Package 'gellipsoid'

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Type Package

Title Generalized Ellipsoids

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Description
Represent generalized geometric ellipsoids with the ``(U,D)'' representation. It allows degenerate
and/or unbounded ellipsoids, together with methods for linear and duality transforma-
tions, and for plotting.
The ideas are described in Friendly, Monette & Fox (2013).

Depends rgl

URL https://github.com/friendly/gellipsoid

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License GPL (>= 2)

Language en-US

LazyLoad yes

RoxygenNote 7.1.2

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gellipsoid-package   *Generalized ellipsoids*

Description

Represents generalized ellipsoids with the "(U,D)" representation, allowing both degenerate and
unbounded ellipsoids, together with methods for linear and duality transformations, and for plotting

Details

Package:	gellipsoid
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License:	GPL (>=2)
LazyLoad:	yes

(U, D) representation of generalized ellipsoids in R^d.

(U is square orthogonal and D diagonal with extended non-negative real numbers, i.e. 0, Inf or a
positive real).

The resulting class of ellipsoids includes degenerate ellipsoids that are flat and/or unbounded. Thus
ellipsoids are naturally defined to include lines, hyperplanes, points, cylinders, etc.

The class is closed under linear and affine transformations (including those between spaces of different
dimensions) and under duality ('inverse') transformations.

Unbounded ellipsoids, e.g. cylinders with elliptical cross-sections, correspond to singular inner
products, i.e. inner products defined by a singular inner product matrix.

Flat ellipsoids correspond to singular variances. The corresponding inner product is defined only
on the supporting subspace.

Ellipsoids that are both flat and unbounded correspond to lines, points, subspaces, hyperplanes, etc.

gell can currently generate the U-D representation from 5 ways of specifying an ellipsoid:

1. From the non-negative definite dispersion (variance) matrix, Sigma: $UD^2U' = \Sigma$, where
   some elements of the diagonal matrix D can be 0. This can only generate bounded ellipsoids,
   possibly flat.

2. From the non-negative definite inner product matrix 'ip': $UW^2U = C$ where some elements of the diagonal matrix W can be 0. Then set D = W^-1 where 0^-1 = Inf. This can only generate fat (non-empty interior) ellipsoids, possibly unbounded.

3. From a subspace spanned by 'span' Let U_1 be an orthonormal basis of Span('span'), let U_2 be an orthonormal basis of the orthogonal complement, the U = [U_1 U_2] and D = diag(c(Inf,...,Inf, 0,..,0)) where the number of Inf's is equal to the number of columns of U_1.

4. From a transformation of the unit sphere given by A(Unit sphere) where $A = UDV'$, i.e. the SVD.

5. (Generalization of 4): (A, d) where A is any matrix and d is a vector of factors corresponding to columns of A. These factors can be 0, positive or Inf. In this case U and D are such that U D(Unit sphere) = A diag(d)(Unit sphere). This is the only representation that can be used for all forms of ellipsoids and in which any ellipsoid can be represented.

Author(s)

Georges Monette and Michael Friendly

Maintainer: Michael Friendly <friendly@yorku.ca>

References

Friendly, M., Monette, G. and Fox, J. (2013). Elliptical Insights: Understanding Statistical Methods through Elliptical Geometry. *Statistical Science*, **28**(1), 1-39.

See Also

None yet

Examples

```
(zsph <- gell(Sigma = diag(3)))  # unit sphere in R^3

(zplane <- gell(span = diag(3)[, 1:2]))  # a plane

dual(zplane)  # line orthogonal to that plane

(zhplane <- gell(center = c(0, 0, 2), span = diag(3)[, 1:2]))  # a hyperplane

dual(zhplane)  # orthogonal line through same center (note that the 'gell'
# object with a center contains more information than the geometric plane)

zorigin <- gell(span = cbind(c(0, 0, 0)))
dual(zorigin)

# signatures of these ellipsoids
signature(zsph)
signature(zhplane)
signature(dual(zhplane))
```

---

basis3d   *Draw Basis Vectors in an rgl Scene*

Description

The function calculates and optionally draws basis vectors to be used as coordinate axes in an rgl
3D display. For geometric diagrams and some data displays, this can be more useful than using
[axis3d](#) or [box3d](#) to provide guides or axes. The `origin` argument permits multiple diagrams or
graphs in the same display, each with their own coordinate axes.

