

# Package ‘fitODBOD’

January 15, 2023

**Type** Package

**Title** Modeling Over Dispersed Binomial Outcome Data Using BMD and ABD

**Version** 1.5.0

**Description** Contains Probability Mass Functions, Cumulative Mass Functions, Negative Log Likelihood value, parameter estimation and modeling data using Binomial Mixture Distributions (BMD) (Manoj et al (2013) <[doi:10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24)>) and Alternate Binomial Distributions (ABD) (Paul (1985) <[doi:10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)>), also Journal article to use the package(<[doi:10.21105/joss.01505](https://doi.org/10.21105/joss.01505)>).

**License** MIT + file LICENSE

**URL** <https://github.com/Amalan-ConStat/fitODBOD>,  
<https://amalan-constat.github.io/fitODBOD/index.html>

**BugReports** <https://github.com/Amalan-ConStat/fitODBOD/issues>

**Depends** R (>= 4.0.0)

**Imports** bbmle, hypergeo, stats

**Suggests** flextable, ggplot2, ggthemes, grid, gridExtra, knitr, reshape2, rmarkdown, spelling, testthat, tibble, viridis

**Encoding** UTF-8

**Language** en-US

**LazyData** TRUE

**RoxygenNote** 7.2.3

**NeedsCompilation** no

**Author** Amalan Mahendran [aut, cre] (<<https://orcid.org/0000-0002-0643-9052>>),  
Pushpakanthie Wijekoon [aut, ctb]  
(<<https://orcid.org/0000-0003-4242-1017>>)

**Maintainer** Amalan Mahendran <[amalan0595@gmail.com](mailto:amalan0595@gmail.com)>

**Repository** CRAN

**Date/Publication** 2023-01-15 11:40:06 UTC

**R topics documented:**

Alcohol_data . . . . .	4
BODextract . . . . .	5
Chromosome_data . . . . .	5
Course_data . . . . .	6
dAddBin . . . . .	7
dBETA . . . . .	9
dBetaBin . . . . .	11
dBetaCorrBin . . . . .	13
dCOMPBin . . . . .	15
dCorrBin . . . . .	17
dGAMMA . . . . .	19
dGammaBin . . . . .	21
dGBeta1 . . . . .	23
dGHGBB . . . . .	25
dGHGBeta . . . . .	27
dGrassiaIBin . . . . .	29
dKUM . . . . .	31
dKumBin . . . . .	33
dLMBin . . . . .	35
dMcGBB . . . . .	37
dMultiBin . . . . .	39
dTRI . . . . .	41
dTriBin . . . . .	44
dUNI . . . . .	46
dUniBin . . . . .	48
Epidemic_Cold . . . . .	49
EstMGFBetaBin . . . . .	50
EstMLEAddBin . . . . .	52
EstMLEBetaBin . . . . .	53
EstMLEBetaCorrBin . . . . .	55
EstMLECOMPBin . . . . .	56
EstMLECorrBin . . . . .	57
EstMLEGammaBin . . . . .	59
EstMLEGHGBB . . . . .	60
EstMLEGrassiaIBin . . . . .	61
EstMLEKumBin . . . . .	62
EstMLELMBin . . . . .	64
EstMLEMcGBB . . . . .	65
EstMLEMultiBin . . . . .	66
EstMLETriBin . . . . .	68
Exam_data . . . . .	69
fitAddBin . . . . .	70
fitBetaBin . . . . .	72
fitBetaCorrBin . . . . .	74
fitBin . . . . .	76
fitCOMPBin . . . . .	77

fitCorrBin . . . . .	79
fitGammaBin . . . . .	81
fitGHGBB . . . . .	83
fitGrassiaIIBin . . . . .	85
fitKumBin . . . . .	87
fitLMBin . . . . .	89
fitMcGBB . . . . .	90
fitMultiBin . . . . .	92
fitTriBin . . . . .	94
Male_Children . . . . .	96
mazBETA . . . . .	97
mazGAMMA . . . . .	99
mazGBeta1 . . . . .	101
mazGHGBeta . . . . .	103
mazKUM . . . . .	105
mazTRI . . . . .	107
mazUNI . . . . .	109
NegLLAddBin . . . . .	111
NegLLBetaBin . . . . .	112
NegLLBetaCorrBin . . . . .	113
NegLLCOMPBin . . . . .	115
NegLLCorrBin . . . . .	116
NegLLGammaBin . . . . .	117
NegLLGHGBB . . . . .	118
NegLLGrassiaIIBin . . . . .	119
NegLLKumBin . . . . .	120
NegLLLMBin . . . . .	121
NegLLMcGBB . . . . .	122
NegLLMultiBin . . . . .	124
NegLLTriBin . . . . .	125
Overdispersion . . . . .	126
pAddBin . . . . .	127
pBETA . . . . .	129
pBetaBin . . . . .	131
pBetaCorrBin . . . . .	133
pCOMPBin . . . . .	135
pCorrBin . . . . .	137
pGAMMA . . . . .	139
pGammaBin . . . . .	141
pGBeta1 . . . . .	142
pGHGBB . . . . .	144
pGHGBeta . . . . .	146
pGrassiaIIBin . . . . .	149
pKUM . . . . .	150
pKumBin . . . . .	152
Plant_DiseaseData . . . . .	154
pLMBin . . . . .	155
pMcGBB . . . . .	157

pMultiBin . . . . .	159
pTRI . . . . .	161
pTriBin . . . . .	163
pUNI . . . . .	165
pUniBin . . . . .	167
Terror_data_ARG . . . . .	168
Terror_data_USA . . . . .	169

<b>Index</b>	<b>170</b>
--------------	------------

---

Alcohol_data	<i>Alcohol data</i>
--------------	---------------------

---

### Description

Lemmens , Knibbe and Tan(1988) described a study of self reported alcohol frequencies. The no of alcohol consumption data in two reference weeks is separately self reported by a randomly selected sample of 399 respondents in the Netherlands in 1983. Number of days a given individual consumes alcohol out of 7 days a week can be treated as a binomial variable. The collection of all such variables from all respondents would be defined as "Binomial Outcome Data".

### Usage

Alcohol\_data

### Format

A data frame with 3 columns and 8 rows.

Days No of Days Drunk

week1 Observed frequencies for week1

week2 Observed frequencies for week2

### Source

Extracted from

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24)

### Examples

```
Alcohol_data$Days      # extracting the binomial random variables
sum(Alcohol_data$week2) # summing all the frequencies in week2
```

---

BODextract

*Binomial Data Extraction from Raw data*

---

### Description

The below function has the ability to extract from the raw data to Binomial Outcome Data. This function simplifies the data into more presentable way to the user.

### Usage

```
BODextract(data)
```

### Arguments

data                    vector of observations

### Details

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of BODextract gives a list format consisting  
RV binomial random variables in vector form  
Freq corresponding frequencies in vector form

### Examples

```
datapoints <- sample(0:10,340,replace=TRUE) #creating a sample set of observations  
BODextract(datapoints)                    #extracting binomial outcome data from observations  
Random.variable <- BODextract(datapoints)$RV #extracting the binomial random variables
```

---

Chromosome\_data

*Chromosome Data*

---

### Description

Data in this example refer to 337 observations on the secondary association of chromosomes in Brassika; n , which is now the number of chromosomes, equals 3 and X is the number of pairs of bivalents showing association.

### Usage

```
Chromosome_data
```

**Format**

A data frame with 2 columns and 4 rows

No.of.Asso No of Associations

fre Observed frequencies

**Source**

Extracted from

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

**Examples**

```
Chromosome_data$No.of.Asso      #extracting the binomial random variables
sum(Chromosome_data$fre)        #summing all the frequencies
```

---

Course\_data

*Course Data*

---

**Description**

The data refer to the numbers of courses taken by a class of 65 students from the first year of the Department of Statistics of Athens University of Economics. The students enrolled in this class attended 8 courses during the first year of their study. The total numbers of successful examinations (including resits) were recorded.

**Usage**

Course\_data

**Format**

A data frame with 2 columns and 9 rows

sub.pass subjects passed

fre Observed frequencies

**Source**

Extracted from

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

**Examples**

```
Course_data$sub.pass      # extracting the binomial random variables
sum(Course_data$fre)     # summing all the frequencies
```

---

dAddBin *Additive Binomial Distribution*

---

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

**Usage**

```
dAddBin(x,n,p,alpha)
```

**Arguments**

x                      vector of binomial random variables.  
n                        single value for no of binomial trials.  
p                        single value for probability of success  
alpha                    single value for alpha parameter.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{AddBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \left( \frac{\alpha}{2} \left( \frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{(1-p)} - \frac{\alpha(n-1)n}{2} \right) + 1 \right)$$

The alpha is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq \alpha \leq \left(\frac{n+(2p-1)^2}{4p(1-p)}\right)^{-1}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-1 < \alpha < 1$$

The mean and the variance are denoted as

$$E_{Addbin}[x] = np$$

$$Var_{Addbin}[x] = np(1-p)(1+(n-1)\alpha)$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dAddBin gives a list format consisting  
 pdf probability function values in vector form.  
 mean mean of Additive Binomial Distribution.  
 var variance of Additive Binomial Distribution.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

Jorge G. Morel and Nagaraj K. Neerchal. *Overdispersion Models in SAS*. SAS Institute, 2012.

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dAddBin(0:10,10,0.58,0.022)$pdf      #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dAddBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
  points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pAddBin(0:10,10,0.58,0.022)         #acquiring the cumulative probability values
```



dBETA

*Beta Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1]

**Usage**

dBETA(p, a, b)

**Arguments**

p                      vector of probabilities.  
a                        single value for shape parameter alpha representing as a.  
b                        single value for shape parameter beta representing as b.

**Details**

The probability density function and cumulative density function of a unit bounded Beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a, b)}$$

;  $0 \leq p \leq 1$ 

$$G_P(p) = \frac{B_p(a, b)}{B(a, b)}$$

;  $0 \leq p \leq 1$ 

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$r = 1, 2, 3, \dots$

Defined as  $B_p(a, b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dBETA gives a list format consisting  
 pdf probability density values in vector form.  
 mean mean of the Beta distribution.  
 var variance of the Beta distribution.

**References**

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.  
 Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

**See Also**

[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3) #acquiring the cumulative probability values
mazBETA(1.4,3,2) #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2
```

```
#only the integer value of moments is taken here because moments cannot be decimal
mazBETA(1.9,5.5,6)
```

---

 dBetaBin

*Beta-Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

### Usage

```
dBetaBin(x,n,a,b)
```

### Arguments

x                      vector of binomial random variables.  
 n                      single value for no of binomial trials.  
 a                      single value for shape parameter alpha representing as a.  
 b                      single value for shape parameter beta representing as b.

### Details

Mixing Beta distribution with Binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x, n+b-x)}{B(a,b)}$$

$$a, b > 0$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as B(a,b) is the beta function.

**Value**

The output of dBetaBin gives a list format consisting

- pdf probability function values in vector form.
- mean mean of the Beta-Binomial Distribution.
- var variance of the Beta-Binomial Distribution.
- over.dis para over dispersion value of the Beta-Binomial Distribution.

**References**

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.  
Available at: doi: [10.1186/1471228858](https://doi.org/10.1186/1471228858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf      #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean    #extracting the mean
dBetaBin(0:10,10,4,.2)$var     #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}

pBetaBin(0:10,10,4,.2) #acquiring the cumulative probability values
```

dBetaCorrBin

*Beta-Correlated Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Correlated Binomial Distribution.

**Usage**

dBetaCorrBin(x,n,cov,a,b)

**Arguments**

x                      vector of binomial random variables.  
n                        single value for no of binomial trials.  
cov                     single value for covariance.  
a                        single value for alpha parameter.  
b                        single value for beta parameter.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{BetaCorrBin}(x) = \binom{n}{x} \frac{B(a+x, b+n-x)}{B(a+b)} \left[ 1 + \frac{cov}{2} \left( \frac{(x(x-1) \prod_{k=1}^4 (a+b+n-k))}{(\prod_{k=1}^2 (x+a-k) \prod_{k=1}^2 (n-x+b-k))} - \frac{(2x(n-1) \prod_{k=1}^3 (a+b+n-k))}{(x+a-1) \prod_{k=1}^2 (n-x+b-k)} + \frac{(n(n-1) \prod_{k=1}^2 (a+b+n-k))}{(\prod_{k=1}^2 (n-x+b-k))} \right) \right]$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < a, b$$

$$-\infty < cov < +\infty$$

$$0 < p < 1$$

$$p = \frac{a}{a+b}$$

$$\Theta = \frac{1}{a+b}$$

The Correlation is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq \text{correlation} \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

where  $fo = \min[(x - (n-1)p - 0.5)^2]$

The mean and the variance are denoted as

$$E_{BetaCorrBin}[x] = np$$

$$Var_{BetaCorrBin}[x] = np(1-p)(n\Theta + 1)(1 + \Theta)^{-1} + n(n-1)cov$$

$$Corr_{BetaCorrBin}[x] = \frac{cov}{p(1-p)}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dBetaCorrBin gives a list format consisting

pdf probability function values in vector form.

mean mean of Beta-Related Binomial Distribution.

var variance of Beta-Related Binomial Distribution.

corr correlation of Beta-Related Binomial Distribution.

mincorr minimum correlation value possible.

maxcorr maximum correlation value possible.

### References

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Related binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],pch=16)
}

dBetaCorrBin(0:10,10,0.001,10,13)$pdf      #extracting the pdf values
dBetaCorrBin(0:10,10,0.001,10,13)$mean    #extracting the mean
```

```

dBetaCorrBin(0:10,10,0.001,10,13)$var      #extracting the variance
dBetaCorrBin(0:10,10,0.001,10,13)$corr     #extracting the correlation
dBetaCorrBin(0:10,10,0.001,10,13)$mincorr  #extracting the minimum correlation value
dBetaCorrBin(0:10,10,0.001,10,13)$maxcorr  #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Related binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],pch=16)
}

pBetaCorrBin(0:10,10,0.001,10,13)      #acquiring the cumulative probability values

```

---

dCOMPBin

*COM Poisson Binomial Distribution*


---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the COM Poisson Binomial Distribution.

## Usage

```
dCOMPBin(x,n,p,v)
```

## Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
v	single value for v.

## Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{COMPBin}(x) = \frac{\binom{n}{x}^v p^x (1-p)^{n-x}}{\sum_{j=0}^n \binom{n}{j}^v p^j (1-p)^{(n-j)}}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dCOMPBin gives a list format consisting  
 pdf probability function values in vector form.  
 mean mean of COM Poisson Binomial Distribution.  
 var variance of COM Poisson Binomial Distribution.

### References

Extracted from

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCOMPBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCOMPBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dCOMPBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
```



```

points(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCOMPBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values

```

---

dCorrBin                      *Correlated Binomial Distribution*

---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

### Usage

```
dCorrBin(x, n, p, cov)
```

### Arguments

x                      vector of binomial random variables.  
n                      single value for no of binomial trials.  
p                      single value for probability of success.  
cov                    single value for covariance.

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{CorrBin}(x) = \binom{n}{x} (p^x)(1-p)^{n-x} \left(1 + \left(\frac{cov}{2p^2(1-p)^2}\right)((x-np)^2 + x(2p-1) - np^2)\right)$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

The Correlation is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

where  $fo = \min[(x - (n-1)p - 0.5)^2]$

The mean and the variance are denoted as

$$E_{CorrBin}[x] = np$$

$$Var_{CorrBin}[x] = n(p(1-p) + (n-1)cov)$$

$$Corr_{CorrBin}[x] = \frac{cov}{p(1-p)}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dCorrBin gives a list format consisting  
 pdf probability function values in vector form.  
 mean mean of Correlated Binomial Distribution.  
 var variance of Correlated Binomial Distribution.  
 corr correlation of Correlated Binomial Distribution.  
 mincorr minimum correlation value possible.  
 maxcorr maximum correlation value possible.

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean    #extracting the mean
```

```

dCorrBin(0:10,10,0.58,0.022)$var      #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr    #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCorrBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values

```

---

dGAMMA

*Gamma Distribution*


---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

## Usage

```
dGAMMA(p, c, l)
```

## Arguments

p	vector of probabilities.
c	single value for shape parameter c.
l	single value for shape parameter l.

## Details

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1}}{\gamma(l)} [\ln(1/p)]^{l-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{I_g(l, c \ln(1/p))}{\gamma(l)}$$

;  $0 \leq p \leq 1$

$$l, c > 0$$

The mean the variance are denoted by

$$E[P] = \left(\frac{c}{c+1}\right)^l$$

$$var[P] = \left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l}$$

The moments about zero is denoted as

$$E[P^r] = \left(\frac{c}{c+r}\right)^l$$

$r = 1, 2, 3, \dots$

Defined as  $\gamma(l)$  is the gamma function Defined as  $Ig(l, c \ln(1/p)) = \int_0^{c \ln(1/p)} t^{l-1} e^{-t} dt$  is the Lower incomplete gamma function

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGAMMA gives a list format consisting

pdf probability density values in vector form.

mean mean of the Gamma distribution.

var variance of Gamma distribution.

### References

Olshen, A. C. Transformations of the Pearson Type III Distribution. Ann. Math. Statist. 9 (1938), no. 3, 176–200.

### See Also

[GammaDist](#)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean #extracting the mean
```

```

dGAMMA(seq(0,1,by=0.01),5,6)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6) #acquiring the cumulative probability values
mazGAMMA(1.4,5,6) #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)

```

---

dGammaBin

*Gamma Binomial Distribution*


---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gamma Binomial Distribution.

## Usage

```
dGammaBin(x,n,c,l)
```

## Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
c	single value for shape parameter c.
l	single value for shape parameter l.

## Details

Mixing Gamma distribution with Binomial distribution will create the the Gamma Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GammaBin}[x] = \binom{n}{x} \sum_{j=0}^{n-x} \binom{n-x}{j} (-1)^j \left(\frac{c}{c+x+j}\right)^l$$

$$c, l > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GammaBin}[x] = \left(\frac{c}{c+1}\right)^l$$

$$Var_{GammaBin}[x] = n^2 \left[ \left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l} \right] + n \left(\frac{c}{c+1}\right)^l \left(1 - \left(\frac{c+1}{c+2}\right)^l\right)$$

$$overdispersion = \frac{\left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l}}{\left(\frac{c}{c+1}\right)^l \left[1 - \left(\frac{c}{c+1}\right)^l\right]}$$

### Value

The output of dGammaBin gives a list format consisting  
 pdf probability function values in vector form.  
 mean mean of the Gamma Binomial Distribution.  
 var variance of the Gamma Binomial Distribution.  
 over.dis para over dispersion value of the Gamma Binomial Distribution.

### References

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Gamma Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGammaBin(0:10,10,4,.2)$pdf #extracting the pdf values
dGammaBin(0:10,10,4,.2)$mean #extracting the mean
dGammaBin(0:10,10,4,.2)$var #extracting the variance
dGammaBin(0:10,10,4,.2)$over.dis para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
```

```

plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
}

pGammaBin(0:10,10,4,.2) #acquiring the cumulative probability values

```

dGBeta1

*Generalized Beta Type-1 Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

**Usage**

dGBeta1(p, a, b, c)

**Arguments**

p                    vector of probabilities.  
a                    single value for shape parameter alpha representing as a.  
b                    single value for shape parameter beta representing as b.  
c                    single value for shape parameter gamma representing as c.

**Details**

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1-p^c)^{b-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} {}_2F_1(a, 1-b; p^c; a+1)$$

$0 \leq p \leq 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$\text{var}[P] = \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a+b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is Beta function. Defined as  ${}_2F_1(a, b; c; d)$  is Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGBeta1 gives a list format consisting  
 pdf probability density values in vector form.  
 mean mean of the Generalized Beta Type-1 Distribution.  
 var variance of the Generalized Beta Type-1 Distribution.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24) .

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1, .2, .3, 1.5, 2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean #extracting the mean
```



```

dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

pGBeta1(0.04,2,3,4)                    #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2)                    #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2 #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)

```

dGHGBB

*Gaussian Hypergeometric Generalized Beta Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

**Usage**

```
dGHGBB(x, n, a, b, c)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha value representing a.
b	single value for shape parameter beta value representing b.
c	single value for shape parameter lambda value representing c.

**Details**

Mixing Gaussian Hypergeometric Generalized Beta distribution with Binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GHGBB}(x) = \frac{1}{{}_2F_1(-n, a; -b - n + 1; c)} \binom{n}{x} \frac{B(x + a, n - x + b)}{B(a, b + n)} (c^x)$$

$$a, b, c > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$\begin{aligned} \text{Var}_{GHGBB}[x] &= nE_{GHGBeta}(1 - E_{GHGBeta}) + n(n-1)\text{Var}_{GHGBeta} \\ \text{overdispersion} &= \frac{\text{var}_{GHGBeta}}{E_{GHGBeta}(1 - E_{GHGBeta})} \end{aligned}$$

Defined as  $B(a, b)$  is the beta function. Defined as  ${}_2F_1(a, b; c; d)$  is the Gaussian Hypergeometric function

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBB gives a list format consisting

pdf probability function values in vector form.

mean mean of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

var variance of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

over.dis para over dispersion value of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

### References

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

### See Also

[hypergeo\\_powerseries](#)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}

dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean    #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var     #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par #extracting the over dispersion value

#plotting the random variables and cumulative probability values
```

```

col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}

pGHGBB(0:7,7,1.3,0.3,1.3)    #acquiring the cumulative probability values

```

dGHGBeta

*Gaussian Hypergeometric Generalized Beta Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

**Usage**

dGHGBeta(p, n, a, b, c)

**Arguments**

p                      vector of probabilities.  
n                        single value for no of binomial trials.  
a                        single value for shape parameter alpha representing as a.  
b                        single value for shape parameter beta representing as b.  
c                        single value for shape parameter lambda representing as c.

**Details**

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

;  $0 \leq p \leq 1$

$a, b, c > 0$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$\text{var}[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  as the beta function. Defined as  ${}_2F_1(a, b; c; d)$  as the Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBeta gives a list format consisting

pdf probability density values in vector form.

mean mean of the Gaussian Hypergeometric Generalized Beta Distribution.

var variance of the Gaussian Hypergeometric Generalized Beta Distribution.

### References

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

### See Also

[hypergeo\\_powerseries](#)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1, .2, .3, 1.5, 2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
```

```

lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659) #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)

```

---

dGrassiaIIBin

*Grassia-II-Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Grassia-II-Binomial Distribution.

### Usage

```
dGrassiaIIBin(x,n,a,b)
```

### Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter a.
b	single value for shape parameter b.

## Details

Mixing Gamma distribution with Binomial distribution will create the the Grassia-II-Binomial distribution, only when  $(1-p)=e^{(-\lambda)}$  of the Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GrassiaIIBin}[x] = \binom{n}{x} \sum_{j=0}^x \binom{x}{j} (-1)^{x-j} (1 + b(n-j))^{-a}$$

$$a, b > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GrassiaIIBin}[x] = \left(\frac{b}{b+1}\right)^a$$

$$Var_{GrassiaIIBin}[x] = n^2 \left[ \left(\frac{b}{b+2}\right)^a - \left(\frac{b}{b+1}\right)^{2a} \right] + n \left(\frac{b}{b+1}\right)^a - \left(\frac{b+1}{b+2}\right)^a$$

$$overdispersion = \frac{\left(\frac{b}{b+2}\right)^a - \left(\frac{b}{b+1}\right)^{2a}}{\left(\frac{b}{b+1}\right)^a \left[1 - \left(\frac{b}{b+1}\right)^a\right]}$$

## Value

The output of dGrassiaIIBin gives a list format consisting

pdf probability function values in vector form.

mean mean of the Grassia II Binomial Distribution.

var variance of the Grassia II Binomial Distribution.

over.dis para over dispersion value of the Grassia II Binomial Distribution.

## References

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Grassia II binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGrassiaIIBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
```

```

points(0:10,dGrassiaIIBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGrassiaIIBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dGrassiaIIBin(0:10,10,4,.2)$mean  #extracting the mean
dGrassiaIIBin(0:10,10,4,.2)$var   #extracting the variance
dGrassiaIIBin(0:10,10,4,.2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <-c (1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGrassiaIIBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGrassiaIIBin(0:10,10,a[i],a[i]),col = col[i])
}

pGrassiaIIBin(0:10,10,4,.2) #acquiring the cumulative probability values

```

---

dKUM

*Kumaraswamy Distribution*


---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between  $[0,1]$ .

## Usage

dKUM(p, a, b)

## Arguments

p                    vector of probabilities.  
a                    single value for shape parameter alpha representing as a.  
b                    single value for shape parameter beta representing as b.

## Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1 - p^a)^{b-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = 1 - (1 - p^a)^b$$

;  $0 \leq p \leq 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$\text{var}[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dKUM gives a list format consisting

pdf probability density values in vector form.

mean mean of the Kumaraswamy distribution.

var variance of the Kumaraswamy distribution.

### References

Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.

Available at : doi: [10.1016/00221694\(80\)900360](https://doi.org/10.1016/00221694(80)900360).

Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.

Available at : doi: [10.1016/j.stamet.2008.04.001](https://doi.org/10.1016/j.stamet.2008.04.001).

### See Also

[Kumaraswamy](#)





n	single value for no of binomial trial
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
it	number of iterations to converge as a proper probability function replacing infinity

### Details

Mixing Kumaraswamy distribution with Binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{KumBin}(x) = ab \binom{n}{x} \sum_{j=0}^{it} (-1)^j \binom{b-1}{j} B(x+a+aj, n-x+1)$$

$$a, b > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$it > 0$$

The mean, variance and over dispersion are denoted as

$$E_{KumBin}[x] = nbB\left(1 + \frac{1}{a}, b\right)$$

$$Var_{KumBin}[x] = n^2b\left(B\left(1 + \frac{2}{a}, b\right) - bB\left(1 + \frac{1}{a}, b\right)^2\right) + nb\left(B\left(1 + \frac{1}{a}, b\right) - B\left(1 + \frac{2}{a}, b\right)\right)$$

$$overdispersion = \frac{(bB\left(1 + \frac{2}{a}, b\right) - (bB\left(1 + \frac{1}{a}, b\right))^2)}{(bB\left(1 + \frac{1}{a}, b\right) - (bB\left(1 + \frac{1}{a}, b\right))^2)}$$

Defined as  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dKumBin gives a list format consisting

pdf probability function values in vector form.

mean mean of the Kumaraswamy Binomial Distribution.

var variance of the Kumaraswamy Binomial Distribution.

over.dis para over dispersion value of the Kumaraswamy Distribution.

## References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. *Chinese Journal of Applied Probability and Statistics*, 27(5), 511-521.

## Examples

```
## Not run:
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5) {
lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

## End(Not run)

dKumBin(0:10,10,4,2)$pdf #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

## Not run:
#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5) {
lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
}

## End(Not run)

pKumBin(0:10,10,4,2) #acquiring the cumulative probability values
```

---

dLMBin

*Lovinson Multiplicative Binomial Distribution*


---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Lovinson Multiplicative Binomial Distribution.

**Usage**

dLMBin(x,n,p,phi)

**Arguments**

x                    vector of binomial random variables.  
 n                    single value for no of binomial trials.  
 p                    single value for probability of success.  
 phi                  single value for phi.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{LMBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(phi)^{x(n-x)}}{f(p, phi, n)}$$

here  $f(p, phi, n)$  is

$$f(p, phi, n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (phi)^{k(n-k)}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < phi$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dLMBin gives a list format consisting

pdf probability function values in vector form.

mean mean of Lovinson Multiplicative Binomial Distribution.

var variance of Lovinson Multiplicative Binomial Distribution.

**References**

Elamir, E.A., 2013. Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation. Journal of Statistical Theory and Applications, 12(1), pp.92-105.

**Examples**

```

#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
      function graph",xlab="Binomial random variable",
      ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dLMBin(0:10,10,.58,10.022)$pdf #extracting the pdf values
dLMBin(0:10,10,.58,10.022)$mean #extracting the mean
dLMBin(0:10,10,.58,10.022)$var #extracting the variance

#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
      function graph",xlab="Binomial random variable",
      ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pLMBin(0:10,10,.58,10.022) #acquiring the cumulative probability values

```

dMcGBB

*McDonald Generalized Beta Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

**Usage**

```
dMcGBB(x, n, a, b, c)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

**Details**

Mixing Generalized Beta Type-1 Distribution with Binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{McGGBB}(x) = \binom{n}{x} \frac{1}{B(a, b)} \left( \sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B\left(\frac{x}{c} + a + \frac{j}{c}, b\right) \right)$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGGBB}[x] = n \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGGBB}[x] = n^2 \left( \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2 \right) + n \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} \right)$$

$$overdispersion = \frac{\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}\right)^2}{\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \left(\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})}\right)^2}$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

**Value**

The output of dMcGGBB gives a list format consisting

pdf probability function values in vector form.

mean mean of McDonald Generalized Beta Binomial Distribution.

var variance of McDonald Generalized Beta Binomial Distribution.

over.dis para over dispersion value of McDonald Generalized Beta Binomial Distribution.

