Package ‘dCovTS’

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Description Computing and plotting the distance covariance and correlation function of a univariate or a multivariate time series. Both versions of biased and unbiased estimators of distance covariance and correlation are provided. Test statistics for testing pairwise independence are also implemented. Some data sets are also included. References include:
a) Edelmann Dominic, Fokianos Konstantinos and Pitsillou Maria (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. International Statistical Review, 87(2): 237--262. <doi:10.1111/insr.12294>.
b) Fokianos Konstantinos and Pitsillou Maria (2018). Testing independence for multivariate time series via the auto-distance correlation matrix. Biometrika, 105(2): 337--352. <doi:10.1093/biomet/asx082>.
c) Fokianos Konstantinos and Pitsillou Maria (2017). Consistent testing for pairwise dependence in time series. Technometrics, 59(2): 262--270. <doi:10.1080/00401706.2016.1156024>.
d) Pitsillou Maria and Fokianos Konstantinos (2016). dCovTS: Distance Covariance/Correlation for Time Series. R Journal, 8(2):324-340. <doi:10.32614/RJ-2016-049>.

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`dCovTS-package` *Distance Covariance and Correlation Theory for Time Series*

Description

Computing and plotting the distance covariance and correlation function of a univariate or a multivariate time series. Both versions of biased and unbiased estimators of distance covariance and correlation are provided. Test statistics for testing pairwise independence are also implemented. Some data sets are also included.

Details

Package:	dCovTS
Type:	Package
Version:	1.3
Date:	2022-06-27
License:	GPL(>=2)

Note

Disclaimer: Dr Maria Pitsillou is the actual creator of this package. Dr Tsagris is the current maintainer.

Improvements: We have modified the codes to run faster, we included the packages Rfast and Rfast2 for fast computations and the "dcov" package that allows for extremely fast computations of the distance correlation/covariance with univariate data.

Author(s)

Michail Tsagris, Maria Pitsillou and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Fokianos, K. and M. Pitsillou. (2018). Testing independence for multivariate time series via the auto-distance correlation matrix. *Biometrika*, 105, 337-352.

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Dehling, H. and T. Mikosch. (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis* 51, 392-413.

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Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

Leucht, A. and M. H. Neumann (2013). Dependent wild bootstrap for degenerate U- and V- statistics. *Journal of Multivariate Analysis*, 117, 257-280.

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Politis, N. P., J. P. Romano and M. Wolf (1999). *Subsampling*. New York: Springer.

Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

Shumway, R. H. and D. S. Stoffer (2011). *Time Series Analysis and Its Applications With R Examples*. New York: Springer. Third Edition.

Szekely, G. J. and M. L. Rizzo (2014). Partial distance correlation with methods for dissimilarities. *The Annals of Statistics*, 42, 2382-2412.

Szekely, G. J., M. L. Rizzo and N. K. Bakirov (2007). Measuring and testing dependence by correlation of distances. *The Annals of Statistics*, 35, 2769-2794, .

Tsay, R. S. (2014). *Multivariate Time Series Analysis with R and Financial Applications*. Hoboken, NJ: Wiley.

Tsay, R. S. (2010). *Analysis of Financial Time Series*. Hoboken, NJ: Wiley. Third edition.

Zhou, Z. (2012). Measuring nonlinear dependence in time series, a distance correlation approach. *Journal of Time Series Analysis*, 33, 438-457.

ADCF *Auto-Distance Correlation Function*

Description

Computes the auto-distance correlation function of a univariate time series. It also computes the bias-corrected estimator of (squared) auto-distance correlation.

Usage

```
ADCF(x, MaxLag = 15, unbiased = FALSE)
```

Arguments

`x`	A numeric vector or univariate time series.
`MaxLag`	Maximum lag order at which to calculate the `ADCF`. Default is 15.
`unbiased`	Logical value. If unbiased = TRUE, the bias-corrected estimator of squared auto-distance correlation is returned. Default value is FALSE.

Details

Distance covariance and correlation firstly introduced by Szekely et al. (2007) are new measures of dependence between two random vectors. Zhou (2012) extended this measure to univariate time series framework.

For a univariate time series, `ADCF` computes the auto-distance correlation function, $R_X(j)$, between $\{X_t\}$ and $\{X_{t+j}\}$, whereas `ADCV` computes the auto-distance covariance function between them, denoted by $V_X(j)$. Formal definition of $R_X(\cdot)$ and $V_X(\cdot)$ can be found in Zhou (2012) and Fokianos and Pitsillou (2017). The empirical auto-distance correlation function, $\hat{R}_X(j)$, is computed as the positive square root of

$$\hat{R}^2_X(j) = \frac{\hat{V}^2_X(j)}{\hat{V}^2_X(0)}, \quad j = 0, \pm 1, \pm 2, \ldots$$

for $\hat{V}^2_X(0) \neq 0$ and zero otherwise, where $\hat{V}_X(\cdot)$ is a function of the double centered Euclidean distance matrices of the sample X_t and its lagged sample X_{t+j} (see `ADCV` for more details). Theoretical properties of this measure can be found in Fokianos and Pitsillou (2017).