Usage

```
basis3d(
  matrix = diag(3),
  origin = c(0, 0, 0),
  scale = 1,
  draw = TRUE,
  label = draw,
  texts = colnames(basis),
  lwd = 3,
  cex = 2,
  ...
)
```

Arguments

matrix	A 3 x 3 matrix, whose *columns* are the basis vectors, typically of unit length.
origin	Origin in the rgl scene of the basis vectors
scale	Scaling factor for the basis vectors
draw	If `TRUE`, the vectors are plotted
label	If `TRUE`, the vectors are labeled at their tips
texts	A character vector of length 3 used for labels. Defaults to `c("x", "y", "z")`.
lwd	Line width for vectors
cex	Character size for labels
...	Other arguments, passed to [segments3d](#) and [text3d](#)

Value

Returns invisibly a 6 x 3 matrix, whose columns are the coordinate axes, `c("x", "y", "z")`. Odd
numbered rows give the coordinates of the `origin`, while even numbered rows give the locations of
the tips of the vectors.

Author(s)

Michael Friendly

See Also

axis3d, box3d

Examples

```
# show two different sets of basis vectors
open3d()
basis3d()
A <- matrix(c(1,2,0.1,
              2,1,0.1,
              0.1,0.1,0.5), 3,3)
basis3d(t(A), col="red")

# scene with two different displays
open3d()
basis3d()
basis3d(origin=c(2,0,0))
```

---

bbox3d   *Find the bounding box of a* rgl::mesh3d *or* rgl::qmesh3d *object*

Description

Ellipsoids are created by **rgl** functions as meshes of points, segments, ... from coordinates in various forms. This function calculates the bounding box, defined as the range of the x, y, and z coordinates.

Usage

```
bbox3d(x, ...)
```

Arguments

x   A mesh3d object

...   ignored

Value

A 2 x 3 matrix, giving the minimum and maximum values in the rows and x, y, z coordinates in the columns.

---

dual | *Dual or 'Inverse' of an ellipsoid*

Description

`dual` produces the orthogonal complement for subspaces or for ellipsoids. This is equivalent to inverting Σ or an inner product `ip` when these are non-singular.

Usage

```
dual(x, ...)

## S3 method for class 'gell'
dual(x, ...)
```

Arguments

x	An object, of class `"gell"`
...	Other arguments, unused for now.

Details

At present, `dual` is only defined for objects of class `"gell"`.

In the (U,D) representation, the dual simply has the columns of U in the reverse order, and the reciprocals of the diagonal elements of D, also in reverse order.

Value

A (U, D) representation of the dual, with components LIST, use

u	Right singular vectors
d	Singular values

Author(s)

Georges Monette

References

Dempster, A. (1969). *Elements of Continuous Multivariate Analysis* Reading, MA: Addison-Wesley.

See Also

[gell](#)

Examples

```
(zplane <- gell(span = diag(3)[,1:2]))  # a plane

dual(zplane)  # line orthogonal to that plane

(zhplane <- gell(center = c(0,0,2), span = diag(3)[,1:2]))  # a hyperplane

dual(zhplane) # orthogonal line through same center (note that the 'gell'
              # object with a center contains more information than the geometric plane)

zorigin <- gell(span = cbind(c(0,0,0)))
dual( zorigin )
```

---

ell3d   *Plot Generalized Ellipsoids in 3D using rgl*

Description

ell3d is a convenience wrapper for [ellipsoid](#) specialized for class "gell" objects. It plots an ellipsoid in the current rgl window.

