Package 'grpSLOPE'

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Type Package

Title Group Sorted L1 Penalized Estimation

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Description Group SLOPE is a penalized linear regression method that is used for adaptive selection of groups of significant predictors in a high-dimensional linear model.

The Group SLOPE method can control the (group) false discovery rate at a user-specified level (i.e., control the expected proportion of irrelevant among all selected groups of predictors).

License GPL-3

URL https://github.com/agisga/grpSLOPE

BugReports https://github.com/agisga/grpSLOPE/issues

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Author Alexej Gossmann [aut, cre],

Damian Brzyski [aut],

Weijie Su [aut],

Malgorzata Bogdan [aut],

Ewout van den Berg [ctb] (Code adapted from 'SLOPE' version 0.1.3, as well as from

http://statweb.stanford.edu/~candes/SortedL1/software.html under GNU GPL-3),

Emmanuel Candes [ctb] (Code adapted from 'SLOPE' version 0.1.3, as well as from http://statweb.stanford.edu/~candes/SortedL1/software.html under GNU GPL-3),

Chiara Sabatti [ctb] (Code adapted from 'SLOPE' version 0.1.3, as well

as from http://statweb.stanford.edu/~candes/SortedL1/software.html under GNU GPL-3),

Evan Patterson [ctb] (Code adapted from 'SLOPE' version 0.1.3, as well as from http://statweb.stanford.edu/~candes/SortedL1/software.html under GNU GPL-3)

Maintainer Alexej Gossmann <alexej.go@googlemail.com>

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admmSolverGroupSLOPE Alternating direction method of multipliers

Description

Compute the coefficient estimates for the Group SLOPE problem.

Usage

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```
admmSolverGroupSLOPE(
   y,
   A,
   group,
   wt,
   lambda,
   rho = NULL,
   max.iter = 10000,
   verbose = FALSE,
   absolute.tol = 1e-04,
   relative.tol = 1e-04,
   z.init = NULL,
```

```
u.init = NULL, ...
)
```

Arguments

У	the response vector
Α	the model matrix
group	A vector describing the grouping structure. It should contain a group id for each predictor variable.
wt	A vector of weights (per coefficient)
lambda	A decreasing sequence of regularization parameters λ
rho	Penalty parameter in the augmented Lagrangian (see Boyd et al., 2011)
max.iter	Maximal number of iterations to carry out
verbose	A logical specifying whether to print output or not
absolute.tol	The absolute tolerance used in the stopping criteria for the primal and dual feasibility conditions (see Boyd et al., 2011, Sec. 3.3.1)
relative.tol	The relative tolerance used in the stopping criteria for the primal and dual feasibility conditions (see Boyd et al., 2011, Sec. 3.3.1)
z.init	An optional initial value for the iterative algorithm
u.init	An optional initial value for the iterative algorithm
	Options passed to prox_sorted_L1

Details

admmSolverGroupSLOPE computes the coefficient estimates for the Group SLOPE model. The employed optimization algorithm is the alternating direction method of multipliers (ADMM).

Value

A list with the entries:

```
    x Solution (n-by-1 matrix)
    status Convergence status: 1 if optimal, 2 if iteration limit reached
    iter Number of iterations of the ADMM method
```

References

S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein (2011) *Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers*. Foundations and Trends in Machine Learning 3 (1).

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Examples

```
set.seed(1)
A <- matrix(runif(100, 0, 1), 10, 10)
grp <- c(0, 0, 1, 1, 2, 2, 2, 2, 2, 3)
wt <-c(2, 2, 2, 2, 5, 5, 5, 5, 5, 1)
   <- c(0, 0, 5, 1, 0, 0, 0, 1, 0, 3)
   <- A %*% x
lam <- 0.1 * (10:7)
result <- admmSolverGroupSLOPE(y = y, A = A, group = grp, wt = wt,
                              lambda=lam, rho = 1, verbose = FALSE)
result$x
            [,1]
# [1,] 0.000000
  [2,] 0.000000
  [3,] 3.856002
  [4,] 2.080742
  [5,] 0.000000
  [6,] 0.000000
  [7,] 0.000000
# [8,] 0.000000
# [9,] 0.000000
# [10,] 3.512829
```

coef.grpSLOPE

Extract model coefficients

Description

Extract the regression coefficients from a grpSLOPE object, either on the scale of the normalized design matrix (i.e., columns centered and scaled to unit norm), or on the original scale.

Usage