If unbiased = TRUE, `ADCF` computes the bias-corrected estimator of the squared auto-distance correlation, $\tilde{R}^2_X(j)$, based on the unbiased estimator of auto-distance covariance function, $\tilde{V}^2_X(j)$. The definition of $\tilde{V}^2_X(j)$ relies on the U-centered matrices proposed by Szekely and Rizzo (2014) (see `ADCV` for a brief description).

`mADCF` computes the auto-distance correlation function of a multivariate time series.

Value

Returns a vector, whose length is determined by `MaxLag`, and contains the biased estimator of `ADCF` or the bias-corrected estimator of squared `ADCF`.

Note

Based on the definition of ADCF, one can observe that $R^2_X(j) = R^2_X(-j)\ \forall\ j$, and so results based on negative lags are omitted.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Fokianos K. and M. Pitsillou (2017). Consistent testing for pairwise dependence in time series. *Technometrics*, 159(2), 262-3270.

Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

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Szekely, G. J. and M. L. Rizzo (2014). Partial distance correlation with methods for dissimilarities. *The Annals of Statistics*, 42, 2382-2412.

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Zhou, Z. (2012). Measuring nonlinear dependence in time series, a distance correlation approach. *Journal of Time Series Analysis* 33, 438-457.

See Also

`ADCFplot`, `ADCV`, `mADCF`

Examples

```
x <- rnorm(100)
ADCF(x)

ADCF(ldeaths, 18)

ADCF(mdeaths, unbiased = TRUE)
```

ADCFplot *Auto-distance correlation plot*

Description

The function plots the estimated auto-distance correlation function obtained by `ADCF` and provides confindence intervals by employing three bootstrap based methods.

Usage

```
ADCFplot(x, MaxLag = 15, alpha = 0.05, b = 499, bootMethod =
c("Wild Bootstrap", "Subsampling", "Independent Bootstrap"), ylim = NULL, main = NULL)
```

Arguments

x	A numeric vector or univariate time series.
MaxLag	The maximum lag order at which to plot ADCF. Default is 15.
alpha	The significance level used to construct the $(1-\alpha)\%$ empirical critical values.
b	The number of bootstrap replications for constructing the $(1-\alpha)\%$ empirical critical values. Default is 499.
bootMethod	A character string indicating the method to use for obtaining the $(1-\alpha)\%$ critical values. Possible choices are "Wild Bootstrap" (the default), "Independent Bootstrap" and "Subsampling".
ylim	A numeric vector of length 2 indicating the y limits of the plot. The default value, NULL, indicates that the range $(0, v)$, where v is the maximum number between 1 and the empirical critical values, should be used.
main	The title of the plot.

Details

Fokianos and Pitsillou (2018) showed that the sample auto-distance covariance function ADCV (and thus ADCF) can be expressed as a V-statistic of order two, which under the null hypothesis of independence is degenerate. Thus, constructing a plot analogous to the traditional autocorrelation plot where the confidence intervals are obtained simultaneously, turns to be a complicated task. To overcome this issue, the $(1-\alpha)\%$ confidence intervals shown in the plot (dotted blue horizontal line) are computed simultaneously via Monte Carlo simulation, and in particular via the independent wild bootstrap approach (Dehling and Mikosch, 1994; Shao, 2010; Leucht and Neumann, 2013). The reader is referred to Fokianos and Pitsillou (2018) for the steps followed. mADCFplot returns an analogous plot of the estimated auto-distance correlation function for a multivariate time series.

One can also compute the pairwise $(1-\alpha)\%$ critical values via the subsampling approach suggested by Zhou (2012, Section 5.1).That is, the critical values are obtained at each lag separately. The block size of the procedure is based on the minimum volatility method proposed by Politis et al. (1999, Section 9.4.2). In addition, the function provides the ordinary independent bootstrap methodology to derive simultaneous $(1-\alpha)\%$ critical values.

Value

A plot of the estimated ADCF values. It also returns a list including:

ADCF	The sample auto-distance correlation function for all lags specified by MaxLag.
bootMethod	The method followed for computing the $(1-\alpha)\%$ confidence intervals of the plot.
critical.value	The critical value shown in the plot.

Note

When the critical values are obtained via the Subsampling methodology, the function returns a plot that starts from lag 1.

The function plots only the biased estimator of ADCF.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Dehling, H. and T. Mikosch (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis*, 51, 392-413.

Dominic, E, K. Fokianos and M. Pitsillou Maria (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

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Leucht, A. and M. H. Neumann (2013). Dependent wild bootstrap for degenerate U- and V- statistics. *Journal of Multivariate Analysis*, 117, 257-280.