## References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of Mcdonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.6)
plot(0,0,main="Mcdonald generalized beta-binomial probability function graph",
xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMcGGB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMcGGB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}

dMcGGB(0:10,10,4,2,1)$pdf           #extracting the pdf values
dMcGGB(0:10,10,4,2,1)$mean         #extracting the mean
dMcGGB(0:10,10,4,2,1)$var         #extracting the variance
dMcGGB(0:10,10,4,2,1)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pMcGGB(0:10,10,a[i],a[i],2),col = col[i])
points(0:10,pMcGGB(0:10,10,a[i],a[i],2),col = col[i])
}

pMcGGB(0:10,10,4,2,1)             #acquiring the cumulative probability values
```

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

**Usage**

```
dMultiBin(x,n,p,theta)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
theta	single value for theta.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{\theta^{x(n-x)}}{f(p, \theta, n)}$$

here  $f(p, \theta, n)$  is

$$f(p, \theta, n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (\theta^{k(n-k)})$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < \theta$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dMultiBin gives a list format consisting

pdf probability function values in vector form.

mean mean of Multiplicative Binomial Distribution.

var variance of Multiplicative Binomial Distribution.



## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}
```

```
dMultiBin(0:10,10,.58,10.022)$pdf #extracting the pdf values
dMultiBin(0:10,10,.58,10.022)$mean #extracting the mean
dMultiBin(0:10,10,.58,10.022)$var #extracting the variance
```

```
#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}
```

```
pMultiBin(0:10,10,.58,10.022) #acquiring the cumulative probability values
```

**Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

**Usage**

dTRI(p, mode)

**Arguments**

p                      vector of probabilities.  
mode                    single value for mode.

**Details**

Setting  $min = 0$  and  $max = 1$   $mode = c$  in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

;  $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

;  $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

;  $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

;  $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a + b + c)}{3} = \frac{(1 + c)}{3}$$

$$var[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1 + c^2 - c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, \dots$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dTRI gives a list format consisting  
 pdf probability density values in vector form.  
 mean mean of the unit bounded Triangular distribution.  
 variance variance of the unit bounded Triangular distribution

**References**

- Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.
- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.
- Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.  
 Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2) .
- Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.  
 Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299) .

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean     #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var      #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)          #acquiring the cumulative probability values
mazTRI(1.4,.3)                       #acquiring the moment about zero values
```

```

mazTRI(2,.3)-mazTRI(1,.3)^2    #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)

```

---

dTriBin                      *Triangular Binomial Distribution*

---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

### Usage

```
dTriBin(x,n,mode)
```

### Arguments

x                      vector of binomial random variables.  
n                      single value for no of binomial trials.  
mode                   single value for mode.

### Details

Mixing unit bounded Triangular distribution with Binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{TriBin}(x) = 2 \binom{n}{x} (c^{-1} B_c(x+2, n-x+1) + (1-c)^{-1} B(x+1, n-x+2) - (1-c)^{-1} B_c(x+1, n-x+2))$$

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{TriBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$

$$\text{overdispersion} = \frac{(1 - c + c^2)}{2(2 + c - c^2)}$$

Defined as  $B_c(a, b) = \int_0^c t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dTriBin gives a list format consisting

pdf probability function values in vector form.

mean mean of the Triangular Binomial Distribution.

var variance of the Triangular Binomial Distribution.

over.dis para over dispersion value of the Triangular Binomial Distribution.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}

dTriBin(0:10,10,.4)$pdf      #extracting the pdf values
dTriBin(0:10,10,.4)$mean    #extracting the mean
dTriBin(0:10,10,.4)$var     #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(7)
```

```

x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}

pTriBin(0:10,10,.4) #acquiring the cumulative probability values

```

---

dUNI

*Uniform Distribution Bounded Between [0,1]*


---

### Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

### Usage

dUNI(p)

### Arguments

p                      vector of probabilities.

### Details

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable  $P$  are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$\text{var}[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$r = 1, 2, 3, \dots$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dUNI gives a list format consisting  
 pdf probability density values in vector form.  
 mean mean of unit bounded uniform distribution.  
 var variance of unit bounded uniform distribution.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

### See Also

[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>

### Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
      xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
      xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))        #acquiring the cumulative probability values

mazUNI(c(1,2,3))              #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)
```

dUniBin

*Uniform Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

**Usage**

```
dUniBin(x,n)
```

**Arguments**

x                      vector of binomial random variables.  
n                      single value for no of binomial trials.

**Details**

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{UniBin}(x) = \frac{1}{n+1}$$

$$n = 1, 2, \dots$$

$$x = 0, 1, 2, \dots, n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of dUniBin gives a list format consisting

pdf probability function values in vector form.

mean mean of the Uniform Binomial Distribution.

var variance of the Uniform Binomial Distribution.

ove.dis para over dispersion value of Uniform Binomial Distribution.



## References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299) .

## Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)

dUniBin(0:300,300)$pdf #extracting the pdf values
dUniBin(0:10,10)$mean #extracting the mean
dUniBin(0:10,10)$var #extracting the variance
dUniBin(0:10,10)$over.dis.para #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15) #acquiring the cumulative probability values
```

---

Epidemic\_Cold

*Family Epidemics*

---

## Description

In this investigation, families of the same size, two parents and three children, living in different circumstances of domestic overcrowding were visited at fortnightly intervals. The date of onset and the clinical nature of upper respiratory infectious experienced by each member of the family were charted on a time scale marked off in days. Family epidemics of acute coryza-or common colds-were thus available for analysis.

## Usage

Epidemic\_Cold

## Format

A data frame with 6 columns and 5 rows

Cases No of Further Cases

Families No of Families  
 Father Father with Status of Introducing Cases  
 Mother Mother with Status of Introducing Cases  
 SChild School Child with Status of Introducing Cases  
 PSChild Pre-School Child with Status of Introducing Cases

### Details

By inspection of the epidemic time charts, it was possible to identify new or primary introductions of illness into the household by the onset of a cold after a lapse of 10 days since the last such case in the same home. Two such cases occurring on the same or succeeding days were classified as multiple primaries. Thereafter, the links in the epidemic chain of spread were defined by an interval of one day or more between successive cases in the same family. These family epidemics could then be described thus 1-2-1, 1-1-1-0, 2-1-0, etc. It must be emphasized that although this method of classification is somewhat arbitrary, it was completed before the corresponding theoretical distributions were worked out and the interval chosen agrees with the distribution of presumptive incubation periods of the common cold seen in field surveys (e.g. Badger, Dingle, Feller, Hodges, Jordan, and Rammelkamp, 1953).

### Source

Extracted from

Heasman, M. A. and Reid, D. D. (1961). "Theory and observation in family epidemics of the common cold." Br. J. pleu. SOC. Med., 15, 12-16.

### Examples

```
Epidemic_Cold$Cases
sum(Epidemic_Cold$SChild)
```

---

EstMGFBetaBin	<i>Estimating the shape parameters a and b for Beta-Binomial Distribution</i>
---------------	---

---

### Description

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the Beta-Binomial distribution when the binomial random variables and corresponding frequencies are given.

### Usage

```
EstMGFBetaBin(x, freq)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.

**Details**

$$a, b > 0$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of EstMGFBetaBin will produce the class mgf format consisting  
a shape parameter of beta distribution representing for alpha  
b shape parameter of beta distribution representing for beta  
min Negative loglikelihood value  
AIC AIC value  
call the inputs for the function  
Methods print, summary, coef and AIC can be used to extract specific outputs.

**References**

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: doi: [10.1186/14712288858](https://doi.org/10.1186/14712288858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi: [10.1094/PHYTO83759](https://doi.org/10.1094/PHYTO83759)

**See Also**

[mle2](#)

**Examples**

```

No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
estimate <- EstMLEBetaBin(No.D.D,Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(estimate)  #extracting the parameters

#estimating the parameters using moment generating function methods
results <- EstMGFBetaBin(No.D.D,Obs.fre.1)

# extract the estimated parameters and summary
coef(results)
summary(results)

AIC(results) #show the AIC value

```

---

EstMLEAddBin	<i>Estimating the probability of success and alpha for Additive Binomial Distribution</i>
--------------	---

---

**Description**

The function will estimate the probability of success and alpha using the maximum log likelihood method for the Additive Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLEAddBin(x, freq)
```

**Arguments**

x                    vector of binomial random variables.  
freq                 vector of frequencies.

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of EstMLEAddBin will produce the class mlAB and ml with a list consisting  
min Negative Log Likelihood value.

p estimated probability of success.

alpha estimated alpha parameter.

AIC AIC value.

call the inputs for the function.

Methods print, summary, coef and AIC can be used to extract specific outputs.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

## Not run:
#estimating the probability value and alpha value
results <- EstMLEAddBin(No.D.D,Obs.fre.1)

#printing the summary of results
summary(results)

#extracting the estimated parameters
coef(results)

## End(Not run)
```

---

EstMLEBetaBin	<i>Estimating the shape parameters a and b for Beta-Binomial Distribution</i>
---------------	---

---

**Description**

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the Beta-Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLEBetaBin(x, freq, a, b, ...)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$a, b > 0$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEBetaBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

**References**

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: doi: [10.1186/14712288858](https://doi.org/10.1186/14712288858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi: [10.1094/PHYTO83759](https://doi.org/10.1094/PHYTO83759)

**See Also**

[mle2](#)

**Examples**

```

No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
estimate <- EstMLEBetaBin(No.D.D,Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(estimate) #extracting the parameters

#estimating the parameters using moment generating function methods
EstMGFBetaBin(No.D.D,Obs.fre.1)

```

---

EstMLEBetaCorrBin	<i>Estimating the covariance, alpha and beta parameter values for Beta-Correlated Binomial Distribution</i>
-------------------	---

---

**Description**

The function will estimate the covariance, alpha and beta parameter values using the maximum log likelihood method for the Beta-Correlated Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLEBetaCorrBin(x, freq, cov, a, b, ...)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
cov	single value for covariance.
a	single value for alpha parameter.
b	single value for beta parameter.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$\begin{aligned}
 x &= 0, 1, 2, \dots \\
 freq &\geq 0 \\
 -\infty &< cov < +\infty \\
 0 &< a, b
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEBetaCorrBin here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

**References**

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

**See Also**

[mle2](#)

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaCorrBin(x=No.D.D,freq=Obs.fre.1,cov=0.0050,a=10,b=10)

bbmle::coef(parameters)      #extracting the parameters
```

---

EstMLECOMPBin	<i>Estimating the probability of success and v parameter for COM Poisson Binomial Distribution</i>
---------------	--

---

**Description**

The function will estimate the probability of success and  $v$  parameter using the maximum log likelihood method for the COM Poisson Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLECOMPBin(x, freq, p, v, ...)
```

**Arguments**

<code>x</code>	vector of binomial random variables.
<code>freq</code>	vector of frequencies.
<code>p</code>	single value for probability of success.
<code>v</code>	single value for $v$ .
<code>...</code>	<code>mle2</code> function inputs except data and estimating parameter.



**Details**

$$\begin{aligned}
 x &= 0, 1, 2, \dots \\
 freq &\geq 0 \\
 0 &< p < 1 \\
 -\infty &< v < +\infty
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLECOMPBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

**References**

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

**Examples**

```

No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECOMPBin(x=No.D.D,freq=Obs.fre.1,p=0.5,v=0.1)

bbmle::coef(parameters)      #extracting the parameters

```

---

EstMLECorrBin	<i>Estimating the probability of success and correlation for Correlated Binomial Distribution</i>
---------------	---

---

**Description**

The function will estimate the probability of success and correlation using the maximum log likelihood method for the Correlated Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLECorrBin(x, freq, p, cov, ...)
```

**Arguments**

<code>x</code>	vector of binomial random variables.
<code>freq</code>	vector of frequencies.
<code>p</code>	single value for probability of success.
<code>cov</code>	single value for covariance.
<code>...</code>	mle2 function inputs except data and estimating parameter.

**Details**

$$\begin{aligned}
 x &= 0, 1, 2, \dots \\
 freq &\geq 0 \\
 0 &< p < 1 \\
 -\infty &< cov < +\infty
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLECorrBin here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

**References**

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.
- Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .
- Jorge G. Morel and Nagaraj K. Neerchal. *Overdispersion Models in SAS*. SAS Institute, 2012.

**See Also**

[mle2](#)

**Examples**

```

No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECorrBin(x=No.D.D,freq=Obs.fre.1,p=0.5,cov=0.0050)

bbmle::coef(parameters)           #extracting the parameters

```

---

EstMLEGammaBin	<i>Estimating the shape parameters <math>c</math> and <math>l</math> for Gamma Binomial distribution</i>
----------------	--

---

### Description

The function will estimate the shape parameters using the maximum log likelihood method for the Gamma Binomial distribution when the binomial random variables and corresponding frequencies are given.

### Usage

```
EstMLEGammaBin(x, freq, c, l, ...)
```

### Arguments

<code>x</code>	vector of binomial random variables.
<code>freq</code>	vector of frequencies.
<code>c</code>	single value for shape parameter $c$ .
<code>l</code>	single value for shape parameter $l$ .
<code>...</code>	mle2 function inputs except data and estimating parameter.

### Details

$$0 < c, l$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

EstMLEGammaBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

### References

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```

No.D.D <- 0:7                #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGammaBin(x=No.D.D,freq=Obs.fre.1,c=0.1,l=0.1)

bbmle::coef(parameters)      #extracting the parameters

```

---

EstMLEGHGBB

*Estimating the shape parameters a,b and c for Gaussian Hypergeometric Generalized Beta Binomial Distribution*


---

**Description**

The function will estimate the shape parameters using the maximum log likelihood method for the Gaussian Hypergeometric Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLEGHGBB(x, freq, a, b, c, ...)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
c	single value for shape parameter lambda representing c.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEGHGBB here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

## References

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1–123.