Usage

```
ell3d(x, ...)

## Default S3 method:
ell3d(x, shape, radius = 1, segments = 40, shade = TRUE, wire = FALSE, ...)

## S3 method for class 'gell'
ell3d(
  x,
  length = 10,
  sides = 30,
  segments = 40,
  shade = TRUE,
  wire = FALSE,
  ...
)
```

Arguments

x	A vector of length 3 giving the center of the ellipsoid for the default method, or a class "gell" object for the gell method.
...	Other arguments, passed to [ellipsoid](#)
shape	A 3 x 3 symmetric matrix giving the shape of the ellipsoid.

radius	radius of the ellipsoid
segments	number of segments in each direction in calculating the mesh3d object
shade	Logical. Whether the 3D ellipsoid should be shaded
wire	Logical. Whether the 3D ellipsoid should be rendered with its wire frame
length	For unbounded, (infinite) ellipsoids, the length to display for infinite dimensions
sides	For unbounded, cylindrical ellipsoids, the number of sides for the elliptical cross sections

Value

Returns the result of the call to ellipsoid

Note

This implementation is subject to change.

Author(s)

Georges Monette

See Also

ellipsoid, gell

Examples

```
# a proper ellipsoid, and its inverse
c1 <- c(0,0,0)
C1 <- matrix(c(6, 2, 1,
               2, 3, 2,
               1, 2, 2), 3,3)

open3d()
zell1 <- gell(center=c1, Sigma=C1)
E1 <- ell3d(zell1, col="blue", alpha=0.1)
# inverse of C1
E1I <- ell3d(dual(zell1), col="blue", alpha=0.1, wire=TRUE)

#open3d()  ## to see this in a new window
# a singular ellipsoid and its inverse
c2 <- c1
C2 <- matrix(c(6, 2, 0,
               2, 3, 0,
               0, 0, 0), 3,3)

E2 <- ell3d( zell2 <-gell( center = c2, Sigma = C2),
             col = 'red', alpha = 0.25, wire = TRUE)
E2I <- ell3d( dual(gell( center = c2, Sigma = C2)),
             length=3, col = 'red', alpha = 0.25, wire = TRUE )
```

```
# signatures
signature(zell1)
signature(zell2)
```

---

ellipsoid   *Calculate an ellipsoid in 3D*

Description

Calculates a [qmesh3d](#) object representing a 3D ellipsoid with given `center` and `shape` matrix. The function allows for degenerate ellipsoids where the `shape` matrix has rank < 3 and plots as an ellipse or a line.

Usage

```
ellipsoid(center, shape, radius = 1, segments = 60, warn.rank = FALSE)
```

Arguments

`center`	A vector of length 3 giving the center of the 3D ellipsoid, typically the mean vector of a data matrix.
`shape`	A 3 x 3 matrix giving the shape of the 3D ellipsoid, typical a covariance matrix of a data matrix.
`radius`	radius of the ellipsoid, with default `radius=1`, giving a standard ellipsoid. For a multivariate sample with dfe degrees of freedom associated with `shape`, an ellipsoid of `level` coverage can be calculated using `radius=sqrt(3 * qf(level, 3, dfe))`.
`segments`	number of line segments to use in each direction in the wire-frame representation of the ellipsoid
`warn.rank`	warn if the `shape` matrix is of rank < 3?

Details

The ellipsoid is calculated by transforming a unit sphere by the Cholesky square root of the `shape` matrix, and translating to the `center`.

The ellipsoid can be plotted with `plot3d`

Value

A qmesh3d object

Author(s)

Michael Friendly and John Fox, extending Duncan Murdoch

Examples

```
## none yet
```

---

gell   *(U, D) Representation of an Ellipsoid in R^p.*

Description

gell provides a set of ways to specify a generalized ellipsoid in R^p, using the (U, D) representation to include all special cases, where U is a square orthogonal matrix, and D is diagonal with extended non-negative real numbers, i.e. 0, Inf or a positive real.

