and plots the estimated PDF/CDF/QF curves.

#### Usage

```
plotEstimator(object, X, ...)
```

# Arguments

An object of class "SPQR" object Χ A row vector indicating covariate values for which the conditional PDF/CDF/QF is computed and plotted. Arguments passed on to predict. SPQR . . . nY An integer number indicating length of grid when Y is not specified. Default: 101. type The function to be predicted; "PDF": probability density function, "CDF": cumulative distribution function, and "QF": the quantile function (default). tau The grid of quantiles for which the quantile function is computed. Default: seq(0.1,0.9,0.1). ci.level The credible level for computing the pointwise credible intervals. The default is 0 indicating no credible intervals should be computed. getAll If TRUE, extracts all posterior samples of the prediction. Default: FALSE.

#### Value

A ggplot object.

plotGOF 9

## **Examples**

plotGOF

goodness-of-fit test for SPQR estimator

# Description

Performs a goodness-of-fit test for the estimated conditional probability density function (PDF) using probability inverse transformation method.

#### Usage

```
plotGOF(object, getAll = FALSE)
```

#### **Arguments**

object An object of class SPQR.

getAll If TRUE and SPQR is fitted with method = "MCMC", plots all posterior samples of

Q-Q lines. Default: FALSE.

#### Value

A ggplot object.

10 plotMCMCtrace

```
plotGOF(fit)
```

plotMCMCtrace

plot MCMC trace plots

# Description

Show trace plot of the log-likelihood or estimates, of a "SPQR" class object fitted using the MCMC method

# Usage

```
plotMCMCtrace(
  object,
  target = c("loglik", "PDF", "CDF", "QF"),
  X = NULL,
  Y = NULL,
  tau = 0.5,
  window = NULL
)
```

# Arguments

object	An object of class SPQR.
target	A character indicating the statistic/estimate for which traceplot should be plotted; "loglik": log-likelihood (default), "PDF": probability density function, "CDF": cumulative density function, "QF": quantile function.
X	If target != "loglik", a row vector specifying the covariate values for which the estimates are computed. Default: NULL.
Υ	If target = "PDF" or target = "CDF" a scalar specifying the response value for which the estimates are computed. Default: NULL.
tau	If target != "QF", a scalar specifying the quantile level for which the estimates are computed. Default: 0.5.
window	A vector specifying the range of index of the MCMC samples for which the traceplot should be plotted. Default is NULL indicating that the whole chain is plotted.

# Value

A ggplot object.

plotQALE 11

#### **Examples**

plotQALE

plot accumulated local effects (ALE)

## **Description**

Computes and plots the quantile ALEs of a SPQR class object. The function plots the ALE main effects across tau for a single covariate using line plots, and the ALE interaction effects between two covariates across tau using contour plots.

#### Usage

```
plotQALE(object, ...)
```

#### **Arguments**

object An object of class "SPQR".
... Arguments passed on to QALE

var.index a numeric scalar or length-two vector of indices of the covariates
for which the ALEs will be calculated. When length(var.index) = 1, the
function computes the main effect for X[,var.index]. When length(var.index)
= 2, the function computes the interaction effect between X[,var.index[1]]
and X[,var.index[2]].

tau The quantiles of interest.

- n.bins the maximum number of intervals into which the covariate range is divided when calculating the ALEs. The actual number of intervals depends on the number of unique values in X[,var.index]. When length(var.index) = 2, n.bins is applied to both covariates.
- ci.level The credible level for computing the pointwise credible intervals for ALE when length(var.index) = 1. The default is 0 indicating no credible intervals should be computed.

12 plotQVI

pred.fun A function that will be used instead of predict.SPQR() for computing predicted quantiles given covariates. This can be useful when the user wants to compare the QALE calculated using SPQR to that using other quantile regression models, or maybe that using the true model in a simulation study.

#### Value

A ggplot object.

## **Examples**

plotQVI

plot variable importance comparison by quantile

## **Description**

Computes the quantile ALE-induced variable importance (VI) measure for each of the covariate specified in var.index, and produces a ranking plot of the covariates using bar plot for each quantile of interest.