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Politis, N. P., J. P. Romano and M. Wolf (1999). *Subsampling*. New York: Springer.

Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

Zhou, Z. (2012). Measuring nonlinear dependence in time series, a distance correlation approach. *Journal of Time Series Analysis*, 33, 438-457.

See Also

ADCF, ADCV, mADCFplot

Examples

```
### x <- rnorm(200)
### ADCFplot(x, bootMethod = "Subs")
```

ADCV *Auto-distance Covariance Function*

Description

Computes the auto-distance covariance function of a univariate time series. It also computes the unbiased estimator of squared auto-distance covariance.

Usage

```
ADCV(x, MaxLag = 15, unbiased = FALSE)
```

Arguments

`x`	A numeric vector or univariate time series.
`MaxLag`	The maximum lag order at which to calculate the `ADCV`. Default is 15.
`unbiased`	A logical value. If unbiased = TRUE, the unbiased estimator of squared auto-distance covariance is returned. Default value is FALSE.

Details

Szekely et al. (2007) proposed distance covariance function between two random vectors. Zhou (2012) extended this measure of dependence to a time series framework by calling it auto-distance covariance function.

`ADCV` computes the sample auto-distance covariance function, $V_X(\cdot)$, between $\{X_t\}$ and $\{X_{t+j}\}$. Formal definition of $V_X(\cdot)$ can be found in Zhou (2012) and Fokianos and Pitsillou (2017).

The empirical auto-distance covariance function, $\hat{V}_X(\cdot)$, is the non-negative square root defined by

$$\hat{V}_X^2(j) = \frac{1}{(n-j)^2} \sum_{r,l=1+j}^{n} A_{rl} B_{rl}, \quad 0 \le j \le (n-1)$$

and $\hat{V}_X^2(j) = \hat{V}_X^2(-j)$, for $-(n-1) \le j < 0$, where $A = A_{rl}$ and $B = B_{rl}$ are Euclidean distances with elements given by

$$A_{rl} = a_{rl} - \bar{a}_{r.} - \bar{a}_{.l} + \bar{a}_{..}$$

with $a_{rl} = |X_r - X_l|$, $\bar{a}_{r.} = \left(\sum_{l=1+j}^{n} a_{rl}\right)/(n-j)$, $\bar{a}_{.l} = \left(\sum_{r=1+j}^{n} a_{rl}\right)/(n-j)$, $\bar{a}_{..} = \left(\sum_{r,l=1+j}^{n} a_{rl}\right)/(n-j)^2$. B_{rl} is given analogously based on $b_{rl} = |Y_r - Y_l|$, where $Y_t = X_{t+j}$. X_t and X_{t+j} are independent if and only if $V_X^2(j) = 0$. See Fokianos and Pitsillou (2017) for more information on theoretical properties of $V_X^2(\cdot)$ including consistency.

If unbiased = TRUE, `ADCV` returns the unbiased estimator of squared auto-distance covariance function, $\tilde{V}_X^2(j)$, proposed by Szekely and Rizzo (2014). In the context of time series data, this is given by

$$\tilde{V}_X^2(j) = \frac{1}{(n-j)(n-j-3)} \sum_{r \ne l} \tilde{A}_{rl} \tilde{B}_{rl},$$

for $n > 3$, where $\tilde{A}_{rl}$ is the (r, l) element of the so-called U-centered matrix $\tilde{A}$, defined by

$$\tilde{A}_{rl} = \frac{1}{n-j-2}\sum_{t=1+j}^{n} a_{rt} - \frac{1}{n-j-2}\sum_{s=1+j}^{n} a_{sl} + \frac{1}{(n-j-1)(n-j-2)}\sum_{t,s=1+j}^{n} a_{ts}, \quad i \neq j,$$

with zero diagonal.

`mADCV` gives the auto-distance covariance function of a multivariate time series.

Value

A vector whose length is determined by `MaxLag` and contains the biased estimator of `ADCV` or the unbiased estimator of squared `ADCV`.

Note

Based on the definition of $\hat{V}_X(\cdot)$, we observe that $\hat{V}_X^2(j) = \hat{V}_X^2(-j)$, and thus results based on negative lags are omitted.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Dominic, E, K. Fokianos and M. Pitsillou Maria (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262. .

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Pitsillou M. and Fokianos K. (2016). dCovTS: Distance Covariance/Correlation for Time Series. *R Journal*, 8, 324-340.

Szekely, G. J. and M. L. Rizzo (2014). Partial distance correlation with methods for dissimilarities. *The Annals of Statistics* 42, 2382-2412.

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Zhou, Z. (2012). Measuring nonlinear dependence in time series, a distance correlation approach. *Journal of Time Series Analysis* 33, 438-457.