## See Also

[hypergeo\\_powerseries](#)

---

[mle2](#)

## Examples

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGHGBB(No.D.D,Obs.fre.1,a=0.1,b=0.2,c=0.5)

bbmle::coef(parameters) #extracting the parameters
```

---

EstMLEGrassiaIIBin     *Estimating the shape parameters a and b for Grassia II Binomial distribution*

---

## Description

The function will estimate the shape parameters using the maximum log likelihood method for the Grassia II Binomial distribution when the binomial random variables and corresponding frequencies are given.

## Usage

```
EstMLEGrassiaIIBin(x, freq, a, b, ...)
```

## Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter a.
b	single value for shape parameter b.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEGrassiaIIBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

**References**

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```
No.D.D <- 0:7                #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGrassiaIIBin(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(parameters)     #extracting the parameters
```

---

EstMLEKumBin	<i>Estimating the shape parameters a and b and iterations for Kumaraswamy Binomial Distribution</i>
--------------	---

---

**Description**

The function will estimate the shape parameters using the maximum log likelihood method for the Kumaraswamy Binomial distribution when the binomial random variables and corresponding frequencies are given

**Usage**

```
EstMLEKumBin(x,freq,a,b,it,...)
```

**Arguments**

<code>x</code>	vector of binomial random variables.
<code>freq</code>	vector of frequencies.
<code>a</code>	single value for shape parameter alpha representing as a.
<code>b</code>	single value for shape parameter beta representing as b.
<code>it</code>	number of iterations to converge as a proper probability function replacing infinity.
<code>...</code>	mle2 function inputs except data and estimating parameter.

**Details**

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

$$it > 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEKumBin here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

**References**

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

**See Also**

[mle2](#)

**Examples**

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters1 <- EstMLEKumBin(x=No.D.D,freq=Obs.fre.1,a=10.1,b=1.1,it=10000)

bbmle::coef(parameters1) #extracting the parameters

## End(Not run)
```

---

 EstMLELMBin

*Estimating the probability of success and theta for Lovinson Multiplicative Binomial Distribution*


---

### Description

The function will estimate the probability of success and phi parameter using the maximum log likelihood method for the Lovinson Multiplicative Binomial distribution when the binomial random variables and corresponding frequencies are given.

### Usage

```
EstMLELMBin(x, freq, p, phi, ...)
```

### Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
phi	single value for phi parameter.
...	mle2 function inputs except data and estimating parameter.

### Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

### Value

EstMLELMBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

### References

Elamir, E.A., 2013. Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation. *Journal of Statistical Theory and Applications*, 12(1), pp.92-105.

### See Also

[mle2](#)



**Examples**

```

No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLELMBin(x=No.D.D,freq=Obs.fre.1,p=0.5,phi=15)

bbmle::coef(parameters)           #extracting the parameters

```

---

EstMLEMcGBB	<i>Estimating the shape parameters a,b and c for McDonald Generalized Beta Binomial distribution</i>
-------------	--

---

**Description**

The function will estimate the shape parameters using the maximum log likelihood method for the McDonald Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given.

**Usage**

```
EstMLEMcGBB(x, freq, a, b, c, ...)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

EstMLEMcGBB here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

## References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of Mcdonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

## See Also

[m1e2](#)

## Examples

```
No.D.D <- 0:7                #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMcGGB(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1,c=0.2)

bbmle::coef(parameters)    #extracting the parameters

## End(Not run)
```

---

EstMLEMultiBin	<i>Estimating the probability of success and theta for Multiplicative Binomial Distribution</i>
----------------	---

---

## Description

The function will estimate the probability of success and theta parameter using the maximum log likelihood method for the Multiplicative Binomial distribution when the binomial random variables and corresponding frequencies are given.

## Usage

```
EstMLEMultiBin(x, freq, p, theta, ...)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
theta	single value for theta parameter.
...	mle2 function inputs except data and estimating parameter.

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < theta$$

**Value**

EstMLEMultiBin here is used as a wrapper for the mle2 function of **bbmle** package therefore output is of class of mle2.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

**See Also**

[mle2](#)

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMultiBin(x=No.D.D,freq=Obs.fre.1,p=0.5,theta=15)

bbmle::coef(parameters)           #extracting the parameters
```

---

 EstMLETriBin

*Estimating the mode value for Triangular Binomial Distribution*


---

### Description

The function will estimate the mode value using the maximum log likelihood method for the Triangular Binomial Distribution when the binomial random variables and corresponding frequencies are given.

### Usage

```
EstMLETriBin(x, freq)
```

### Arguments

x	vector of binomial random variables.
freq	vector of frequencies.

### Details

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of EstMLETriBin will produce the classes of m1 and m1TB format consisting min Negative log likelihood value.

mode Estimated mode value.

AIC AIC value.

call the inputs for the function.

Methods print, summary, coef and AIC can be used to extract specific outputs.

## References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

## Examples

```
No.D.D <- 0:7 #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

## Not run:
#estimating the mode value and extracting the mode value
results <- EstMLETriBin(No.D.D,Obs.fre.1)

# extract the mode value and summary
coef(results)
summary(results)

AIC(results) #show the AIC value

## End(Not run)
```

---

Exam\_data

*Exam Data*

---

## Description

In an examination, there were 9 questions set on a particular topic. Each question is marked out of a total of 20 and in assessing the final class of a candidate, particular attention is paid to the total number of questions for which he has an "alpha", i.e., at least 15 out of 20, as well as his total number of marks. His number of alpha's is a rough indication of the "quality" of his exam performance. Thus, the distribution of alpha's over the candidates is of interest. There were 209 candidates attempting questions from this section of 9 questions and a total of 326 alpha's was awarded. So we treat 9 as the "litter size", and the dichotomous response is whether or not he got an alpha on the question.

## Usage

Exam\_data

**Format**

A data frame with 2 columns and 10 rows

No.of.alpha No of Alphas

fre Observed frequencies

**Source**

Extracted from

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

**Examples**

```
Exam_data$No.of.alpha      #extracting the binomial random variables
sum(Exam_data$fre)         #summing all the frequencies
```

---

fitAddBin

*Fitting the Additive Binomial Distribution when binomial random variable, frequency, probability of success and alpha are given*

---

**Description**

The function will fit the Additive Binomial distribution when random variables, corresponding frequencies, probability of success and alpha are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom value so that it can be seen if this distribution fits the data.

**Usage**

```
fitAddBin(x,obs.freq,p,alpha)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
alpha	single value for alpha.

**Details**

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-1 < alpha < 1$$

**Value**

The output of fitAddBin gives the class format fitAB and fit consisting a list

bin.ran.var binomial random variables.

obs.freq corresponding observed frequencies.

exp.freq corresponding expected frequencies.

statistic chi-squared test statistics.

df degree of freedom.

p.value probability value by chi-squared test statistic.

fitAB fitted probability values of dAddBin.

NegLL Negative Log Likelihood value.

p estimated probability value.

alpha estimated alpha parameter value.

AIC AIC value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

**Examples**

```

No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding the frequencies

## Not run:
#assigning the estimated probability value
paddbin <- EstMLEAddBin(No.D.D,Obs.fre.1)$p

#assigning the estimated alpha value
alphaaddbin <- EstMLEAddBin(No.D.D,Obs.fre.1)$alpha

#fitting when the random variable,frequencies,probability and alpha are given
results <- fitAddBin(No.D.D,Obs.fre.1,paddbin,alphaaddbin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)

## End(Not run)

```

---

fitBetaBin	<i>Fitting the Beta-Binomial Distribution when binomial random variable, frequency and shape parameters a and b are given</i>
------------	---

---

**Description**

The function will fit the Beta-Binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

**Usage**

```
fitBetaBin(x,obs.freq,a,b)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.



**Details**

$$0 < a, b$$
$$x = 0, 1, 2, \dots, n$$
$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitBetaBin gives the class format fitBB and fit consisting a list bin.ran.var binomial random variables.

obs.freq corresponding observed frequencies.

exp.freq corresponding expected frequencies.

statistic chi-squared test statistics.

df degree of freedom.

p.value probability value by chi-squared test statistic.

fitBB fitted values of dBetaBin.

NegLL Negative Log Likelihood value.

a estimated value for alpha parameter as a.

b estimated value for alpha parameter as b.

AIC AIC value.

over.dis.para over dispersion value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: doi: [10.1186/14712288858](https://doi.org/10.1186/14712288858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi: [10.1094/PHYTO83759](https://doi.org/10.1094/PHYTO83759)

**See Also**

[mle2](#)

**Examples**

```

No.D.D <- 0:7 #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaBin(No.D.D,Obs.fre.1,0.1,0.1)

bbmle::coef(parameters) #extracting the parameters a and b
aBetaBin <- bbmle::coef(parameters)[1] #assigning the parameter a
bBetaBin <- bbmle::coef(parameters)[2] #assigning the parameter b

#fitting when the random variable,frequencies,shape parameter values are given.
fitBetaBin(No.D.D,Obs.fre.1,aBetaBin,bBetaBin)

#estimating the parameters using moment generating function methods
results <- EstMGFBetaBin(No.D.D,Obs.fre.1)
results

aBetaBin1 <- results$a #assigning the estimated a
bBetaBin1 <- results$b #assigning the estimated b

#fitting when the random variable,frequencies,shape parameter values are given.
BB <- fitBetaBin(No.D.D,Obs.fre.1,aBetaBin1,bBetaBin1)

#extracting the expected frequencies
fitted(BB)

#extracting the residuals
residuals(BB)

```

---

fitBetaCorrBin	<i>Fitting the Beta-Correlated Binomial Distribution when binomial random variable, frequency, covariance, alpha and beta parameters are given</i>
----------------	--

---

**Description**

The function will fit the Beta-Correlated Binomial Distribution when random variables, corresponding frequencies, covariance, alpha and beta parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

**Usage**

```
fitBetaCorrBin(x,obs.freq,cov,a,b)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
cov	single value for covariance.
a	single value for alpha parameter.
b	single value for beta parameter.

**Details**

$$\begin{aligned}
 &obs.freq \geq 0 \\
 &x = 0, 1, 2, .. \\
 &-\infty < cov < +\infty \\
 &0 < a, b
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitBetaCorrBin gives the class format fitBCB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.
- df degree of freedom.
- p.value probability value by chi-squared test statistic
- corr Correlation value.
- fitBCB fitted probability values of dBetaCorrBin.
- NegLL Negative Log Likelihood value.
- a estimated shape parameter value a.
- b estimated shape parameter value b.
- cov estimated covariance value.
- AIC AIC value.
- call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.  
Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

**Examples**

```

No.D.D <- 0:7                #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaCorrBin(x=No.D.D,freq=Obs.fre.1,cov=0.0050,a=10,b=10)

covBetaCorrBin <- bbmle::coef(parameters)[1]
aBetaCorrBin <- bbmle::coef(parameters)[2]
bBetaCorrBin <- bbmle::coef(parameters)[3]

#fitting when the random variable,frequencies,covariance, a and b are given
results <- fitBetaCorrBin(No.D.D,Obs.fre.1,covBetaCorrBin,aBetaCorrBin,bBetaCorrBin)
results

#extract AIC value
AIC(results)

#extract fitted values
fitted(results)

```

---

fitBin

*Fitting the Binomial Distribution when binomial random variable, frequency and probability value are given*


---

**Description**

The function will fit the Binomial distribution when random variables, corresponding frequencies and probability value are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom so that it can be seen if this distribution fits the data.

**Usage**

```
fitBin(x, obs.freq, p=0)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability.

**Details**

$$x = 0, 1, 2, \dots$$

$$0 \leq p \leq 1$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of fitBin gives the class format fitB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics value.
- df degree of freedom.
- p.value probability value by chi-squared test statistic.
- fitB fitted probability values of dbinom.
- phat estimated probability value.
- call the inputs of the function.

### Examples

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#fitting when the random variable,frequencies are given.
fitBin(No.D.D,Obs.fre.1)
```

---

fitCOMPBin	<i>Fitting the COM Poisson Binomial Distribution when binomial random variable, frequency, probability of success and v parameter are given</i>
------------	---

---

### Description

The function will fit the COM Poisson Binomial Distribution when random variables, corresponding frequencies, probability of success and v parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

### Usage

```
fitCOMPBin(x,obs.freq,p,v)
```

**Arguments**

<code>x</code>	vector of binomial random variables.
<code>obs.freq</code>	vector of frequencies.
<code>p</code>	single value for probability of success.
<code>v</code>	single value for $v$ .

**Details**

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitCOMPBin gives the class format fitCPB and fit consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitCPB` fitted probability values of dCOMPBin.

`NegLL` Negative Log Likelihood value.

`p` estimated probability value.

`v` estimated  $v$  parameter value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

**References**

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

**Examples**

```

No.D.D <- 0:7 #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECOMPBin(x=No.D.D,freq=Obs.fre.1,p=0.5,v=0.050)

pCOMPBin <- bbmle::coef(parameters)[1]
vCOMPBin <- bbmle::coef(parameters)[2]

#fitting when the random variable,frequencies,probability and v parameter are given
results <- fitCOMPBin(No.D.D,Obs.fre.1,pCOMPBin,vCOMPBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)

```

---

fitCorrBin	<i>Fitting the Correlated Binomial Distribution when binomial random variable, frequency, probability of success and covariance are given</i>
------------	---

---

**Description**

The function will fit the Correlated Binomial Distribution when random variables, corresponding frequencies, probability of success and covariance are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

**Usage**

```
fitCorrBin(x,obs.freq,p,cov)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
cov	single value for covariance.

**Details**

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitCorrBin gives the class format fitCB and fit consisting a list bin.ran.var binomial random variables.

obs.freq corresponding observed frequencies.

exp.freq corresponding expected frequencies.

statistic chi-squared test statistics.

df degree of freedom.

p.value probability value by chi-squared test statistic.

corr Correlation value.

fitCB fitted probability values of dCorrBin.

NegLL Negative Log Likelihood value.

AIC AIC value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.