#### Usage

```
plotQVI(object, var.index = NULL, var.names = NULL, ...)
```

## Arguments

object	An object of class SPQR.
var.index	A vector specifying the index of the covariates for which VI measures should be computed. Default is NULL indicating all covariates are considered.
var.names	The names of the covariates to appear in the bar plots. Default is NULL and the the function will use generic names generated by parse(text=paste0("X[",var.index,"]")).
	Arguments passed on to QALE
	tau The quantiles of interest.

predict.SPQR 13

n.bins the maximum number of intervals into which the covariate range is divided when calculating the ALEs. The actual number of intervals depends on the number of unique values in X[,var.index]. When length(var.index) = 2, n.bins is applied to both covariates.

- ci.level The credible level for computing the pointwise credible intervals for ALE when length(var.index) = 1. The default is 0 indicating no credible intervals should be computed.
- pred.fun A function that will be used instead of predict.SPQR() for computing predicted quantiles given covariates. This can be useful when the user wants to compare the QALE calculated using SPQR to that using other quantile regression models, or maybe that using the true model in a simulation study.

#### Value

A ggplot object.

## **Examples**

predict.SPQR

predict method for class SPQR

## **Description**

Computes the predicted values for different functions based on the fitted "SPQR" object.

# Usage

```
## S3 method for class 'SPQR'
predict(
  object,
  X,
  Y = NULL,
  nY = 101,
```

14 predict.SPQR

```
type = c("QF", "PDF", "CDF"),
tau = seq(0.1, 0.9, 0.1),
ci.level = 0,
getAll = FALSE,
...
)
```

# Arguments

object	An object of class SPQR.
X	The covariate vector/matrix for which the predictions are computed.
Υ	The response vector for which the predictions are computed. Default is NULL indicating that a equi-distant grid vector on $[0,1]$ of length nY is used.
nY	An integer number indicating length of grid when Y is not specified. Default: 101.
type	The function to be predicted; "PDF": probability density function, "CDF": cumulative distribution function, and "QF": the quantile function (default).
tau	The grid of quantiles for which the quantile function is computed. Default: $seq(0.1,0.9,0.1)$ .
ci.level	The credible level for computing the pointwise credible intervals. The default is 0 indicating no credible intervals should be computed.
getAll	If TRUE, extracts all posterior samples of the prediction. Default: FALSE.
	Other arguments.

#### Value

A named array containing all predicted values.

print.SPQR 15

print.SPQR

print method for class SPQR

## **Description**

Summarizes and print the output produced by SPQR() in an organized way.

## Usage

```
## S3 method for class 'SPQR'
print(x, ...)
```

# Arguments

```
    An object of class SPQR
    Arguments passed on to print.summary.SPQR
    showModel If TRUE, prints the detailed NN architecture by layer.
```

#### **Details**

This is equivalent to the function call print.summary.SPQR(summary.SPQR(object), ...).

#### Value

No return value, called for side effects.

16 QALE

#### **Description**

Print the output produced by summary.SPQR().

# Usage

```
## S3 method for class 'summary.SPQR'
print(x, showModel = FALSE, ...)
```

#### **Arguments**

```
x An object of class summary. SPQRshowModel If TRUE, prints the detailed NN architecture by layer.... Other arguments.
```

#### Value

No return value, called for side effects.

#### **Examples**

**QALE** 

quantile accumulated local effects (ALE)

## **Description**

Computes the quantile ALEs of a SPQR class object. The function plots the ALE main effects across tau using line plots for a single covariate, and the ALE interaction effects across tau using contour plots for two covariates .

QALE 17

#### Usage

```
QALE(
   object,
   var.index,
   tau = seq(0.1, 0.9, 0.1),
   n.bins = 40,
   ci.level = 0,
   getAll = FALSE,
   pred.fun = NULL
)
```

#### **Arguments**

object An object of class SPQR.

var.index a numeric scalar or length-two vector of indices of the covariates for which the

ALEs will be calculated. When length(var.index) = 1, the function computes the main effect for X[,var.index]. When length(var.index) = 2, the func-

tion computes the interaction effect between X[, var.index[1]] and X[, var.index[2]].

tau The quantiles of interest.

n.bins the maximum number of intervals into which the covariate range is divided when

calculating the ALEs. The actual number of intervals depends on the number of unique values in X[,var.index]. When length(var.index) = 2, n.bins is

applied to both covariates.

ci.level The credible level for computing the pointwise credible intervals for ALE when

length(var.index) = 1. The default is 0 indicating no credible intervals should

be computed.

getAll If TRUE and length(var.index) = 1, extracts all posterior samples of ALE.

pred. fun A function that will be used instead of predict. SPQR() for computing pre-

dicted quantiles given covariates. This can be useful when the user wants to compare the QALE calculated using SPQR to that using other quantile regression models, or maybe that using the true model in a simulation study.