See Also

`ADCF`, `mADCV`

Examples

```
x <- rnorm(500)
ADCV(x, 18)

ADCV(BJsales, 25)
```

ibmSp500 *Monthly returns of IBM and S&P 500 composite index*

Description

The monthly returns of the stocks of International Business Machines (IBM) and the S&P 500 composite index from January 1926 to December 2011.

Usage

```
ibmSp500
```

Format

A data frame with 1,032 observations on the following 3 variables.

`date` a numeric vector

`ibm` a numeric vector

`sp` a numeric vector

Source

The data is a combination of two datasets:

- The first 612 observations are in Tsay (2010).
- The rest 420 observations are in Tsay (2014).

References

Tsay, R. S. (2010). *Analysis of Financial Time Series*. Hoboken, NJ: Wiley. Third edition.

Tsay, R. S. (2014). *Multivariate Time Series Analysis with R and Financial Applications*. Hoboken, NJ: Wiley.

Examples

```
### attach(ibmSp500)
### series <- tail(ibmSp500[, 2:3], 400)
### lseries <- log(series + 1)
### mADCFplot(lseries, MaxLag = 12)
### mADCFplot(lseries^2, MaxLag = 12)
```

`kernelFun` *Several kernel functions*

Description

Computes several kernel functions(truncated, Bartlett, Daniell, QS, Parzen). These kernels are for constructing test statistics for testing pairwise independence.

Usage

```
kernelFun(type, z)
```

Arguments

`type`	A character string which indicates the name of the smoothing kernel. `kernelFun` can be: 'truncated', 'bartlett', 'daniell', 'QS', 'parzen'. No default is given.
`z`	A real number.

Details

`kernelFun` computes several kernel functions including truncated, Bartlett, Daniell, QS and Parzen.

The exact definition of each of the above functions are given below:

- Truncated
$$k(z) = \begin{cases} 1, & |z| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$
- Bartlett
$$k(z) = \begin{cases} 1 - |z|, & |z| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$
- Daniell
$$k(z) = \frac{\sin(\pi z)}{\pi z}, z \in \Re - \{0\}$$
- QS
$$k(z) = (9/5\pi^2 z^2)\{\sin(\sqrt{5/3}\pi z)/\sqrt{5/3}\pi z - \cos(\sqrt{5/3}\pi z)\}, z \in \Re$$
- Parzen
$$k(z) = \begin{cases} 1 - 6(\pi z/6)^2 + 6|\pi z/6|^3, & |z| \le 3/\pi, \\ 2(1 - |\pi z/6|)^3, & 3/\pi \le |z| \le 6/\pi, \\ 0, & \text{otherwise} \end{cases}$$

All these kernel functions are mainly used to smooth the generalized spectral density function, firstly introduced by Hong (1999). Assumptions and theoretical properties of these functions can be found in Hong (1996;1999) and Fokianos and Pitsillou (2017).

Value

A value that lies in the interval $[-1, 1]$.

Author(s)

Maria Pitsillou and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

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Hong, Y. (1999). Hypothesis testing in time series via the empirical characteristic function: A generalized spectral density approach. *Journal of the American Statistical Association*, 94, 1201-1220.

Examples

```
k1 <- kernelFun( "bartlett", z = 1/3 )
k2 <- kernelFun( "bar", z = 1/5 )
k3 <- kernelFun( "dan", z = 0.5 )
```

---

mADCF *Auto-Distance Correlation Matrix*

Description

Computes the auto-distance correlation matrix of a multivariate time series.

Usage

```
mADCF(x, lags, unbiased = FALSE, output = TRUE)
```

Arguments

`x`	Multivariate time series.
`lags`	The lag order at which to calculate the `mADCF`. No default is given. This can be a single number or a vector of numbers with different lag orders.
`unbiased`	A logical value. If unbiased = TRUE, the individual elements of auto-distance correlation matrix correspond to the bias-corrected estimators of squared auto-distance correlation functions. Default value is FALSE.
`output`	A logical value. If output=FALSE, no output is given. Default value is TRUE.

Details

If $\mathbf{X}_t = (X_{t;1}, \ldots, X_{t;d})'$ is a multivariate time series of dimension d, then `mADCF` computes the sample auto-distance correlation matrix, $\hat{R}(\cdot)$, of $\mathbf{X}_t$. It is defined by

$$\hat{R}(j) = [\hat{R}_{rm}(j)]_{r,m=1}^{d}, \quad j = 0, \pm 1, \pm 2, \ldots,$$

where $\hat{R}_{rm}(j)$ is the biased estimator of the so-called pairwise auto-distance correlation function between $X_{t;r}$ and $X_{t+j;m}$ given by the positive square root of

$$\hat{R}^2_{rm}(j) = \frac{\hat{V}^2_{rm}(j)}{\hat{V}_{rr}(0)\hat{V}_{mm}(0)}$$

for $\hat{V}_{rr}(0)\hat{V}_{mm}(0) \neq 0$ and zero otherwise.