**Examples**

```

No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECorrBin(x=No.D.D,freq=Obs.fre.1,p=0.5,cov=0.0050)

pCorrBin <- bbmle::coef(parameters)[1]
covCorrBin <- bbmle::coef(parameters)[2]

#fitting when the random variable,frequencies,probability and covariance are given
results <- fitCorrBin(No.D.D,Obs.fre.1,pCorrBin,covCorrBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)

```

---

fitGammaBin

*Fitting the Gamma Binomial distribution when binomial random variable, frequency and shape parameters are given*


---

**Description**

The function will fit the Gamma Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

**Usage**

```
fitGammaBin(x,obs.freq,c,l)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
c	single value for shape parameter c.
l	single value for shape parameter l.

**Details**

$$0 < c, l$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitGammaBin gives the class format fitGaB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.
- df degree of freedom.
- p.value probability value by chi-squared test statistic.
- fitMB fitted values of dGammaBin.
- NegLL Negative Log Likelihood value.
- c estimated value for shape parameter c.
- l estimated value for shape parameter l.
- AIC AIC value.
- over.dis.para over dispersion value.
- call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGammaBin(x=No.D.D,freq=Obs.fre.1,c=0.1,l=0.1)

cGBin <- bbmle::coef(parameters)[1]         #assigning the estimated c
lGBin <- bbmle::coef(parameters)[2]         #assigning the estimated l
```

```

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGammaBin(No.D.D,Obs.fre.1,cGBin,lGBin)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

```

---

fitGHGBB	<i>Fitting the Gaussian Hypergeometric Generalized Beta Binomial Distribution when binomial random variable, frequency and shape parameters a,b and c are given</i>
----------	---

---

### Description

The function will fit the Gaussian Hypergeometric Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

### Usage

```
fitGHGBB(x,obs.freq,a,b,c)
```

### Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
c	single value for shape parameter lambda representing c.

### Details

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitGHGBB gives the class format fitGB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.
- df degree of freedom.
- p.value probability value by chi-squared test statistic.
- fitGB fitted values of dGHGBB.
- NegLL Negative Loglikelihood value.
- a estimated value for alpha parameter as a.
- b estimated value for beta parameter as b.
- c estimated value for gamma parameter as c.
- AIC AIC value.
- over.dis.para over dispersion value.
- call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

**See Also**

[hypergeo\\_powerseries](#)

---

[mle2](#)

**Examples**

```
No.D.D <- 0:7          #assigning the random variables
Obs.freq.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGHGBB(No.D.D,Obs.freq.1,0.1,20,1.3)

bbmle::coef(parameters)          #extracting the parameters
aGHGBB <- bbmle::coef(parameters)[1] #assigning the estimated a
bGHGBB <- bbmle::coef(parameters)[2] #assigning the estimated b
cGHGBB <- bbmle::coef(parameters)[3] #assigning the estimated c
```

```

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGHGBB(No.D.D,Obs.fre.1,aGHGBB,bGHGBB,cGHGBB)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

```

---

fitGrassiaIIBin	<i>Fitting the Grassia II Binomial distribution when binomial random variable, frequency and shape parameters are given</i>
-----------------	---

---

### Description

The function will fit the Grassia II Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

### Usage

```
fitGrassiaIIBin(x,obs.freq,a,b)
```

### Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
a	single value for shape parameter a.
b	single value for shape parameter b.

### Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitGrassiaIIBin gives the class format fitGrIIB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.
- df degree of freedom.
- p.value probability value by chi-squared test statistic.
- fitGrIIB fitted values of dGrassiaIIBin.
- NegLL Negative Log Likelihood value.
- a estimated value for shape parameter a.
- b estimated value for shape parameter b.
- AIC AIC value.
- over.dis.para over dispersion value.
- call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

**References**

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```
No.D.D <- 0:7          #assigning the random variables
Obs.freq.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGrassiaIIBin(x=No.D.D,freq=Obs.freq.1,a=0.1,b=0.1)

aGIIBin <- bbmle::coef(parameters)[1]         #assigning the estimated a
bGIIBin <- bbmle::coef(parameters)[2]         #assigning the estimated b

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGrassiaIIBin(No.D.D,Obs.freq.1,aGIIBin,bGIIBin)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)
```

---

fitKumBin	<i>Fitting the Kumaraswamy Binomial Distribution when binomial random variable, frequency and shape parameters a and b, iterations parameter it are given</i>
-----------	---

---

### Description

The function will fit the Kumaraswamy Binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

### Usage

```
fitKumBin(x,obs.freq,a,b,it)
```

### Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
it	number of iterations to converge as a proper probability function replacing infinity.

### Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots, n$$

$$obs.freq \geq 0$$

$$it > 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of fitKumBin gives the class format fitKB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.

df degree of freedom.

p. value probability value by chi-squared test statistic.

fitKB fitted values of dKumBin.

NegLL Negative Log Likelihood value.

a estimated value for alpha parameter as a.

b estimated value for beta parameter as b.

it estimated it value for iterations.

AIC AIC value.

over.dis.para over dispersion value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

## References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

## See Also

[mle2](#)

## Examples

```
No.D.D <- 0:7 #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEKumBin(x=No.D.D,freq=Obs.fre.1,a=10.1,b=1.1,it=10000)

bbmle::coef(parameters) #extracting the parameters
aKumBin <- bbmle::coef(parameters)[1] #assigning the estimated a
bKumBin <- bbmle::coef(parameters)[2] #assigning the estimated b
itKumBin <- bbmle::coef(parameters)[3] #assigning the estimated iterations

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitKumBin(No.D.D,Obs.fre.1,aKumBin,bKumBin,itKumBin*100)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

## End(Not run)
```



---

fitLMBin	<i>Fitting the Lovinson Multiplicative Binomial Distribution when binomial random variable, frequency, probability of success and theta parameter are given</i>
----------	---

---

### Description

The function will fit the Lovinson Multiplicative Binomial distribution when random variables, corresponding frequencies, probability of success and phi parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom value so that it can be seen if this distribution fits the data.

### Usage

```
fitLMBin(x, obs.freq, p, phi)
```

### Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
phi	single value for phi parameter.

### Details

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

### Value

The output of fitLMBin gives the class format fitLMB and fit consisting a list

- bin.ran.var binomial random variables.
- obs.freq corresponding observed frequencies.
- exp.freq corresponding expected frequencies.
- statistic chi-squared test statistics.
- df degree of freedom.
- p.value probability value by chi-squared test statistic.
- fitLMB fitted probability values of dLMBin.
- NegLL Negative Log Likelihood value.

p estimated probability value.  
 phi estimated phi parameter value.  
 AIC AIC value.  
 call the inputs of the function.  
 Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

## References

Elamir, E.A., 2013. Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation. *Journal of Statistical Theory and Applications*, 12(1), pp.92-105.

## See Also

[mle2](#)

## Examples

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLELMBin(x=No.D.D,freq=Obs.fre.1,p=0.1,phi=.3)

pLMBin=bbmle::coef(parameters)[1]    #assigning the estimated probability value
phiLMBin <- bbmle::coef(parameters)[2] #assigning the estimated phi value

#fitting when the random variable,frequencies,probability and phi are given
results <- fitLMBin(No.D.D,Obs.fre.1,pLMBin,phiLMBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

---

fitMcGBB

*Fitting the McDonald Generalized Beta Binomial distribution when binomial random variable, frequency and shape parameters are given*

---

## Description

The function will fit the McDonald Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

**Usage**

```
fitMcGGBB(x, obs.freq, a, b, c)
```

**Arguments**

x                    vector of binomial random variables.  
 obs.freq            vector of frequencies.  
 a                    single value for shape parameter alpha representing a.  
 b                    single value for shape parameter beta representing b.  
 c                    single value for shape parameter gamma representing c.

**Details**

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitMcGGBB gives the class format fitMB and fit consisting a list

bin.ran.var binomial random variables.

obs.freq corresponding observed frequencies.

exp.freq corresponding expected frequencies.

statistic chi-squared test statistics.

df degree of freedom.

p.value probability value by chi-squared test statistic.

fitMB fitted values of dMcGGBB.

NegLL Negative Log Likelihood value.

a estimated value for alpha parameter as a.

b estimated value for beta parameter as b.

c estimated value for gamma parameter as c.

AIC AIC value.

over.dis.para over dispersion value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

## References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of Mcdonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

## See Also

[mle2](#)

## Examples

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)          #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMcGGB(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1,c=3.2)

aMcGGB <- bbmle::coef(parameters)[1]          #assigning the estimated a
bMcGGB <- bbmle::coef(parameters)[2]          #assigning the estimated b
cMcGGB <- bbmle::coef(parameters)[3]          #assigning the estimated c

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitMcGGB(No.D.D,Obs.fre.1,aMcGGB,bMcGGB,cMcGGB)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

## End(Not run)
```

---

fitMultiBin

*Fitting the Multiplicative Binomial Distribution when binomial random variable, frequency, probability of success and theta parameter are given*

---

**Description**

The function will fit the Multiplicative Binomial distribution when random variables, corresponding frequencies, probability of success and theta parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom value so that it can be seen if this distribution fits the data.

**Usage**

```
fitMultiBin(x,obs.freq,p,theta)
```

**Arguments**

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
theta	single value for theta parameter.

**Details**

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < theta$$

**Value**

The output of fitMultiBin gives the class format fitMuB and fit consisting a list  
bin.ran.var binomial random variables.

obs.freq corresponding observed frequencies.

exp.freq corresponding expected frequencies.

statistic chi-squared test statistics.

df degree of freedom.

p.value probability value by chi-squared test statistic.

fitMuB fitted probability values of dMultiBin.

NegLL Negative Log Likelihood value.

p estimated probability value.

theta estimated theta parameter value.

AIC AIC value.

call the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

## See Also

[mle2](#)

## Examples

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMultiBin(x=No.D.D,freq=Obs.fre.1,p=0.1,theta=.3)

pMultiBin <- bbmle::coef(parameters)[1]    #assigning the estimated probability value
thetaMultiBin <- bbmle::coef(parameters)[2] #assigning the estimated theta value

#fitting when the random variable,frequencies,probability and theta are given
results <- fitMultiBin(No.D.D,Obs.fre.1,pMultiBin,thetaMultiBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

---

fitTriBin

*Fitting the Triangular Binomial Distribution when binomial random variable, frequency and mode value are given*

---

## Description

The function will fit the Triangular Binomial distribution when random variables, corresponding frequencies and mode parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

**Usage**

```
fitTriBin(x,obs.freq,mode)
```

**Arguments**

*x* vector of binomial random variables.  
*obs.freq* vector of frequencies.  
*mode* single value for mode.

**Details**

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots$$

$$0 < mode < 1$$

$$obs.freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitTriBin gives the class format fitTB and fit consisting a list

*bin.ran.var* binomial random variables.

*obs.freq* corresponding observed frequencies.

*exp.freq* corresponding expected frequencies.

*statistic* chi-squared test statistics value.

*df* degree of freedom.

*p.value* probability value by chi-squared test statistic.

*fitTB* fitted probability values of dTriBin.

*NegLL* Negative Log Likelihood value.

*mode* estimated mode value.

*AIC* AIC value.

*over.dis.para* over dispersion value.

*call* the inputs of the function.

Methods summary, print, AIC, residuals and fitted can be used to extract specific outputs.

## References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

## Examples

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

modeTriBin <- EstMLETriBin(No.D.D,Obs.fre.1)$mode #assigning the extracted the mode value

#fitting when the random variable,frequencies,mode value are given.
results <- fitTriBin(No.D.D,Obs.fre.1,modeTriBin)
results

#extract AIC value
AIC(results)

#extract fitted values
fitted(results)
```

---

Male\_Children

*Male children data*

---

## Description

The number of male children among the first 12 children of family size 13 in 6115 families taken from the hospital records in the nineteenth century Saxony (Sokal & Rohlf(1994), Lindsey (1995), p. 59). The thirteenth child is ignored to assuage the effect of families non-randomly stopping when a desired gender is reached.

## Usage

Male\_Children

## Format

A data frame with 2 columns and 13 rows.

No\_of\_Males No of Male children among first 12 children of family size 13

freq Observed frequencies for corresponding male children



**Source**

Extracted from

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

**Examples**

```
Male_Children$No_of_Males # extracting the binomial random variables
sum(Male_Children$freq)   # summing all the frequencies
```

---

 mazBETA

*Beta Distribution*


---

**Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1].

**Usage**

```
mazBETA(r, a, b)
```

**Arguments**

r                    vector of moments.  
 a                    single value for shape parameter alpha representing as a.  
 b                    single value for shape parameter beta representing as b.

**Details**

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a, b)}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a, b)}{B(a, b)}$$

;  $0 \leq p \leq 1$

$a, b > 0$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$r = 1, 2, 3, \dots$

Defined as  $B_p(a, b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of mazBETA gives the moments about zero in vector form.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

### See Also

[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>

### Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf #extracting the pdf values
```

```

dBETA(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var  #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3) #acquiring the cumulative probability values
mazBETA(1.4,3,2) #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2

#only the integer value of moments is taken here because moments cannot be decimal
mazBETA(1.9,5.5,6)

```

---

mazGAMMA

*Gamma Distribution*


---

### Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

### Usage

```
mazGAMMA(r, c, l)
```

### Arguments

r	vector of moments.
c	single value for shape parameter c.
l	single value for shape parameter l.

### Details

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1}}{\gamma(l)} [\ln(1/p)]^{l-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{Ig(l, c \ln(1/p))}{\gamma(l)}$$

;  $0 \leq p \leq 1$

$l, c > 0$

The mean the variance are denoted by

$$E[P] = \left(\frac{c}{c+1}\right)^l$$

$$var[P] = \left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l}$$

The moments about zero is denoted as

$$E[P^r] = \left(\frac{c}{c+r}\right)^l$$

$r = 1, 2, 3, \dots$

Defined as  $\gamma(l)$  is the gamma function. Defined as  $Ig(l, \ln(1/p)) = \int_0^{\ln(1/p)} t^{l-1} e^{-t} dt$  is the Lower incomplete gamma function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of mazGAMMA gives the moments about zero in vector form.

### References

Olshen, A. C. Transformations of the Pearson Type III Distribution. Ann. Math. Statist. 9 (1938), no. 3, 176–200.