Value

x If length(var.index) = 1, a (n.bins+1)-length vector specifying the ordered

predictor values at which the ALE plot function is calculated. These are the break points for the n.bins intervals into which the predictor range is divided, plus the lower boundary of the first interval and the upper boundary of the last interval. If length(var.index) = 2, a list of two such vectors, the first containing the X[,var.index[1]] values and the second containing the X[,var.index[2]]

values at which the ALE plot function is calculated.

ALE If length(var.index) = 1, a (n.bins+1) by length(tau) matrix of predicted ALE values for every combination of x and tau. If length(var.index) = 2, a 3-dimensional array of predicted ALE values. Each slice corresponds to a quantile. Within each slice, the rows correspond to X[,var.index[1]] and the

columns correspond to X[,var.index[2]].

18 save.SPQR

#### **Examples**

save.SPQR

save fitted SPQR model

#### **Description**

Save SPQR object in a designated directory. If SPQR is fitted with method = "MCMC" then only a .SPQR file is saved; otherwise, a .pt file storing the fitted torch model is also saved.

#### Usage

```
save.SPQR(object, name = stop("`name` must be specified"), path = NULL)
```

## **Arguments**

object An object of class SPQR.

name The name of the saved object excluding extension.

path The path to save the object. Default is the current working directory.

#### Value

No return value, called for side effects.

```
set.seed(919)
n <- 200
X <- rbinom(n, 1, 0.5)
Y <- rnorm(n, X, 0.8)
fit <- SPQR(X = X, Y = Y, method = "MCMC", normalize = TRUE, verbose = FALSE)
# save.SPQR(fit, name = "mcmc_fit")</pre>
```

SPQR 19

SPQR Fitting SPQR models

# Description

Main function of the package. Fits SPQR using the maximum likelihood estimation (MLE), maximum *a posterior* (MAP) or Markov chain Monte Carlo (MCMC) method. Returns an object of S3 class SPQR.

# Usage

```
SPQR(
   X,
   Y,
   n.knots = 10,
   n.hidden = 10,
   activation = c("tanh", "relu", "sigmoid"),
   method = c("MLE", "MAP", "MCMC"),
   prior = c("ARD", "GP", "GSM"),
   hyperpar = list(),
   control = list(),
   normalize = FALSE,
   verbose = TRUE,
   seed = NULL,
   ...
)
```

## **Arguments**

Χ	The covariate matrix (without intercept column)
Υ	The response vector.
n.knots	The number of basis functions. Default: 10.
n.hidden	A vector specifying the number of hidden neurons in each hidden layer. Default: 10.
activation	The hidden layer activation. Either "tanh" (default) or "relu".
method	Method for estimating SPQR. One of "MLE", "MAP" (default) or "MCMC".
prior	The prior model for variance hyperparameters. One of " $GP$ ", "ARD" (default) or " $GSM$ ".
hyperpar	A list of named hyper-prior hyperparameters to use instead of the default values, including a_lambda, b_lambda, a_sigma and b_sigma. The default value is 0.001 for all four hyperparameters.
control	A list of named and method-dependent parameters that allows finer control of the behavior of the computational approaches.

1. Parameters for MLE and MAP methods

20 SPQR

• use.GPU If TRUE GPU computing will be used if torch::cuda\_is\_available() returns TRUE. Default: FALSE.

