

# Package ‘eel’

October 13, 2022

**Type** Package

**Title** Extended Empirical Likelihood

**Version** 1.1

**Date** 2015-08-30

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**Description** Compute the extended empirical log likelihood ratio (Tsao & Wu, 2014) for the mean and parameters defined by estimating equations.

**Depends** emplik, rootSolve

**License** GPL (>= 2)

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2015-09-01 17:31:11

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 eel-package

*Extended Empirical Likelihood*


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## Description

Compute the extended empirical log likelihood ratio (Tsao & Wu, 2014) for the mean and parameters defined by estimating equations.

## Details

Index of help topics:

EEL	Extended empirical log likelihood ratio for the mean
EEL_est	Extended empirical log likelihood ratio for parameters defined by estimating equations
EMLogLR	Original empirical log likelihood ratio
eel-package	Extended Empirical Likelihood
exp_factor	Calculating expansion factor for EEL for the mean
exp_factor_est	Calculating expansion factor for EEL for parameters defined by estimating equations
prime_image	Calculating prime-image based on similarity mapping for the mean
prime_image_est	Calculating prime-image based on similarity mapping for parameters defined by estimating equations
print.EEL	Printing EEL objects
summary.EEL	Summarizing EEL objects

The extended empirical log likelihood ratio for the mean is computed by calling the function `EEL()`, and that for the parameter defined estimating equations is computed by calling the function `EEL_est()`. This package requires pre-installation of two packages "emplik" and "rootSolve". These are needed for computing the prime image of a point  $\theta$  as well as the final extended empirical log likelihood ratio value as described in Tsao and Wu (2013, 2014). Only the first-order EEL discussed Tsao and Wu (2013, 2014) is included in this package.

## Author(s)

Fan Wu and Yu Zhang

Maintainer: Yu Zhang <yuz@uvic.ca>

## References

Tsao, M. (2013). Extending the empirical likelihood by domain expansion. *The Canadian Journal of Statistics*, 41 (2), 257-274.

Tsao, M., & Wu, F. (2013). Empirical likelihood on the full parameter space. *Annals of Statistics*, 0 (00), 1-21. doi: 10.1214/13-AOS1143

Tsao, M., & Wu, F. (2014). Extended empirical likelihood for estimating equations. *Biometrika*, 1-8. doi: 10.1093/biomet/asu014

### See Also

[EMLogLR](#), [EEL](#), [EEL\\_est](#), [exp\\_factor](#), [prime\\_image](#), [prime\\_image\\_est](#), [exp\\_factor\\_est](#),

### Examples

```
# EXAMPLE: computing the EEL for the mean of a bivariate random variable
# Generating a sample of n=40 bivariate observations.
# For this example, we do this through a univariate normal random number generator.

uninorm<- rnorm(40*2,5,1)
multnorm<-matrix(uninorm,ncol=2)

# To calculate the EEL for a point theta=c(5,2), use
EEL(x=multnorm,theta=c(5,2))

# an example to use the EEL_est in the case of estimating equation

# generate regression dataset
# random variable x
dmx2<-runif(100,min=0,max=100)
dmx<-matrix(0,100,2)
dmx[,1]=1
dmx[,2]=dmx2

# set the initial beta value
beta0<-c(1,2)

# generate random errors and calculate the response variable
errdata<-rnorm(100,0,1)
ydata<-dmx%%beta0+errdata

# calculate the maximum empirical likelihood estimates
beta_lse<-solve(t(dmx)%*%dmx)%*(t(dmx)%*%ydata)

num=EEL_est(x=dmx,theta=c(1,2),theta_tilda=beta_lse,
"gx<-matrix(0,nrow=100,ncol=2)
for(i in 1:2){gx[,i]<-dmx[,i]*(ydata-dmx%%as.matrix(theta))}
gx")
summary(num)
```

**Description**

Calculate the extended empirical log likelihood ratio for a multi-dimensional mean

**Usage**

```
EEL(x, theta)
## Default S3 method:
EEL(x, theta)
```

**Arguments**

x	Data matrix.
theta	The value at which the extended empirical likelihood is to be evaluated.

**Value**

An object of class EEL, basically a list including elements

theta	the value at which the extended empirical likelihood is to be evaluated;
prime	the prime-image inside the convex hull for the point theta;
estimating equation	the estimating equation here is "x-theta";
expansion	the value of the expansion factor gamma;
oel_log	the original empirical log likelihood ratio value;
eel_log	the extended empirical log likelihood ratio value.