### See Also

[GammaDist](#)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean #extracting the mean
dGAMMA(seq(0,1,by=0.01),5,6)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
```

```

a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6) #acquiring the cumulative probability values
mazGAMMA(1.4,5,6) #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)

```

---

mazGBeta1

*Generalized Beta Type-1 Distribution*


---

### Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

### Usage

```
mazGBeta1(r, a, b, c)
```

### Arguments

r	vector of moments
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

### Details

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1 - p^c)^{b-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} {}_2F_1(a, 1 - b; p^c; a + 1)$$

$0 \leq p \leq 1$

$a, b, c > 0$

The mean and the variance are denoted by

$$E[P] = \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}\right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a+b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is Beta function. Defined as  ${}_2F_1(a, b; c; d)$  is Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output mazGBeta1 gives the moments about zero in vector form.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of Mcdonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf #extracting the pdf values
```

```

dGBeta1(seq(0,1,by=0.01),2,3,1)$mean #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var #extracting the variance

pGBeta1(0.04,2,3,4) #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2) #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2 #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)

```

mazGHGBeta

*Gaussian Hypergeometric Generalized Beta Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

**Usage**

```
mazGHGBeta(r, n, a, b, c)
```

**Arguments**

r                    vector of moments.  
n                    single value for no of binomial trials.  
a                    single value for shape parameter alpha representing as a.  
b                    single value for shape parameter beta representing as b.  
c                    single value for shape parameter lambda representing as c.

**Details**

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

;  $0 \leq p \leq 1$

$a, b, c > 0$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$\text{var}[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  as the beta function. Defined as  ${}_2F_1(a, b; c; d)$  as the Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

## Value

The output of mazGHGBeta give the moments about zero in vector form.

## References

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

## See Also

[hypergeo\\_powerseries](#)

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1, .2, .3, 1.5, 2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
```



```

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659) #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)

```

---

mazKUM

*Kumaraswamy Distribution*


---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1].

## Usage

```
mazKUM(r, a, b)
```

## Arguments

r	vector of moments.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

## Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = 1 - (1 - p^a)^b$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$\text{var}[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

## Value

The output of mazKUM gives the moments about zero in vector form.

## References

Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.

Available at : doi: [10.1016/00221694\(80\)900360](https://doi.org/10.1016/00221694(80)900360).

Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.

Available at : doi: [10.1016/j.stamet.2008.04.001](https://doi.org/10.1016/j.stamet.2008.04.001).

## See Also

[Kumaraswamy](#)

## Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}
```

```

dKUM(seq(0,1,by=0.01),2,3)$pdf #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pKUM(seq(0,1,by=0.01),2,3) #acquiring the cumulative probability values
mazKUM(1.4,3,2) #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2 #acquiring the variance for a=2,b=3

#only the integer value of moments is taken here because moments cannot be decimal
mazKUM(1.9,5.5,6)

```

---

mazTRI

*Triangular Distribution Bounded Between [0,1]*


---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

## Usage

```
mazTRI(r,mode)
```

## Arguments

r	vector of moments.
mode	single value for mode.

## Details

Setting  $min = 0$  and  $max = 1$   $mode = c$  in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

;  $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

;  $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

;  $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

;  $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, \dots$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of mazTRI give the moments about zero in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299) .

**Examples**

```

#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean     #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var      #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)          #acquiring the cumulative probability values
mazTRI(1.4,.3)                       #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2         #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)

```

mazUNI

*Uniform Distribution Bounded Between [0,1]***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

**Usage**

```
mazUNI(r)
```

**Arguments**

r                      vector of moments

**Details**

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable  $P$  are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$$r = 1, 2, 3, \dots$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of mazUNI gives the moments about zero in vector form.

**References**

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

**See Also**

[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>

**Examples**

```

#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))        #acquiring the cumulative probability values

mazUNI(c(1,2,3))              #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)

```

---

NegLLAddBin

*Negative Log Likelihood value of Additive Binomial distribution*


---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLAddBin(x, freq, p, alpha)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
alpha	single value for alpha parameter.

**Details**

$$\begin{aligned}
 &freq \geq 0 \\
 &x = 0, 1, 2, \dots \\
 &0 < p < 1 \\
 &-1 < \alpha < 1
 \end{aligned}$$

**Value**

The output of NegLLAddBin will produce a single numeric value.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
NegLLAddBin(No.D.D,Obs.fre.1,.5,.03)        #acquiring the negative log likelihood value
```

---

NegLLBetaBin

*Negative Log Likelihood value of Beta-Binomial Distribution*

---

**Description**

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b.

**Usage**

```
NegLLBetaBin(x, freq, a, b)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.



**Details**

$$0 < a, b$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLBetaBin will produce a single numeric value.

**References**

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: doi: [10.1186/1471228858](https://doi.org/10.1186/1471228858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi: [10.1094/PHTO83759](https://doi.org/10.1094/PHTO83759)

**Examples**

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

NegLLBetaBin(No.D.D,Obs.fre.1,.3,.4) #acquiring the negative log likelihood value
```

---

NegLLBetaCorrBin	<i>Negative Log Likelihood value of Beta-Correlated Binomial distribution</i>
------------------	---

---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLBetaCorrBin(x, freq, cov, a, b)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
cov	single value for covariance.
a	single value for alpha parameter.
b	single value for beta parameter.

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$-\infty < cov < +\infty$$

$$0 < a, b$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLBetaCorrBin will produce a single numeric value.

**References**

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

NegLLBetaCorrBin(No.D.D,Obs.fre.1,0.001,9.03,10)  #acquiring the negative log likelihood value
```

---

NegLLCOMPBin	<i>Negative Log Likelihood value of COM Poisson Binomial distribution</i>
--------------	---

---

### Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

### Usage

```
NegLLCOMPBin(x, freq, p, v)
```

### Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
v	single value for v.

### Details

$$\begin{aligned}
 &freq \geq 0 \\
 &x = 0, 1, 2, \dots \\
 &0 < p < 1 \\
 &-\infty < v < +\infty
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of NegLLCOMPBin will produce a single numeric value.

### References

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

### Examples

```

No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

NegLLCOMPBin(No.D.D,Obs.fre.1,.5,.03)   #acquiring the negative log likelihood value

```

NegLLCorrBin

*Negative Log Likelihood value of Correlated Binomial distribution***Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLCorrBin(x, freq, p, cov)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
cov	single value for covariance.

**Details**

$$\begin{aligned}freq &\geq 0 \\x &= 0, 1, 2, .. \\0 &< p < 1 \\-\infty &< cov < +\infty\end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLCorrBin will produce a single numeric value.

**References**

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.
- Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990) .
- Jorge G. Morel and Nagaraj K. Neerchal. *Overdispersion Models in SAS*. SAS Institute, 2012.

**Examples**

```

No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

NegLLCorrBin(No.D.D,Obs.fre.1,.5,.03)      #acquiring the negative log likelihood value

```

---

NegLLGammaBin	<i>Negative Log Likelihood value of Gamma Binomial Distribution</i>
---------------	---

---

**Description**

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters  $l$  and  $c$ .

**Usage**

```
NegLLGammaBin(x, freq, c, l)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
c	single value for shape parameter $c$ .
l	single value for shape parameter $l$ .

**Details**

$$0 < l, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLGammaBin will produce a single numeric value.

**References**

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```

No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

NegLLGammaBin(No.D.D,Obs.fre.1,.3,.4) #acquiring the negative log likelihood value

```

---

NegLLGHGBB	<i>Negative Log Likelihood value of Gaussian Hypergeometric Generalized Beta Binomial Distribution</i>
------------	--

---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

**Usage**

```
NegLLGHGBB(x, freq, a, b, c)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
c	single value for shape parameter lambda representing c.

**Details**

$$0 < a, b, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLGHGBB will produce a single numeric value.

**References**

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

**See Also**

[hypergeo\\_powerseries](#)

**Examples**

```
No.D.D <- 0:7                #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
NegLLGHGBB(No.D.D,Obs.fre.1,.2,.3,1) #acquiring the negative log likelihood value
```

---

NegLLGrassiaIIBin	<i>Negative Log Likelihood value of Grassia II Binomial Distribution</i>
-------------------	--

---

**Description**

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters  $l$  and  $c$ .

**Usage**

```
NegLLGrassiaIIBin(x,freq,a,b)
```

**Arguments**

$x$	vector of binomial random variables.
$freq$	vector of frequencies.
$a$	single value for shape parameter $a$ .
$b$	single value for shape parameter $b$ .

**Details**

$$0 < a, b$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLGrassiaIIBin will produce a single numeric value.

**References**

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

**Examples**

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

NegLLGrassiaIIBin(No.D.D,Obs.fre.1,.3,.4) #acquiring the negative log likelihood value
```

---

 NegLLKumBin

*Negative Log Likelihood value of Kumaraswamy Binomial Distribution*

---

**Description**

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b and iterations it.

**Usage**

```
NegLLKumBin(x,freq,a,b,it=25000)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
it	number of iterations to converge as a proper probability function replacing infinity.



**Details**

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

$$it > 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLKumBin will produce a single numeric value.

**References**

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

## Not run:
NegLLKumBin(No.D.D,Obs.fre.1,1.3,4.4) #acquiring the negative log likelihood value

## End(Not run)
```

---

NegLLLMBin	<i>Negative Log Likelihood value of Lovinson Multiplicative Binomial distribution</i>
------------	---

---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLLMBin(x, freq, p, phi)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
phi	single value for phi parameter.

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

**Value**

The output of NegLLLMBin will produce a single numeric value.

**References**

Elamir, E.A., 2013. Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation. *Journal of Statistical Theory and Applications*, 12(1), pp.92-105.

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

NegLLLMBin(No.D.D,Obs.fre.1,.5,3)  #acquiring the negative log likelihood value
```

---

NegLLMcGBB

*Negative Log Likelihood value of McDonald Generalized Beta Binomial Distribution*

---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

**Usage**

```
NegLLMcGBB(x, freq, a, b, c)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

**Details**

$$0 < a, b, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**Value**

The output of NegLLMcGBB will produce a single numeric value.

**References**

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

**Examples**

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

NegLLMcGBB(No.D.D,Obs.fre.1,.2,.3,1)  #acquiring the negative log likelihood value
```

---

`NegLLMultiBin`*Negative Log Likelihood value of Multiplicative Binomial distribution*

---

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLMultiBin(x, freq, p, theta)
```

**Arguments**

<code>x</code>	vector of binomial random variables.
<code>freq</code>	vector of frequencies.
<code>p</code>	single value for probability of success.
<code>theta</code>	single value for theta parameter.

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < theta$$

**Value**

The output of `NegLLMultiBin` will produce a single numeric value.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

**Examples**

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

NegLLMultiBin(No.D.D,Obs.fre.1,.5,3)  #acquiring the negative log likelihood value
```

---

NegLLTriBin                      *Negative Log Likelihood value of Triangular Binomial Distribution*

---

**Description**

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the mode value.

**Usage**

```
NegLLTriBin(x,freq,mode)
```

**Arguments**

x	vector of binomial random variables.
freq	vector of frequencies.
mode	single value for mode.

**Details**

$$0 < mode = c < 1$$

$$x = 0, 1, 2, , \dots$$

$$freq \geq 0$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of NegLLTriBin will produce a single numeric value.

## References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

## Examples

```
No.D.D <- 0:7      #assigning the Random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

NegLLTriBin(No.D.D,Obs.fre.1,.023) #acquiring the Negative log likelihood value
```

---

Overdispersion

*Overdispersion*

---

## Description

After fitting the distribution using this function we can extract the overdispersion value. This function works for `fitTriBin`, `fitBetaBin`, `fitKumBin`, `fitGHGBB` and `fitMcGBB` for Binomial Mixture Distributions. Similarly, Alternate Binomial Distributions also support this function for `fitAdBin`, `fitBetaCorrBin`, `fitCOMPBin`, `fitCorrBin` and `fitMultiBin`.

## Usage

```
Overdispersion(object)
```

## Arguments

`object`                    An object from one of the classes of `fitTB`, `fitBB`, `fitKB`, `fitGB`, `fitMB`.

## Details

**Note** : Only objects from classes of above mentioned classes can be used.

## Value

The output of `Overdispersion` gives a single value which is the overdispersion.

**Examples**

```

No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#estimating mode value for given data
results<-EstMLETriBin(No.D.D,Obs.fre.1)
results
mode<-results$mode

#fitting the Triangular Binomial distribution for estimated parameters
TriBin<-fitTriBin(No.D.D,Obs.fre.1,mode)
TriBin

#extracting the overdispersion
Overdispersion(TriBin)

```

pAddBin

*Additive Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

**Usage**

```
pAddBin(x,n,p,alpha)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
alpha	single value for alpha parameter.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{AddBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \left( \frac{\alpha}{2} \left( \frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{(1-p)} - \frac{\alpha n(n-1)}{2} \right) + 1 \right)$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-1 < \alpha < 1$$

The alpha is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq \alpha \leq \left(\frac{n+(2p-1)^2}{4p(1-p)}\right)^{-1}$$

The mean and the variance are denoted as

$$E_{Addbin}[x] = np$$

$$Var_{Addbin}[x] = np(1-p)(1+(n-1)\alpha)$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pAddBin gives cumulative probability values in vector form.

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990)

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dAddBin(0:10,10,0.58,0.022)$pdf      #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
```



```

dAddBin(0:10,10,0.58,0.022)$var      #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pAddBin(0:10,10,0.58,0.022)        #acquiring the cumulative probability values

```

pBETA

*Beta Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1].

**Usage**

pBETA(p, a, b)

**Arguments**

p                    vector of probabilities.  
a                    single value for shape parameter alpha representing as a.  
b                    single value for shape parameter beta representing as b.

**Details**

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

;  $0 \leq p \leq 1$

$a, b > 0$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$r = 1, 2, 3, \dots$

Defined as  $B_p(a, b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pBETA gives the cumulative density values in vector form.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

### See Also

[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>

### Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf #extracting the pdf values
```

```

dBETA(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3) #acquiring the cumulative probability values
mazBETA(1.4,3,2) #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2

#only the integer value of moments is taken here because moments cannot be decimal
mazBETA(1.9,5.5,6)

```

---

pBetaBin

*Beta-Binomial Distribution*


---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

## Usage

```
pBetaBin(x,n,a,b)
```

## Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

## Details

Mixing Beta distribution with Binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x, n+b-x)}{B(a,b)}$$

$$a, b > 0$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as B(a, b) is the beta function.

### Value

The output of pBetaBin gives cumulative probability values in vector form.

### References

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: doi: [10.1186/14712288858](https://doi.org/10.1186/14712288858).

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: doi: [10.1016/01679473\(96\)900158](https://doi.org/10.1016/01679473(96)900158).