- 1r The learning rate used by the Adam optimizer, i.e., torch::optim\_adam().
- dropout A length two vector specifying the dropout probabilities in the input and hidden layers respectively. The default is c(0,0) indicating no dropout.
- batchnorm If TRUE batch normalization will be used after each hidden activation.
- epochs The number of passes of the entire training dataset in gradient descent optimization. If early.stopping.epochs is used then this is the maximum number of passes. Default: 200.
- batch.size The size of mini batches for gradient calculation. Default: 128.
- valid.pct The fraction of data used as validation set. Default: 0.2.
- early.stopping.epochs The number of epochs before stopping if the validation loss does not decrease. Default: 10.
- print.every.epochs The number of epochs before next training progress in printed. Default: 10.
- save.path The path to save the fitted torch model. By default a folder named "SPQR\_model" is created in the current working directory to store the model.
- save.name The name of the file to save the fitted torch model. Default is "SPQR.model.pt".

#### 2. Parameters for MCMC method

These parameters are similar to those in rstan::stan(). Detailed explanations can be found in the Stan reference manual.

- algorithm The sampling algorithm; "HMC": Hamiltonian Monte Carlo with dual-averaging, "NUTS": No-U-Turn sampler (default).
- iter The number of MCMC iterations (including warmup). Default: 2000.
- warmup The number of warm-up/burn-in iterations for step-size and mass matrix adaptation. Default: 500.
- thin The number of iterations before saving next post-warmup samples.
   Default: 1.
- stepsize The discretization interval/step-size  $\epsilon$  of leap-frog integrator. Default is NULL which indicates that it will be adaptively selected during warm-up iterations.
- metric The type of mass matrix; "unit": diagonal matrix of ones, "diag": diagonal matrix with positive diagonal entries estimated during warmup iterations (default), "dense": a dense, symmetric positive definite matrix with entries estimated during warm-up iterations.
- delta The target Metropolis acceptance rate. Default: 0.9.
- max. treedepth The maximum tree depth in NUTS. Default: 6.
- int.time The integration time in HMC. The number of leap-frog steps is calculated as  $L_{\epsilon} = \lfloor t/\epsilon \rfloor$ . Default: 0.3.

normalize If TRUE, all covariates will be normalized to take values between [0,1].

summary.SPQR 21

```
verbose If TRUE (default), training progress will be printed.
seed Random number generation seed.
... other parameters to pass to control.
```

#### Value

An object of class SPQR. A list containing mostly internal model fitting information to be used by helper functions.

#### References

Xu SG, Reich BJ (2021). *Bayesian Nonparametric Quantile Process Regression and Estimation of Marginal Quantile Effects*. Biometrics. doi: 10.1111/biom.13576

## **Examples**

summary.SPQR

summary method for class SPQR

## **Description**

summarizes the output produced by SPQR() and structures them in a more organized way to be examined by the user.

## Usage

```
## S3 method for class 'SPQR'
summary(object, ...)
```

#### **Arguments**

```
object An object of class SPQR.
... Other arguments.
```

22 summary.SPQR

# Value

An object of class summary . SPQR. A list containing summary information of the fitted model.

method	The estimation method
time	The elapsed time
prior	If method = "MAP" or method = "MCMC", the hyperprior model for the variance hyperparameters
model	If method = "MLE" or method = "MAP", the fitted torch model. If method = "MCMC", the posterior samples of neural network parameters
loss	If method = "MLE" or method = "MAP", the train and validation loss
optim.info	If method = "MLE" or method = "MAP", configuration information of the Adam routine
elpd	If method = "MCMC", the expected log-predictive density
diagnostics	If method = "MCMC", diagnostic information of the MCMC chain

# **Index**

```
autoplot.SPQR, 2
coef.SPQR, 3
{\tt createFolds.SPQR,4}
createFolds.SPQR(), 5, 7
cv.SPQR, 5
load. SPQR, 7
plotEstimator, 8
plotEstimator(), 2, 3
plotGOF, 9
plotGOF(), 2, 3
{\tt plotMCMCtrace}, {\tt 10}
plotMCMCtrace(), 2, 3
plotQALE, 11
plotQALE(), 2, 3
plotQVI, 12
plotQVI(), 2, 3
predict.SPQR, 8, 13
print.SPQR, 15
print.summary.SPQR, 15, 16
QALE, 11, 12, 16
save. SPQR, 18
SPQR, 19
\verb|summary.SPQR|, 21|
```