**Author(s)**

Yu Zhang & Fan Wu

**See Also**

[EMLogLR](#), [exp\\_factor](#), [prime\\_image](#), [print.EEL](#), [summary.EEL](#), [EEL\\_est](#)

**Examples**

```
# EXAMPLE: computing the EEL for the mean of a bivariate random variable
# Generating a sample of n=40 bivariate observations.
# For this example, we do this through a univariate normal random number generator.

uninorm<- rnorm(40*2,5,1)
multnorm<-matrix(uninorm,ncol=2)

# To calculate the EEL for a point theta=c(5,3), use
EEL(x=multnorm,theta=c(5,3))
```

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EEL_est	<i>Extended empirical log likelihood ratio for parameters defined by estimating equations</i>
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---

**Description**

Calculate the extended empirical log likelihood ratio for parameters defined by estimating equations

**Usage**

```
EEL_est(x, theta, theta_tilda, equation)
## Default S3 method:
EEL_est(x, theta, theta_tilda, equation)
```

**Arguments**

x	Data matrix.
theta	Value at which the EEL for the parameters defined by estimating equations will be evaluated.
theta_tilda	The maximum empirical likelihood estimator of the unknown parameter.
equation	The estimating equation, must be put inside quotation marks and has to be a function of theta.

**Value**

An object of class EEL, basically a list including elements

theta	value at which the EEL for the parameters defined by estimating equations will be evaluated;
prime	the prime-image inside the convex hull for the point theta;
estimating equation	the estimating equation;
expansion	the value of the expansion factor gamma;
oel_log	the original empirical log likelihood ratio value;
eel_log	the extended empirical log likelihood ratio value.

**Author(s)**

Yu Zhang

**See Also**

[EMLogLR](#), [exp\\_factor\\_est](#), [prime\\_image\\_est](#), [print.EEL](#), [summary.EEL](#), [eel-package](#), [EEL](#)

**Examples**

```
# EXAMPLE: computing the EEL for the mean of a bivariate random variable
# Generating a sample of n=40 bivariate observations.
# For this example, we do this through a univariate normal random number generator.

uninorm<- rnorm(40*2,5,1)
multnorm<-matrix(uninorm,ncol=2)

# To calculate the EEL for a point theta=c(5,3), use
theta_tilda=colMeans(multnorm-as.vector(c(5,3)))
EEL_est(x=multnorm,theta=c(5,3),theta_tilda, "x-theta")
```

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EMLogLR

*Original empirical log likelihood ratio*


---

**Description**

The function extracts the empirical log likelihood ratio value produced by `el.test()` function from package "emplik".

**Usage**

```
EMLogLR(x, mean)
```

**Arguments**

x	Data matrix.
mean	The mean value to be evaluated.

**Value**

The function will return a numerical value representing the original empirical log likelihood ratio.

**Author(s)**

Yu Zhang & Fan Wu

**See Also**

[EEL](#)

**Examples**

```
x=rnorm(50,0,1)
# find the empirical log likelihood ratio at point 0
EMLogLR(x,0)
```

---

`exp_factor`*Calculating expansion factor for EEL for the mean*

---

**Description**

The function calculates the first order expansion factor of EEL for the mean.

**Usage**

```
exp_factor(x, theta)
## Default S3 method:
exp_factor(x, theta)
```

**Arguments**

<code>x</code>	Data matrix of interest.
<code>theta</code>	The value to be evaluated.

**Details**

The first order expansion factor for calculating EEL is defined as

$$\gamma(n, l(\theta)) = 1 + \frac{l(\theta)}{2n}.$$

**Value**

The function will return a numerical value representing the value of the expansion factor calculated.

**Author(s)**

Fan Wu & Yu Zhang

**References**

Tsao, M. (2013). Extending the empirical likelihood by domain expansion. *The Canadian Journal of Statistics*, 41 (2), 257-274.

**See Also**

[EEL](#), [prime\\_image](#), [exp\\_factor\\_est](#)

**Examples**

```
x=rnorm(400,0,3)
exp_factor(x,0)
```

---

exp_factor_est	<i>Calculating expansion factor for EEL for parameters defined by estimating equations</i>
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### Description

The function calculates the first order expansion factor of EEL for the parameters defined by estimating equations.

### Usage

```
exp_factor_est(x, theta, equation)
## Default S3 method:
exp_factor_est(x, theta, equation)
```

### Arguments

x	Data Matrix.
theta	The value to be evaluated.
equation	The estimating equation by which the parameters are defined.

### Details

The first order expansion factor for calculating EEL is defined as

$$\gamma(n, l(\theta)) = 1 + \frac{l(\theta)}{2n}.$$

The estimating equation input has to be a function of theta.

### Value

The function will return a numerical value representing the value of the expansion factor calculated.

### Author(s)

Yu Zhang and Fan Wu

### References

Tsao, M. (2013). Extending the empirical likelihood by domain expansion. *The Canadian Journal of Statistics*, 41 (2), 257-274.

### See Also

[eel-package](#), [exp\\_factor](#), [EEL\\_est](#), [prime\\_image\\_est](#)



**Examples**

```
#generate data with theoretical mean 2 and standard deviation 1
x=rnorm(100,2,1)
exp_factor_est(x=x, theta=2, equation="x-theta")
```

---

prime_image	<i>Calculating prime-image based on similarity mapping for the mean</i>
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---

**Description**

The function calculates the prime image of the given point based on the similarity mapping defined in the EEL calculation.