Usage

```
gell(x, ...)

## Default S3 method:
gell(x, center = 0, Sigma, ip, span, A, u, d = 1, epsfac = 2, ...)

## S3 method for class 'gell'
gell(x, ...)
```

Arguments

x	An object
...	Other arguments
center	A vector specifying the center of the ellipsoid
Sigma	A square, symmetric, non-negative definite dispersion (variance) matrix
ip	A square, symmetric, non-negative definite inner product matrix. See Details.
span	A subspace with a given span. See Details.
A	A matrix giving a linear transformation of the unit sphere.
u	A U matrix
d	Diagonal elements of a D matrix
epsfac	Factor of .Machine$double.eps used to distinguish zero vs. positive singular values

Details

The resulting class of ellipsoids includes degenerate ellipsoids that are flat and/or unbounded. Thus ellipsoids are naturally defined to include lines, hyperplanes, points, cylinders, etc.

`gell` can currently generate the (U, D) representation from 5 ways of specifying an ellipsoid:

1. From the non-negative definite dispersion (variance) matrix, $Sigma : UD^2U' = Sigma$, where some elements of the diagonal matrix D can be 0. This can only generate bounded ellipsoids, possibly flat.

2. From the non-negative definite inner product matrix $'ip' : UW^2U = C$ where some elements of the diagonal matrix W can be 0. Then set D = W^-1 where 0^-1 = Inf. This can only generate fat (non-empty interior) ellipsoids, possibly unbounded.

3. From a subspace spanned by 'span' Let U_1 be an orthonormal basis of Span('span'), let U_2 be an orthonormal basis of the orthogonal complement, the $U = [U_1 U_2]$ and $D = diag(c(Inf, ..., Inf, 0, .., 0))$ where the number of Inf's is equal to the number of columns of U_1.

4. From a transformation of the unit sphere given by A(Unit sphere) where A = UDV', i.e. the SVD.

5. (Generalization of 4): A, d where A is any matrix and d is a vector of factors corresponding to columns of A. These factors can be 0, positive or Inf. In this case U and D are such that U D(Unit sphere) = A diag(d)(Unit sphere). This is the only representation that can be used for all forms of ellipsoids and in which any ellipsoid can be represented.

Value

A (U, D) representation of the ellipsoid, with components

center	center
u	Right singular vectors
d	Singular values

Author(s)

Georges Monette

References

Friendly, M., Monette, G. and Fox, J. (2013). Elliptical Insights: Understanding Statistical Methods through Elliptical Geometry. *Statistical Science*, **28**(1), 1-39.

See Also

dual, gmult, signature,

Examples

```
gell(Sigma = diag(3))    # the unit sphere

(zplane <- gell(span = diag(3)[,1:2]))    # a plane
```

---

gmult   *Linear Transformation of a Generalized Ellipsoid*

Description

Linear transformation of a generalized ellipsoid, including projections to subspaces.

Usage

```
gmult(A, G, epsfac = 2)
```

Arguments

A	A matrix describing a linear transformation, conforming to the U component of G for matrix multiplication.
G	A gell object
epsfac	Factor of .Machine$double.eps used to distinguish zero vs. positive singular values

Details

The matrix A can be non-singular, for a standard linear transformation, or singular, for a projection to a subspace.

Value

gell object

Note

This implementation should be changed so that it provides an S3 method for class "gell" objects for which it was intended.