$\hat{V}_{rm}(j)$ is the (r, m) element of the corresponding `mADCV` matrix at lag j. Formal definition and more details can be found in Fokianos and Pitsillou (2017).

If unbiased = TRUE, `mADCF` returns a matrix that contains the bias-corrected estimators of squared pairwise auto-distance correlation functions.

Value

If `lags` is a single number then the function will return a matrix. If `lags` is a vector of many values the function will return an array. For either case, the matrix (matrices) will contain either the biased estimators of the pairwise auto-distance correlation functions or the bias-corrected estimators of squared pairwise auto-distance correlation functions at lag, j, determined by the argument `lags`.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

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Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

Pitsillou M. and Fokianos K. (2016). dCovTS: Distance Covariance/Correlation for Time Series. *R Journal*, 8, 324-340.

See Also

`ADCF`, `mADCV`

Examples

```
x <- matrix( rnorm(200), ncol = 2 )

mADCF(x, 2)

mADCF(x, -2)

mADCF(x, lags = 4, unbiased = TRUE)
```

---

mADCFplot — *Distance cross-correlation plot*

Description

The function computes and plots the estimator of the auto-distance correlation matrix `mADCF`.

Usage

```
mADCFplot(x, MaxLag = 15, alpha = 0.05, b = 499,
          bootMethod = c("Wild Bootstrap", "Independent Bootstrap"),
          ylim = NULL)
```

Arguments

`x`	A multivariate time series.
`MaxLag`	The maximum lag order at which to plot `mADCF`. Default is 15.
`alpha`	The significance level used to construct the $(1-\alpha)\%$ empirical critical values.
`b`	The number of bootstrap replications for constructing the $(1-\alpha)\%$ empirical critical values. Default is 499.
`bootMethod`	A character string indicating the method to use for obtaining the $(1-\alpha)\%$ critical values. Possible choices are "Wild Bootstrap" (the default) and "Independent Bootstrap".
`ylim`	A numeric vector of length 2 indicating the `y` limits of the plot. The default value, NULL, indicates that the range $(0, v)$, where v is the maximum number between 1 and the empirical critical values, should be used.

Details

The $(1-\alpha)\%$ confidence intervals shown in the plot (dotted blue horizontal line) are computed simultaneously based on the independent wild bootstrap approach (Dehling and Mikosch, 1994; Shao, 2010; Leucht and Neumann, 2013), since the elements of `mADCV` (and thus `mADCF`) can be expressed as degenerate V-statistics of order 2. More details can be found in Fokianos and Pitsillou (2017).

In addition, `mADCFplot` provides the option of independent bootstrap to compute the simultaneous $(1-\alpha)\%$ critical values.

Value

A plot of the estimated `mADCF` matrices. The function also returns a list including

`matrices`	Sample distance correlation matrices starting from lag 0.
`bootMethod`	The method followed for computing the $(1-\alpha)\%$ confidence intervals of the plot.
`critical.value`	The critical value shown in the plot.

Note

The function plots only the biased estimator of ADCF matrix.

Author(s)

Maria Pitsillou and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Dehling, H. and T. Mikosch (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis*, 51, 392-413.

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Fokianos K. and M. Pitsillou (2017). Consistent testing for pairwise dependence in time series. *Technometrics*, 159, 262-3270.

Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

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Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

See Also

`mADCF`, `mADCV`

Examples

```
### x <- matrix( rnorm(200), ncol = 2 )
### mADCFplot(x, 12, ylim = c(0, 0.5) )
### mADCFplot(x, 12, b = 100)
```

mADCFtest *Distance Correlation test of independence in multivariate time series*

Description

A multivariate test of independence based on auto-distance correlation matrix proposed by Fokianos and Pitsillou (2017).

Usage

```
mADCFtest(x, type = c("truncated", "bartlett", "daniell", "QS", "parzen"), p,
          b = 0, parallel = FALSE, bootMethod = c("Wild Bootstrap",
          "Independent Bootstrap"))
```

Arguments

x	multivariate time series.
type	A character string which indicates the smoothing kernel. Possible choices are 'truncated' (the default), 'bartlett', 'daniell', 'QS', 'parzen'.
p	The bandwidth, whose choice is determined by $p = cn^{\lambda}$ for $c > 0$ and $\lambda \in (0, 1)$.
b	The number of bootstrap replicates of the test statistic. It is a positive integer. If b=0 (the default), then no p-value is returned.
parallel	A logical value. By default, parallel=FALSE. If parallel=TRUE, bootstrap computation is distributed to multiple cores, which typically is the maximum number of available CPUs and is detecting directly from the function.
bootMethod	A character string indicating the method to use for obtaining the empirical p-value of the test. Possible choices are "Wild Bootstrap" (the default) and "Independent Bootstrap".