**Usage**

```
prime_image(theta_tilda, theta, x)
## Default S3 method:
prime_image(theta_tilda, theta, x)
```

**Arguments**

theta_tilda	Sample mean or the maximum empirical likelihood estimate of the estimating equations.
theta	Value to be evaluated.
x	Data matrix.

**Details**

The prime image was found by solving the equation

$$f(\zeta'') = \zeta'$$

See the reference paper for details.

**Value**

The function returns a vector, with the same length as the mean, representing the prime image of the point theta, based on the similarity mapping defined in EEL calculation.

**Author(s)**

Fan Wu and Yu Zhang

**References**

Tsao, M. (2013). Extending the empirical likelihood by domain expansion. *The Canadian Journal of Statistics*, 41 (2), 257-274.

Tsao, M., & Wu, F. (2013). Empirical likelihood on the full parameter space. *Annals of Statistics*, 0 (00), 1-21. doi: 10.1214/13-AOS1143

**See Also**

[EEL](#), [exp\\_factor](#), [eel-package](#), [prime\\_image\\_est](#)

**Examples**

```
x=rnorm(50,6,2)
# find the prime image of true mean
prime_image(theta_tilda=mean(x), theta=6, x=x)
```

---

prime_image_est	<i>Calculating prime-image based on similarity mapping for parameters defined by estimating equations</i>
-----------------	---

---

**Description**

The function calculates the prime image of a given point based on the similarity mapping defined in the EEL calculation.

**Usage**

```
prime_image_est(theta_tilda, theta, x, equation)
## Default S3 method:
prime_image_est(theta_tilda, theta, x, equation)
```

**Arguments**

theta_tilda	The maximum empirical likelihood estimates for parameters defined by estimating equations.
theta	The value to be evaluated.
x	Data Matrix.
equation	The estimating equation by which the parameters are defined, must be put as a function of theta.

**Details**

The prime image was found by solving the equation

$$f(\zeta'') = \zeta'.$$

See the reference paper for details.

**Value**

The function returns a vector, with the same dimension as the mean, representing the prime image of the point theta, based on the similarity mapping defined in EEL calculation.

**Author(s)**

Yu Zhang

**References**

Tsao, M. (2013). Extending the empirical likelihood by domain expansion. *The Canadian Journal of Statistics*, 41 (2), 257-274.

Tsao, M., & Wu, F. (2013). Empirical likelihood on the full parameter space. *Annals of Statistics*, 0 (00), 1-21. doi: 10.1214/13-AOS1143

**See Also**[EEL](#), [exp\\_factor\\_est](#), [EEL\\_est](#), [prime\\_image](#)**Examples**

```
x=rnorm(50,6,2)
# find the prime image of true mean
prime_image_est(theta_tilda=mean(x),theta=6,x=x,"x-theta")
```

---

`print.EEL`*Printing EEL objects*

---

**Description**

The function prints the extended empirical log likelihood ratio.

**Usage**

```
## S3 method for class 'EEL'
print(x,...)
```

**Arguments**

<code>x</code>	EEL object.
<code>...</code>	Further arguments passed to or from other methods.

**Value**

The function prints the extended empirical log likelihood ratio value of the EEL object.

**Author(s)**

Yu Zhang

**See Also**[EEL](#)**Examples**

```
# EXAMPLE: computing the EEL for the mean of a bivariate random variable
# Generating a sample of n=40 bivariate observations.
# For this example, we do this through a univariate normal random number generator.

 uninorm<- rnorm(40*2,5,1)
 multnorm<-matrix(uninorm,ncol=2)

# To calculate the EEL for a point theta=c(5,3), use
obj=EEL(x=multnorm,theta=c(5,3))

print(obj)
```

summary.EEL

*Summarizing EEL objects***Description**

The function prints a summary of EEL objects.

**Usage**

```
## S3 method for class 'EEL'
summary(object,...)
```

**Arguments**

object            An EEL object.  
...                Additional arguments affecting the summary produced.

**Value**

The function prints a summary of EEL objects including

theta	the point at which the EEL is to be evaluated;
estimating equation	the estimating equation at which the parameters are defined;
log oel ratio	empirical log likelihood ratio for the point theta;
prime image	prime-image of theta defined by the similarity mapping in EEL calculation;
expansion factor	value of the expansion factor
	$\gamma(x, \theta)$
	;
log eel ratio	value of the extended empirical log likelihood ratio.

**Author(s)**

Yu Zhang

**See Also**

[EEL](#), [print.EEL](#), [EEL\\_est](#)

**Examples**

```
# EXAMPLE: computing the EEL for the mean of a bivariate random variable
# Generating a sample of n=40 bivariate observations.
# For this example, we do this through a univariate normal random number generator.

uninorm<- rnorm(40*2,5,1)
multnorm<-matrix(uninorm,ncol=2)

# To calculate the EEL for a point theta=c(5,3), use
obj=EEL(x=multnorm,theta=c(5,3))

summary(obj)
```

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