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi: [10.1094/PHTO83759](https://doi.org/10.1094/PHTO83759)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf      #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean    #extracting the mean
dBetaBin(0:10,10,4,.2)$var     #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis.para #extracting the over dispersion value
```

```

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}

pBetaBin(0:10,10,4,.2) #acquiring the cumulative probability values

```

pBetaCorrBin

*Beta-Correlated Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Correlated Binomial Distribution.

**Usage**

```
pBetaCorrBin(x,n,cov,a,b)
```

**Arguments**

x                    vector of binomial random variables.  
n                    single value for no of binomial trials.  
cov                  single value for covariance.  
a                    single value for alpha parameter  
b                    single value for beta parameter.

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{BetaCorrBin}(x) = \binom{n}{x} \frac{B(a+x, b+n-x)}{B(a+b)} \left[ 1 + \frac{cov}{2} \left( \frac{(x(x-1) \prod_{k=1}^4 (a+b+n-k))}{(\prod_{k=1}^2 (x+a-k) \prod_{k=1}^2 (n-x+b-k))} - \frac{(2x(n-1) \prod_{k=1}^3 (a+b+n-k))}{((x+a-1) \prod_{k=1}^2 (n-x+b-k))} + \frac{(n(n-1) \prod_{k=1}^2 (a+b+n-k))}{(\prod_{k=1}^2 (n-x+b-k))} \right) \right]$$

$$x = 0, 1, 2, 3, \dots, n$$

$$\begin{aligned}
 n &= 1, 2, 3, \dots \\
 -\infty &< cov < +\infty \\
 0 &< a, b \\
 0 &< p < 1 \\
 p &= \frac{a}{a+b} \\
 \Theta &= \frac{1}{a+b}
 \end{aligned}$$

The Correlation is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

where  $fo = \min(x - (n-1)p - 0.5)^2$

The mean and the variance are denoted as

$$\begin{aligned}
 E_{BetaCorrBin}[x] &= np \\
 Var_{BetaCorrBin}[x] &= np(1-p)(n\Theta + 1)(1 + \Theta)^{-1} + n(n-1)cov \\
 Corr_{BetaCorrBin}[x] &= \frac{cov}{p(1-p)}
 \end{aligned}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pBetaCorrBin gives cumulative probability values in vector form.

### References

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

### Examples

```

#plotting the random variables and probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Related binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],pch=16)
}

```

```

}

dBetaCorrBin(0:10,10,0.001,10,13)$pdf      #extracting the pdf values
dBetaCorrBin(0:10,10,0.001,10,13)$mean    #extracting the mean
dBetaCorrBin(0:10,10,0.001,10,13)$var     #extracting the variance
dBetaCorrBin(0:10,10,0.001,10,13)$corr    #extracting the correlation
dBetaCorrBin(0:10,10,0.001,10,13)$mincorr #extracting the minimum correlation value
dBetaCorrBin(0:10,10,0.001,10,13)$maxcorr #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],pch=16)
}

pBetaCorrBin(0:10,10,0.001,10,13)      #acquiring the cumulative probability values

```

---

pCOMPBin

*COM Poisson Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the COM Poisson Binomial Distribution.

### Usage

```
pCOMPBin(x,n,p,v)
```

### Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
v	single value for v.

### Details

The probability function and cumulative function can be constructed and are denoted below  
The cumulative probability function is the summation of probability function values.

$$P_{COMPBin}(x) = \frac{\binom{n}{x}^v p^x (1-p)^{n-x}}{\sum_{j=0}^n \binom{n}{j}^v p^j (1-p)^{n-j}}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pCOMPBin gives cumulative probability values in vector form.

### References

Extracted from

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. *Statistics & Probability Letters*, 87, pp.158-166.

Available at: doi: [10.1016/j.spl.2014.01.019](https://doi.org/10.1016/j.spl.2014.01.019)

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCOMPBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCOMPBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dCOMPBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
```



```

lines(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCOMPBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values

```

---

pCorrBin                      *Correlated Binomial Distribution*

---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

### Usage

```
pCorrBin(x, n, p, cov)
```

### Arguments

x                      vector of binomial random variables.  
n                      single value for no of binomial trials.  
p                      single value for probability of success.  
cov                    single value for covariance.

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{CorrBin}(x) = \binom{n}{x} (p^x)(1-p)^{n-x} \left(1 + \left(\frac{cov}{2p^2(1-p)^2}\right)((x-np)^2 + x(2p-1) - np^2)\right)$$

$x = 0, 1, 2, 3, \dots, n$      $n = 1, 2, 3, \dots$      $0 < p < 1$      $-\infty < cov < +\infty$

The Correlation is in between

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{1-p}{p}\right) \leq cov \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

where  $fo = \min(x - (n-1)p - 0.5)^2$

The mean and the variance are denoted as

$$E_{CorrBin}[x] = np$$

$$Var_{CorrBin}[x] = n(p(1-p) + (n-1)cov)$$

$$Corr_{CorrBin}[x] = \frac{cov}{p(1-p)}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pCorrBin gives cumulative probability values in vector form.

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

Jorge G. Morel and Nagaraj K. Neerchal. *Overdispersion Models in SAS*. SAS Institute, 2012.

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dCorrBin(0:10,10,0.58,0.022)$var     #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr    #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCorrBin(0:10,10,0.58,0.022)        #acquiring the cumulative probability values
```

pGAMMA

Gamma Distribution

**Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

**Usage**

pGAMMA(p, c, l)

**Arguments**

p                      vector of probabilities.  
 c                      single value for shape parameter c.  
 l                      single value for shape parameter l.

**Details**

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1} [\ln(1/p)]^{l-1}}{\gamma(l)}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{I\gamma(l, c \ln(1/p))}{\gamma(l)}$$

;  $0 \leq p \leq 1$

$$l, c > 0$$

The mean the variance are denoted by

$$E[P] = \left(\frac{c}{c+1}\right)^l$$

$$var[P] = \left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l}$$

The moments about zero is denoted as

$$E[P^r] = \left(\frac{c}{c+r}\right)^l$$

$r = 1, 2, 3, \dots$

Defined as  $\gamma(l)$  is the gamma function. Defined as  $I\gamma(l, c \ln(1/p)) = \int_0^{c \ln(1/p)} t^{l-1} e^{-t} dt$  is the Lower incomplete gamma function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pGAMMA gives the cumulative density values in vector form.

**References**

Olshen, A. C. Transformations of the Pearson Type III Distribution. Ann. Math. Statist. 9 (1938), no. 3, 176–200.

**See Also**

[GammaDist](#)

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean #extracting the mean
dGAMMA(seq(0,1,by=0.01),5,6)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6) #acquiring the cumulative probability values
mazGAMMA(1.4,5,6) #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)
```

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Gamma Binomial Distribution.

**Usage**

pGammaBin(x, n, c, l)

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
c	single value for shape parameter c.
l	single value for shape parameter l.

**Details**

Mixing Gamma distribution with Binomial distribution will create the the Gamma Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GammaBin}[x] = \binom{n}{x} \sum_{j=0}^{n-x} \binom{n-x}{j} (-1)^j \left(\frac{c}{c+x+j}\right)^l$$

$$c, l > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GammaBin}[x] = \left(\frac{c}{c+1}\right)^l$$

$$Var_{GammaBin}[x] = n^2 \left[ \left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l} \right] + n \left(\frac{c}{c+1}\right)^l \left[ 1 - \left(\frac{c+1}{c+2}\right)^l \right]$$

$$overdispersion = \frac{\left(\frac{c}{c+2}\right)^l - \left(\frac{c}{c+1}\right)^{2l}}{\left(\frac{c}{c+1}\right)^l \left[ 1 - \left(\frac{c}{c+1}\right)^l \right]}$$

**Value**

The output of pGammaBin gives cumulative probability values in vector form.

## References

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. Australian Journal of Statistics, 19(2), pp.108-114.

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Gamma-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGammaBin(0:10,10,4,.2)$pdf      #extracting the pdf values
dGammaBin(0:10,10,4,.2)$mean    #extracting the mean
dGammaBin(0:10,10,4,.2)$var     #extracting the variance
dGammaBin(0:10,10,4,.2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
}

pGammaBin(0:10,10,4,.2) #acquiring the cumulative probability values
```

---

pGBeta1

*Generalized Beta Type-1 Distribution*

---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

## Usage

pGBeta1(p, a, b, c)

**Arguments**

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

**Details**

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1 - p^c)^{b-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} {}_2F_1(a, 1 - b; p^c; a + 1)$$

$0 \leq p \leq 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$\text{var}[P] = \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a + b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is Beta function. Defined as  ${}_2F_1(a, b; c; d)$  is Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output pGBeta1 gives the cumulative density values in vector form.

## References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of Mcdonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf      #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean    #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var     #extracting the variance

pGBeta1(0.04,2,3,4)                      #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2)                      #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2  #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)
```

---

pGHGBB

*Gaussian Hypergeometric Generalized Beta Binomial Distribution*

---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

## Usage

pGHGBB(x, n, a, b, c)



**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha value representing a.
b	single value for shape parameter beta value representing b.
c	single value for shape parameter lambda value representing c.

**Details**

Mixing Gaussian Hypergeometric Generalized Beta distribution with Binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GHGBB}(x) = \frac{1}{{}_2F_1(-n, a; -b - n + 1; c)} \binom{n}{x} \frac{B(x + a, n - x + b)}{B(a, b + n)} (c^x)$$

$$a, b, c > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$Var_{GHGBB}[x] = nE_{GHGBeta}(1 - E_{GHGBeta}) + n(n - 1)Var_{GHGBeta}$$

$$overdispersion = \frac{var_{GHGBeta}}{E_{GHGBeta}(1 - E_{GHGBeta})}$$

Defined as  $B(a, b)$  is the beta function. Defined as  ${}_2F_1(a, b; c; d)$  is the Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pGHGBB gives cumulative probability function values in vector form.

**References**

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1-123.

**See Also**

[hypergeo\\_powerseries](#)

**Examples**

```
#plotting the random variables and probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}

dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean    #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var     #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}

pGHGBB(0:7,7,1.3,0.3,1.3)      #acquiring the cumulative probability values
```

---

pGHGBeta

*Gaussian Hypergeometric Generalized Beta Distribution*

---

**Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

**Usage**

pGHGBeta(p, n, a, b, c)

**Arguments**

p	vector of probabilities.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter lambda representing as c.

**Details**

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

;  $0 \leq p \leq 1$

$$a, b, c > 0$$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a, b)} \frac{{}_2F_1(-n, a; -b - n + 1; 1)}{{}_2F_1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  as the beta function. Defined as  ${}_2F_1(a, b; c; d)$  as the Gaussian Hypergeometric function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pGHGBeta gives the cumulative density values in vector form.

## References

Rodriguez-Avi, J., Conde-Sanchez, A., Saez-Castillo, A. J., & Olmo-Jimenez, M. J. (2007). A generalization of the beta-binomial distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 56(1), 51-61.

Pearson, J., 2009. Computation of Hypergeometric Functions. *Transformation*, (September), p.1–123.

## See Also

[hypergeo\\_powerseries](#)

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659) #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)
```

---

pGrassiaIIBin                      *Grassia-II-Binomial Distribution*

---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Grassia-II-Binomial Distribution.

### Usage

pGrassiaIIBin(x,n,a,b)

### Arguments

x                                      vector of binomial random variables.  
n                                        single value for no of binomial trials.  
a                                        single value for shape parameter a.  
b                                        single value for shape parameter b.

### Details

Mixing Gamma distribution with Binomial distribution will create the the Grassia-II-Binomial distribution, only when  $(1-p)=e^{(-\lambda)}$  of the Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GrassiaIIBin}[x] = \binom{n}{x} \sum_{j=0}^x \binom{x}{j} (-1)^{x-j} (1 + b(n-j))^{-a}$$

$$a, b > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GrassiaIIBin}[x] = \left(\frac{b}{b+1}\right)^a$$

$$Var_{GrassiaIIBin}[x] = n^2 \left[ \left(\frac{b}{b+2}\right)^a - \left(\frac{b}{b+1}\right)^{2a} \right] + n \left(\frac{b}{b+1}\right)^a \left[ 1 - \left(\frac{b+1}{b+2}\right)^a \right]$$

$$overdispersion = \frac{\left(\frac{b}{b+2}\right)^a - \left(\frac{b}{b+1}\right)^{2a}}{\left(\frac{b}{b+1}\right)^a \left[ 1 - \left(\frac{b}{b+1}\right)^a \right]}$$

### Value

The output of pGrassiaIIBin gives cumulative probability values in vector form.

## References

Grassia, A., 1977. On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions. *Australian Journal of Statistics*, 19(2), pp.108-114.

## Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.3,0.4,0.5,0.6,0.8)
plot(0,0,main="Grassia II binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGrassiaIIBin(0:10,10,2*a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGrassiaIIBin(0:10,10,2*a[i],a[i])$pdf,col = col[i],pch=16)
}

dGrassiaIIBin(0:10,10,4,.2)$pdf      #extracting the pdf values
dGrassiaIIBin(0:10,10,4,.2)$mean    #extracting the mean
dGrassiaIIBin(0:10,10,4,.2)$var    #extracting the variance
dGrassiaIIBin(0:10,10,4,.2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(0.3,0.4,0.5,0.6)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGrassiaIIBin(0:10,10,2*a[i],a[i]),col = col[i])
points(0:10,pGrassiaIIBin(0:10,10,2*a[i],a[i]),col = col[i])
}

pGrassiaIIBin(0:10,10,4,.2) #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between  $[0,1]$ .

## Usage

pKUM(p, a, b)

**Arguments**

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

**Details**

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = 1 - (1-p)^b$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$var[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pKUM gives the cumulative density values in vector form.

**References**

Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.

Available at : doi: [10.1016/00221694\(80\)900360](https://doi.org/10.1016/00221694(80)900360).

Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.

Available at : doi: [10.1016/j.stamet.2008.04.001](https://doi.org/10.1016/j.stamet.2008.04.001).

**See Also**[Kumaraswamy](#)**Examples**

```

#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dKUM(seq(0,1,by=0.01),2,3)$pdf #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pKUM(seq(0,1,by=0.01),2,3) #acquiring the cumulative probability values

mazKUM(1.4,3,2) #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2 #acquiring the variance for a=2,b=3

#only the integer value of moments is taken here because moments cannot be decimal
mazKUM(1.9,5.5,6)

```

---

pKumBin

*Kumaraswamy Binomial Distribution*

---

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Kumaraswamy Binomial Distribution.

**Usage**