```
## S3 method for class 'grpSLOPE'
coef(object, scaled = TRUE, ...)
```

Arguments

object	A grpSL0PE object
scaled	Should the coefficients be returned for the normalized version of the design matrix?
	Potentially further arguments passed to and from methods

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Details

If scaled is set to TRUE, then the coefficients are returned for the normalized version of the design matrix, which is the scale on which they were computed. If scaled is set to FALSE, then the coefficients are transformed to correspond to the original (unaltered) design matrix. In case that scaled = FALSE, an estimate for the intercept term is returned with the other coefficients. In case that scaled = TRUE, the estimate of the intercept is always equal to zero, and is not explicitly provided.

Value

A named vector of regression coefficients where the names signify the group that each entry belongs to

Examples

```
set.seed(1)
  <- matrix(rnorm(100^2), 100, 100)
grp <- rep(rep(letters[1:20]), each=5)</pre>
   <- c(rep(1, 20), rep(0, 80))
   <- A %*% b + rnorm(10)
result <- grpSLOPE(X=A, y=y, group=grp, fdr=0.1)
head(coef(result), 8)
                                     a_4
       a_1
                 a_2
                           a_3
                                               a_5
                                                        b_1
                                                                  b_2
 7.942177 7.979269 8.667013 8.514861 10.026664 8.963364 10.037355 10.448692
head(coef(result, scaled = FALSE), 8)
# (Intercept) a_1
                              a_2
                                         a_3
                                                   a_4
                                                              a_5
                                                                        b_1
                                                                                   b_2
# -0.4418113 0.8886878 0.8372108 0.8422089 0.8629597 0.8615827 0.9323849 0.9333445
```

getGroupID

Get a groupID object

Description

Mostly intended for internal use.

Usage

```
getGroupID(group)
```

Arguments

group

A vector describing the grouping structure. It should contain a group id for each predictor variable.

Value

An object of class groupID, which is a list, whose members are vectors of indices corresponding to each group. The names of the list members are the corresponding group names.

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Examples

```
group <- c("A", "A", 2, 9, "A", 9, 9, 2, "A")
group.id <- getGroupID(group)
group.id
# $A
# [1] 1 2 5 9
#
# $`2`
# [1] 3 8
#
# $`9`
# [1] 4 6 7
#
# attr(,"class")
# [1] "groupID"</pre>
```

grpSLOPE

Group SLOPE (Group Sorted L-One Penalized Estimation)

Description

Performs selection of significant groups of predictors and estimation of the corresponding coefficients using the Group SLOPE method (see Brzyski et. al., 2016).

Usage