Details

mADCFtest performs a test of multivariate independence. In particular, the function computes a test statistic for testing whether the data are independent and identically distributed (i.i.d). The p-value of the test is obtained via resampling method. Possible choices are the independent wild bootstrap (Dehling and Mikosch, 1994; Shao, 2010; Leucht and Neumann, 2013) and the independent bootstrap, with b replicates. The observed statistic is given by

$$\sum_{j=1}^{n-1}(n-j)k^2(j/p)\mathrm{tr}\{\hat{V}^*(j)\hat{D}^{-1}\hat{V}(j)\hat{D}^{-1}\}$$

where $\hat{D}^{-1} = \mathrm{diag}\{\hat{V}_{11}(0), \ldots, \hat{V}_{dd}(0)\}$ with d denoting the dimension of the multivariate time series and $\hat{V}_{rm}(0)$ is obtained from the elements of the corresponding matrix mADCV. $\hat{V}^*(\cdot)$ denotes the complex conjugate matrix of $\hat{V}(\cdot)$ obtained from mADCV, and tr$\{A\}$ denotes the trace of a matrix

A. $k(\cdot)$ is a kernel function computed by kernelFun and p is a bandwidth or lag order whose choice is further discussed in Fokianos and Pitsillou (2017).

Under the null hypothesis of independence and some further assumptions about the kernel function $k(\cdot)$, the standardized version of the test statistic follows $N(0,1)$ asymptotically and it is consistent. More details of the asymptotic properties of the statistic can be found in Fokianos and Pitsillou (2017).

mADCVtest performs the same test based on the auto-distance covariance matrix mADCV.

Value

An object of class htest which is a list containing:

method	The description of the test.
statistic	The observed value of the test statistic.
replicates	The bootstrap replicates of the test statistic (if $b = 0$ then replicates=NULL).
p.value	The p-value of the test (if $b = 0$ then p.value=NA).
bootMethod	The method followed for computing the p-value of the test.
data.name	A description of the data (data name, kernel type, type, bandwidth, p, and the number of bootstrap replicates, b).

Note

The computation of the test statistic is only based on the biased estimator of auto-distance covariance matrix.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Dehling, H. and T. Mikosch (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis*, 51, 392-413.

Fokianos K. and Pitsillou M. (2018). Testing independence for multivariate time series via the auto-distance correlation matrix. *Biometrika*, 105, 337-352.

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Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

Leucht, A. and M. H. Neumann (2013). Dependent wild bootstrap for degenerate U- and V- statistics. *Journal of Multivariate Analysis*, 117, 257-280.

Pitsillou M. and Fokianos K. (2016). dCovTS: Distance Covariance/Correlation for Time Series. *R Journal*, 8, 324-340.

Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

See Also

mADCF, mADCV, mADCVtest

Examples

```
x <- matrix( rnorm(200), ncol = 2 )
n <- length(x)
c <- 3
lambda <- 0.1
p <- ceiling(c * n^lambda)
mF <- mADCFtest(x, type = "truncated", p = p, b = 500, parallel = FALSE)
```

---

mADCV *Auto-Distance Covariance Matrix*

Description

Computes the sample auto-distance covariance matrices of a multivariate time series.

Usage

```
mADCV(x, lags, unbiased = FALSE, output = TRUE)
```

Arguments

x	Multivariate time series.
lags	The lag order at which to calculate the mADCV. No default is given.
unbiased	A logical value. If unbiased = TRUE, the individual elements of auto-distance covariance matrix correspond to the unbiased estimators of squared auto-distance covariance functions. Default value is FALSE.
output	A logical value. If output=FALSE, no output is given. Default value is TRUE.

Details

Suppose that $\mathbf{X}_t = (X_{t;1}, \ldots, X_{t;d})'$ is a multivariate time series of dimension d. Then, mADCV computes the $d \times d$ sample auto-distance covariance matrix, $\hat{V}(\cdot)$, of $\mathbf{X}_t$ given by

$$\hat{V}(j) = [\hat{V}_{rm}(j)]_{r,m=1}^{d}, \quad j = 0, \pm 1, \pm 2, \ldots,$$

where $\hat{V}_{rm}(j)$ denotes the biased estimator of the pairwise auto-distance covariance function between $X_{t;r}$ and $X_{t+j;m}$. The definition of $\hat{V}_{rm}(j)$ is given analogously as in the univariate case (see ADCV). Formal definitions and theoretical properties of auto-distance covariance matrix can be found in Fokianos and Pitsillou (2018).

If unbiased = TRUE, mADCV computes the matrix, $\tilde{V}^{(2)}(j)$, whose elements correspond to the unbiased estimators of squared pairwise auto-distance covariance functions, namely

$$\tilde{V}^{(2)}(j) = [\tilde{V}^{2}_{rm}(j)]_{r,m=1}^{d}, \quad j = 0, \pm 1, \pm 2, \ldots.$$

The definition of $\tilde{V}^{2}_{rm}(\cdot)$ is defined analogously as explained in the univariate case (see ADCV).