Author(s)

Georges Monette

See Also

[gell](), [dual](), [signature]()

Examples

```
(zplane <- gell(span = diag(3)[,1:2]))  # a plane

dual(zplane)  # orthogonal line
(zplane2 <- gmult( cbind( c(1,1,1), c(1,-1,0), c(1,0,-1)), zplane))

# correctly wipes out one dimension
(zplane3 <- gmult( cbind( c(1,0,0), c(1,0,0), c(0,0,1)), zplane))
```

---

isBounded   *Tests for Classes of Generalized Ellipsoids*

Description

These functions provide tests for classes of generalized ellipsoids in the (U, D) representation, based on the numbers of positive, zero and infinite singular values in D.

Usage

```
isBounded(x, ...)

## S3 method for class 'gell'
isBounded(x, ...)

isFat(x, ...)

isFlat(x, ...)

isUnbounded(x, ...)
```

Arguments

x   A class "gell" object
...   Other arguments, not used.

Details

They are included here mainly as computational definitions of the terms 'bounded' for an ellipsoid with finite extent, 'fat', for a bounded ellipsoid with non-empty interior, 'flat', for degenerate (singular) ellipsoids in R^p with empty interior.

Value

TRUE or FALSE

Author(s)

Georges Monette

References

Friendly, M., Monette, G. and Fox, J. (2013). Elliptical Insights: Understanding Statistical Methods through Elliptical Geometry. *Statistical Science*, **28**(1), 1-39.

See Also

[signature](#)

Examples

```
# None yet
```

---

par3d.save   *Save and Restore the par3d Parameters for an rgl Scene*

Description

Use this function to make a manually manipulated rgl view reproducible, within sessions or across sessions. Within an R session, simply use par3d.save to record the par3d parameters for the view(s) to named objects. Across sessions, use the filename argument to save these to .rds files

Usage

```
par3d.save(
  params = c("userMatrix", "scale", "zoom", "FOV"),
  filename,
  dev = rgl.cur()
)

par3d.restore(parms, filename)
```

Arguments

params	A list of par3d parameters to save
filename	Name of a .rds file to save to or restore from
dev	The rgl device. Currently unused
parms	A list of par3d parameters to restore

Value

par3d.save returns a list with the current values of the par3d parameters named in params.

Author(s)

Michael Friendly

See Also

[par3d](par3d)

Examples

```
## Not run:
library(rgl)
open3d()
# ...
parms <- par3d.save()
# ....
par3d.restore(parms)


## End(Not run)
```

---

signature   *Signature of a Generalized Ellipsoid*

Description

Calculates the signature of a generalized ellipsoid, a vector of length 3 giving the number of positive, zero and infinite singular values in the (U, D) representation

Usage

```
signature(G)
```

Arguments

G   A class ″gell″ objects

Value

A vector of length 3, with named components pos, zero and inf

Author(s)

Georges Monette

References

Friendly, M., Monette, G. and Fox, J. (2013). Elliptical Insights: Understanding Statistical Methods through Elliptical Geometry. *Statistical Science*, **28**(1), 1-39.

See Also

isBounded

Examples

```
(zsph <- gell(Sigma = diag(3)))  # unit sphere in R^3

(zplane <- gell(span = diag(3)[,1:2]))  # a plane

dual(zplane)  # line orthogonal to that plane

(zhplane <- gell(center = c(0,0,2), span = diag(3)[,1:2]))  # a hyperplane

dual(zhplane) # orthogonal line through same center (note that the 'gell'
              # object with a center contains more information than the geometric plane)

zorigin <- gell(span = cbind(c(0,0,0)))
dual( zorigin )

# signatures of these ellipsoids
signature(zsph)
signature(zhplane)
signature(dual(zhplane))
```

UD   *SVD, modified to return U, and D extended with 0's*

Description

Calculates U and D in the (U, D) representation of a generalized ellipsoid. This uses the SVD, modified to return U, and D extended with 0's for singular matrices.

Usage

```
UD(x)
```

Arguments

x   A matrix

Value

A list with the following components:

u   Right singular vectors

d   Singular values

Author(s)

Georges Monette

See Also

svd

Examples

```
# None yet
```

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