```
pKumBin(x,n,a,b,it=25000)
```



**Arguments**

- x                      vector of binomial random variables.
- n                      single value for no of binomial trial.
- a                      single value for shape parameter alpha representing a.
- b                      single value for shape parameter beta representing b.
- it                      number of iterations to converge as a proper probability function replacing infinity.

**Details**

Mixing Kumaraswamy distribution with Binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{KumBin}(x) = ab \binom{n}{x} \sum_{j=0}^{it} (-1)^j \binom{b-1}{j} B(x+a+aj, n-x+1)$$

$$a, b > 0$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$it > 0$$

The mean, variance and over dispersion are denoted as

$$E_{KumBin}[x] = nbB(1 + \frac{1}{a}, b)$$

$$Var_{KumBin}[x] = (n^2)b(B(1 + \frac{2}{a}, b) - bB(1 + \frac{1}{a}, b)^2) + nb(B(1 + \frac{1}{a}, b) - B(1 + \frac{2}{a}, b))$$

$$overdispersion = \frac{(bB(1 + \frac{2}{a}, b) - (bB(1 + \frac{1}{a}, b))^2)}{(bB(1 + \frac{1}{a}, b) - (bB(1 + \frac{1}{a}, b))^2)}$$

Defined as  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of pKumBin gives cumulative probability values in vector form.

**References**

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

## Examples

```
## Not run:
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5) {
lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

## End(Not run)

dKumBin(0:10,10,4,2)$pdf #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

## Not run:
#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5) {
lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
}

## End(Not run)
pKumBin(0:10,10,4,2) #acquiring the cumulative probability values
```

---

 Plant\_DiseaseData

*Plant Disease Incidence data*


---

## Description

Cochran(1936) provided a data that comprise the number of tomato spotted wilt virus(TSWV) infected tomato plants in the field trials in Australia. The field map was divided into 160 'quadrats'. 9 tomato plants in each quadrat. then the numbers of TSWV infected tomato plants were counted in each quadrat. Number of infected plants out of 9 plants per quadrat can be treated as a binomial variable. the collection of all such responses from all 160 quadrats would form "binomial outcome data" below provided is a data set similar to Cochran plant disease incidence data. Marcus R(1984). orange trees infected with citrus tristeza virus (CTV) in an orchard in central Israel. We divided the field map into 84 "quadrats" of 4 rows x 3 columns and counted the total number (1981 + 1982) of infected trees out of a maximum of  $n = 12$  in each quadrat

**Usage**

```
Plant_DiseaseData
```

**Format**

A data frame with 2 columns and 10 rows

```
Dis.plant Diseased Plants
```

```
fre Observed frequencies
```

**Source**

Extracted from

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. *Phytopathology*, 83(9), p.759.

Available at: doi: [10.1094/Phyto83759](https://doi.org/10.1094/Phyto83759).

**Examples**

```
Plant_DiseaseData$Dis.plant      # extracting the binomial random variables
sum(Plant_DiseaseData$fre)       # summing all the frequencies
```

---

pLMBin

*Lovinson Multiplicative Binomial Distribution*

---

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Lovinson Multiplicative Binomial Distribution.

**Usage**

```
pLMBin(x,n,p,phi)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
phi	single value for phi.

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{LMBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(phi)^{x(n-x)}}{f(p, phi, n)}$$

here  $f(p, phi, n)$  is

$$f(p, phi, n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (phi)^{k(n-k)}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < phi$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pLMBin gives cumulative probability values in vector form.

### References

Elamir, E.A., 2013. Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation. Journal of Statistical Theory and Applications, 12(1), pp.92-105.

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
      function graph",xlab="Binomial random variable",
      ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dLMBin(0:10,10,.58,10.022)$pdf #extracting the pdf values
```

```

dLMBin(0:10,10,.58,10.022)$mean #extracting the mean
dLMBin(0:10,10,.58,10.022)$var #extracting the variance

#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
      function graph",xlab="Binomial random variable",
      ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pLMBin(0:10,10,.58,10.022) #acquiring the cumulative probability values

```

pMcGGB

*McDonald Generalized Beta Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

**Usage**

```
pMcGGB(x, n, a, b, c)
```

**Arguments**

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

**Details**

Mixing Generalized Beta Type-1 Distribution with Binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{McGGB}(x) = \binom{n}{x} \frac{1}{B(a, b)} \left( \sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B\left(\frac{x}{c} + a + \frac{j}{c}, b\right) \right)$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGGBB}[x] = n \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGGBB}[x] = n^2 \left( \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2 \right) + n \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} \right)$$

$$overdispersion = \frac{\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2}{\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2}$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

### Value

The output of pMcGGBB gives cumulative probability function values in vector form.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. *International Journal of Statistics and Probability*, 2(2), pp.24-41.

Available at: doi: [10.5539/ijsp.v2n2p24](https://doi.org/10.5539/ijsp.v2n2p24).

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roозegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. *Communications in Statistics - Simulation and Computation*, (May), pp.0-0.

Available at: doi: [10.1080/03610918.2015.1088024](https://doi.org/10.1080/03610918.2015.1088024).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.6)
plot(0,0,main="McDonald generalized beta-binomial probability function graph",
xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMcGGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMcGGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}

dMcGGBB(0:10,10,4,2,1)$pdf           #extracting the pdf values
dMcGGBB(0:10,10,4,2,1)$mean         #extracting the mean
dMcGGBB(0:10,10,4,2,1)$var         #extracting the variance
```

```

dMcGBB(0:10,10,4,2,1)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
points(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
}

pMcGBB(0:10,10,4,2,1) #acquiring the cumulative probability values

```

---

pMultiBin

*Multiplicative Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

### Usage

```
pMultiBin(x,n,p,theta)
```

### Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
theta	single value for theta.

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(\theta)^{x(n-x)}}{f(p, \theta, n)}$$

here  $f(p, \theta, n)$  is

$$f(p, \theta, n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (\theta)^{k(n-k)}$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < \theta$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pMultiBin gives cumulative probability values in vector form.

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi: [10.1080/03610928508828990](https://doi.org/10.1080/03610928508828990).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}
```

```
dMultiBin(0:10,10,.58,10.022)$pdf #extracting the pdf values
dMultiBin(0:10,10,.58,10.022)$mean #extracting the mean
dMultiBin(0:10,10,.58,10.022)$var #extracting the variance
```

```
#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
```



```

for (i in 1:5)
{
lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pMultiBin(0:10,10,.58,10.022)    #acquiring the cumulative probability values

```

---

pTRI

*Triangular Distribution Bounded Between [0,1]*


---

### Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

### Usage

pTRI(p,mode)

### Arguments

p                    vector of probabilities.  
mode                 single value for mode.

### Details

Setting  $min = 0$  and  $max = 1$   $mode = c$  in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

;  $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

;  $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

;  $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

;  $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a + b + c)}{3} = \frac{(1 + c)}{3}$$

$$\text{var}[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1 + c^2 - c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, \dots$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pTRI gives the cumulative density values in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In *Advances in Mathematical and Statistical Modeling*. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean     #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var      #extracting the variance
```

```

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2    #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)

```

---

pTriBin

*Triangular Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

### Usage

```
pTriBin(x,n,mode)
```

### Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
mode	single value for mode

### Details

Mixing unit bounded Triangular distribution with Binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{TriBin}(x) = 2 \binom{n}{x} (c^{-1} B_c(x+2, n-x+1) + (1-c)^{-1} B(x+1, n-x+2) - (1-c)^{-1} B_c(x+1, n-x+2))$$

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{TriBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$

$$overdispersion = \frac{(1-c+c^2)}{2(2+c-c^2)}$$

Defined as  $B_c(a, b) = \int_0^c t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pTriBin gives cumulative probability function values in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi: [10.1007/9780817646264\\_2](https://doi.org/10.1007/9780817646264_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

### Examples

```
#plotting the random variables and probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}
```

```

dTriBin(0:10,10,.4)$pdf      #extracting the pdf values
dTriBin(0:10,10,.4)$mean    #extracting the mean
dTriBin(0:10,10,.4)$var     #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}

pTriBin(0:10,10,.4) #acquiring the cumulative probability values

```

pUNI

*Uniform Distribution Bounded Between [0,1]***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

**Usage**

pUNI(p)

**Arguments**

p                      vector of probabilities.

**Details**

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable  $P$  are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$\text{var}[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$r = 1, 2, 3, \dots$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pUNI gives the cumulative density values in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley.

### See Also

[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>

### Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))        #acquiring the cumulative probability values

mazUNI(c(1,2,3))              #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)
```

---

pUniBin

*Uniform Binomial Distribution*


---

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

### Usage

pUniBin(x,n)

### Arguments

x                      vector of binomial random variables.  
n                        single value for no of binomial trials.

### Details

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{UniBin}(x) = \frac{1}{n+1}$$

$$n = 1, 2, \dots$$

$$x = 0, 1, 2, \dots, n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

**NOTE** : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of pUniBin gives cumulative probability function values in vector form.

## References

Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.

Available at: doi: [10.9734/BJMCS/2014/12299](https://doi.org/10.9734/BJMCS/2014/12299).

## Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)

dUniBin(0:300,300)$pdf #extracting the pdf values
dUniBin(0:10,10)$mean #extracting the mean
dUniBin(0:10,10)$var #extracting the variance
dUniBin(0:10,10)$over.dis.para #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15) #acquiring the cumulative probability values
```

---

Terror\_data\_ARG

*Terror Data ARG*

---

## Description

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the United States and 65 ones in Argentina.

## Usage

Terror\_data\_ARG

## Format

A data frame with 2 columns and 9 rows

Incidents No of Incidents Occurred

fre Observed frequencies



**Source**

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

**Examples**

```
Terror_data_ARG$Incidents      #extracting the binomial random variables
sum(Terror_data_ARG$fre)       #summing all the frequencies
```

---

Terror_data_USA	<i>Terror Data USA</i>
-----------------	------------------------

---

**Description**

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the United States and 65 ones in Argentina.

**Usage**

```
Terror_data_USA
```

**Format**

A data frame with 2 columns and 9 rows

Incidents No of Incidents Occurred

fre Observed frequencies

**Source**

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

**Examples**

```
Terror_data_USA$Incidents      #extracting the binomial random variables
sum(Terror_data_USA$fre)       #summing all the frequencies
```

# Index

## \* datasets

- Alcohol\_data, 4
  - Chromosome\_data, 5
  - Course\_data, 6
  - Epidemic\_Cold, 49
  - Exam\_data, 69
  - Male\_Children, 96
  - Plant\_DiseaseData, 154
  - Terror\_data\_ARG, 168
  - Terror\_data\_USA, 169
- Alcohol\_data, 4
- Beta, 10, 98, 130
- BODextract, 5
- Chromosome\_data, 5
- Course\_data, 6
- dAddBin, 7
- dBETA, 9
- dBetaBin, 11
- dBetaCorrBin, 13
- dCOMPBin, 15
- dCorrBin, 17
- dGAMMA, 19
- dGammaBin, 21
- dGBeta1, 23
- dGHGBB, 25
- dGHGBeta, 27
- dGrassiaIIBin, 29
- dKUM, 31
- dKumBin, 33
- dLMBin, 35
- dMcGGBB, 37
- dMultiBin, 39
- dTRI, 41
- dTriBin, 44
- dUNI, 46
- dUniBin, 48
- Epidemic\_Cold, 49
- EstMGFBetaBin, 50
- EstMLEAddBin, 52
- EstMLEBetaBin, 53
- EstMLEBetaCorrBin, 55
- EstMLECOMPBin, 56
- EstMLECorrBin, 57
- EstMLEGammaBin, 59
- EstMLEGHGBB, 60
- EstMLEGrassiaIIBin, 61
- EstMLEKumBin, 62
- EstMLELMBin, 64
- EstMLEMcGGBB, 65
- EstMLEMultiBin, 66
- EstMLETriBin, 68
- Exam\_data, 69
- fitAddBin, 70
- fitBetaBin, 72
- fitBetaCorrBin, 74
- fitBin, 76
- fitCOMPBin, 77
- fitCorrBin, 79
- fitGammaBin, 81
- fitGHGBB, 83
- fitGrassiaIIBin, 85
- fitKumBin, 87
- fitLMBin, 89
- fitMcGGBB, 90
- fitMultiBin, 92
- fitTriBin, 94
- GammaDist, 20, 100, 140
- hypergeo\_powerseries, 26, 28, 61, 84, 104, 119, 146, 148
- Kumaraswamy, 32, 106, 152
- Male\_Children, 96
- mazBETA, 97

- mazGAMMA, [99](#)
- mazGBeta1, [101](#)
- mazGHGBeta, [103](#)
- mazKUM, [105](#)
- mazTRI, [107](#)
- mazUNI, [109](#)
- mIe2, [51](#), [54](#), [56](#), [58](#), [61](#), [63](#), [64](#), [66](#), [67](#), [73](#), [84](#),  
[88](#), [90](#), [92](#), [94](#)
  
- NegLLAddBin, [111](#)
- NegLLBetaBin, [112](#)
- NegLLBetaCorrBin, [113](#)
- NegLLCOMPBin, [115](#)
- NegLLCorrBin, [116](#)
- NegLLGammaBin, [117](#)
- NegLLGHGBB, [118](#)
- NegLLGrassiaIIBin, [119](#)
- NegLLKumBin, [120](#)
- NegLLLMBin, [121](#)
- NegLLMcGBB, [122](#)
- NegLLMultiBin, [124](#)
- NegLLTriBin, [125](#)
  
- Overdispersion, [126](#)
  
- pAddBin, [127](#)
- pBETA, [129](#)
- pBetaBin, [131](#)
- pBetaCorrBin, [133](#)
- pCOMPBin, [135](#)
- pCorrBin, [137](#)
- pGAMMA, [139](#)
- pGammaBin, [141](#)
- pGBeta1, [142](#)
- pGHGBB, [144](#)
- pGHGBeta, [146](#)
- pGrassiaIIBin, [149](#)
- pKUM, [150](#)
- pKumBin, [152](#)
- Plant\_DiseaseData, [154](#)
- pLMBin, [155](#)
- pMcGBB, [157](#)
- pMultiBin, [159](#)
- pTRI, [161](#)
- pTriBin, [163](#)
- pUNI, [165](#)
- pUniBin, [167](#)
  
- Terror\_data\_ARG, [168](#)
  
- Terror\_data\_USA, [169](#)
  
- Uniform, [47](#), [110](#), [166](#)