Value

If `lags` is a single number then the function will return a matrix. If `lags` is a vector of many values the function will return an array. For either case, the matrix (matrices) will contain either the biased estimators of the pairwise auto-distance covariance functions or the unbiased estimators of squared pairwise auto-distance covariance functions at lag, j, determined by the argument `lags`.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Fokianos K. and Pitsillou M. (2018). Testing independence for multivariate time series via the auto-distance correlation matrix. *Biometrika*, 105, 337-352.

Huo, X. and G. J. Szekely. (2016). Fast Computing for Distance Covariance. *Technometrics*, 58, 435-447.

Pitsillou M. and Fokianos K. (2016). dCovTS: Distance Covariance/Correlation for Time Series. *R Journal*, 8, 324-340.

See Also

`ADCV`, `mADCF`

Examples

```
x <- matrix( rnorm(200), ncol = 2 )
mADCV(x, lags = 1)
mADCV(x, lags = 15)

y <- as.ts(swiss)
mADCV(y, 15)
mADCV(y, 15, unbiased = TRUE)
```

`mADCVtest` *Distance covariance test of independence in multivariate time series*

Description

A test of independence based on auto-distance covariance matrix in multivariate time series proposed by Fokianos a nd Pitsillou (2017).

Usage

```
mADCVtest(x, type = c("truncated", "bartlett", "daniell", "QS", "parzen"), p,
          b = 0, parallel = FALSE, bootMethod = c("Wild Bootstrap",
          "Independent Bootstrap"))
```

Arguments

x	Multivariate time series.
type	A character string which indicates the smoothing kernel. Possible choices are 'truncated' (the default), 'bartlett', 'daniell', 'QS', 'parzen'.
p	The bandwidth, whose choice is determined by $p = cn^{\lambda}$ for $c > 0$ and $\lambda \in (0, 1)$.
b	The number of bootstrap replicates of the test statistic. It is a positive integer. If b=0 (the default), then no p-value is returned.
parallel	A logical value. By default, parallel=FALSE. If parallel=TRUE, bootstrap computation is distributed to multiple cores, which typically is the maximum number of available CPUs and is detecting directly from the function.
bootMethod	A character string indicating the method to use for obtaining the empirical p-value of the test. Possible choices are "Wild Bootstrap" (the default) and "Independent Bootstrap".

Details

mADCVtest tests whether the vector series are independent and identically distributed (i.i.d). The p-value of the test is obtained via resampling scheme. Possible choices are the independent wild bootstrap (Dehling and Mikosch, 1994; Shao, 2010; Leucht and Neumann, 2013) and independent bootstrap, with b replicates. The observed statistic is

$$\sum_{j=1}^{n-1}(n-j)k^2(j/p)\mathrm{tr}\{\hat{V}^*(j)\hat{V}(j)\}$$

where $\hat{V}^*(\cdot)$ denotes the complex conjugate matrix of $\hat{V}(\cdot)$ obtained from mADCV, and tr$\{A\}$ denotes the trace of a matrix A, which is the sum of the diagonal elements of A. $k(\cdot)$ is a kernel function computed by kernelFun and p is a bandwidth or lag order whose choice is further discussed in Fokianos and Pitsillou (2017).

Under the null hypothesis of independence and some further assumptions about the kernel function $k(\cdot)$, the standardized version of the test statistic follows $N(0, 1)$ asymptotically and it is consistent. More details of the asymptotic properties of the statistic can be found in Fokianos and Pitsillou (2017).

mADCFtest performs the same test based on the distance correlation matrix mADCF.

Value

An object of class htest which is a list including:

method	The description of the test.
statistic	The observed value of the test statistic.
replicates	Bootstrap replicates of the test statistic (if $b = 0$ then replicates=NULL).
p.value	The p-value of the test (if $b = 0$ then p.value=NA).
bootMethod	The method followed for computing the p-value of the test.
data.name	The description of the data (data name, kernel type, type, bandwidth, p, and the number of bootstrap replicates b).

Note

The computation of the test statistic is only based on the biased estimator of auto-distance covariance matrix.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Edelmann, D, K. Fokianos. and M. Pitsillou. (2019). An Updated Literature Review of Distance Correlation and Its Applications to Time Series. *International Statistical Review*, 87, 237-262.

Dehling, H. and T. Mikosch (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis*, 51, 392-413.

Fokianos K. and Pitsillou M. (2018). Testing independence for multivariate time series via the auto-distance correlation matrix. *Biometrika*, 105, 337-352.

Fokianos K. and M. Pitsillou (2017). Consistent testing for pairwise dependence in time series. *Technometrics*, 159, 262-3270.

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Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

See Also

mADCV, mADCF, mADCFtest

Examples

```
x <- matrix( rnorm(200), ncol = 2 )
n <- length(x)
c <- 3
lambda <- 0.1
p <- ceiling(c * n^lambda)
mF <- mADCVtest(x, type = "bar", p = p, b = 500, parallel = FALSE)
```

MortTempPart — *Cardiovascular mortality, temperature and pollution data in Los Angeles County*

Description

Cardiovascular mortality data measured daily in Los Angeles County over the 10 year period 1970-1979. Temperature series and pollutant particulate series corresponding to mortality data are also given.

Usage

```
MortTempPart
```

Format

A data frame with 508 observations on the following 3 variables.

`cmort` A numeric vector.

`tempr` A numeric vector.

`part` A numeric vector.

References

Shumway R. H. and D. S. Stoffer (2011). *Time Series Analysis and Its Applications With R Examples*. New York: Springer. Third Edition.

Examples

```
### mADCFplot(MortTempPart[1:100, ], MaxLag = 10)
```

UnivTest — *Testing for independence in univariate time series*

Description

A test of pairwise independence for univariate time series.

Usage

```
UnivTest(x, type = c("truncated", "bartlett", "daniell", "QS", "parzen"),
         testType = c("covariance", "correlation"), p, b = 0, parallel = FALSE,
         bootMethod = c("Wild Bootstrap", "Independent Bootstrap"))
```

Arguments

x	A numeric vector or univariate time series.
type	A character string which indicates the smoothing kernel. Possible choices are 'truncated' (the default), 'bartlett', 'daniell', 'QS', 'parzen'.
testType	A character string indicating the type of the test to be used. Allowed values are 'covariance' (default) for using the distance covariance function and 'correlation' for using the distance correlation function.
p	The bandwidth, whose choice is determined by $p = cn^{\lambda}$ for $c > 0$ and $\lambda \in (0, 1)$.
b	The number of bootstrap replicates of the test statistic. It is a positive integer. If b=0 (the default), then no p-value is returned.
parallel	A logical value. By default, parallel=FALSE. If parallel=TRUE, bootstrap computation is distributed to multiple cores, which typically is the maximum number of available CPUs and is detecting directly from the function.
bootMethod	A character string indicating the method to use for obtaining the empirical p-value of the test. Possible choices are "Wild Bootstrap" (the default) and "Independent Bootstrap".

Details

`UnivTest` performs a test on the null hypothesis of independence in univariate time series. The p-value of the test is obtained via resampling method. Possible choices are the independent wild bootstrap (Dehling and Mikosch, 1994; Shao, 2010; Leucht and Neumann, 2013) (default option) and the ordinary independent bootstrap, with `b` replicates. If `typeTest` = 'covariance' then, the observed statistic is

$$\sum_{j=1}^{n-1} (n-j) k^2(j/p) \hat{V}_X^2(j),$$

otherwise

$$\sum_{j=1}^{n-1} (n-j) k^2(j/p) \hat{R}_X^2(j),$$

where $k(\cdot)$ is a kernel function computed by `kernelFun` and `p` is a bandwidth or lag order whose choice is further discussed in Fokianos and Pitsillou (2017).

Under the null hypothesis of independence and some further assumptions about the kernel function $k(\cdot)$, the standardizedversion of the test statistic follows $N(0, 1)$ asymptotically and it is consistent. More details of the asymptotic properties of the statistic can be found in Fokianos and Pitsillou (2017).

Value

An object of class `htest` which is a list including:

method	The description of the test.
statistic	The observed value of the test statistic.
replicates	Bootstrap replicates of the test statistic (if $b = 0$ then `replicates`=NULL).

p.value The p-value of the test (if $b = 0$ then p.value=NA).

bootMethod The method followed for computing the p-value of the test.

data.name Description of data (the data name, kernel type, type, bandwidth, p, and the number of bootstrap replicates b).

Note

The observed statistics of the tests are only based on the biased estimators of distance covariance and correlation functions.

Author(s)

Maria Pitsillou, Michail Tsagris and Konstantinos Fokianos.

References

Dehling, H. and T. Mikosch (1994). Random quadratic forms and the bootstrap for U-statistics. *Journal of Multivariate Analysis*, 51, 392-413.

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Pitsillou M. and Fokianos K. (2016). dCovTS: Distance Covariance/Correlation for Time Series. R Journal, 8, 324-340.

Shao, X. (2010). The dependent wild bootstrap. *Journal of the American Statistical Association*, 105, 218-235.

See Also

ADCF, ADCV

Examples

```
dat <- tail(ibmSp500[, 2], 100)
n2 <- length(dat)
c2 <- 3
lambda2 <- 0.1
p2 <- ceiling(c2 * n2^lambda2)
testCov <- UnivTest(dat, type = "par", testType = "covariance", p = p2,
                    b = 500, parallel = FALSE)
testCor <- UnivTest(dat, type = "par", testType = "correlation", p = p2,
                    b = 500, parallel = FALSE)
```

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