```
grpSLOPE(
   X,
   y,
   group,
   fdr,
   lambda = "corrected",
   sigma = NULL,
   verbose = FALSE,
   orthogonalize = NULL,
   normalize = TRUE,
   max.iter = 10000,
   dual.gap.tol = 1e-06,
   infeas.tol = 1e-06,
   x.init = NULL,
   ...
)
```

Arguments

X The model matrix y The response variable grpSLOPE 7

group A vector describing the grouping structure. It should contain a group id for each

predictor variable.

fdr Target group false discovery rate (gFDR)

lambda Method used to obtain the regularizing sequence lambda. Possible values are

"max", "mean", and "corrected" (default). See lambdaGroupSLOPE for detail. Alternatively, any non-increasing sequence of the correct length can be passed.

sigma Noise level. If ommited, estimated from the data, using Procedure 2 in Brzyski

et. al. (2016).

verbose A logical specifying whether to print output or not

orthogonalize Whether to orthogonalize the model matrix within each group. Do not set man-

ually unless you are certain that your data is appropriately pre-processed.

normalize Whether to center the input data and re-scale the columns of the design matrix

to have unit norms. Do not disable this unless you are certain that your data are

appropriately pre-processed.

max.iter See proximalGradientSolverGroupSLOPE.
dual.gap.tol See proximalGradientSolverGroupSLOPE.
infeas.tol See proximalGradientSolverGroupSLOPE.
x.init See proximalGradientSolverGroupSLOPE.

... Options passed to prox_sorted_L1

Details

Multiple methods are available to generate the regularizing sequence lambda, see lambdaGroupSLOPE for detail. The model matrix is transformed by orthogonalization within each group (see Section 2.1 in Brzyski et. al., 2016), and penalization is imposed on $\|X_{I_i}\beta_{I_i}\|$. When orthogonalize = TRUE, due to within group orthogonalization, the solution vector beta cannot be computed, if a group submatrix does not have full column rank (e.g., if there are more predictors in a selected group than there are observations). In that case only the solution vector c of the transformed (orthogonalized) model is returned. Additionally, in any case the vector group. norms is returned with its ith entry being $\|X_{I_i}\beta_{I_i}\|$, i.e., the overall effect of each group. Note that all of these results are returned on the scale of the normalized versions of X and y. However, original.scale contains the regression coefficients transformed to correspond to the original (unaltered) X and y. In that case, an estimate for the intercept term is also returned with the other coefficients in original.scale (while on the normalized scale the estimate of the intercept is always equal to zero, and is not explicitly provided in the grpSLOPE output).

Value

A list with members:

beta Solution vector. See Details.

c Solution vector of the transformed model. See Details.

group.norms Overall effect of each group. See Details.

selected Names of selected groups (i.e., groups of predictors with at least one non-zero coefficient estimate)

```
optimal Convergence status
```

iter Iterations of the proximal gradient method

lambda Regularizing sequence

lambda.method Method used to construct the regularizing sequence

sigma (Estimated) noise level

group The provided grouping structure (corresponding to beta)

group.c Grouping structure of the transformed model (corresponding to c)

original.scale A list containing the estimated intercept and regression coefficients on the original scale. See Details.

References

- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2016) *Group SLOPE adaptive selection of groups of predictors*, https://arxiv.org/abs/1610.04960
- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2019) *Group SLOPE adaptive selection of groups of predictors*. Journal of the American Statistical Association 114 (525): 419–33.

Examples

```
# generate some data
set.seed(1)
A <- matrix(rnorm(100^2), 100, 100)
grp \leftarrow rep(rep(1:20), each=5)
   <- c(runif(20), rep(0, 80))
# (i.e., groups 1, 2, 3, 4, are truly significant)
   <- A %*% b + rnorm(10)
fdr <- 0.1 # target false discovery rate
# fit a Group SLOPE model
result <- grpSLOPE(X=A, y=y, group=grp, fdr=fdr)</pre>
result$selected
# [1] "1" "2" "3" "4" "14"
result$sigma
# [1] 0.7968632
head(result$group.norms)
                             3
                                                            6
       1 2
# 2.905449 5.516103 8.964201 10.253792 0.000000 0.000000
```

lambdaGroupSLOPE

Regularizing sequence for Group SLOPE

Description

Generate the regularizing sequence lambda for the Group SLOPE problem according to one of multiple methods (see Details).

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Usage

```
lambdaGroupSLOPE(method, fdr, group, wt, n.obs = NULL)
```

Arguments

method	Possible values are "max", "mean", and "corrected". See under Details.
fdr	Target group false discovery rate (gFDR)
group	A vector describing the grouping structure. It should contain a group id for each predictor variable.
wt	A named vector of weights, one weight per group of predictors (named according to names as in vector group)
n.obs	Number of observations (i.e., number of rows in A); required only if method is "corrected"

Details

Multiple methods are available to generate the regularizing sequence lambda:

- "max" lambdas as in Theorem 2.5 in Brzyski et. al. (2016). Provalby controls gFDR in orthogonal designs.
- "mean" lambdas of equation (2.16) in Brzyski et. al. (2016). Applicable for gFDR control in orthogonal designs. Less conservative than "max".
- "corrected" lambdas of Procedure 1 in Brzyski et. al. (2016); in the special case that all
 group sizes are equal and wt is a constant vector, Procedure 6 of Brzyski et. al. (2016) is
 applied. Applicable for gFDR control when predictors from different groups are stochastically
 independent.

References

- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2016) *Group SLOPE adaptive selection of groups of predictors*, https://arxiv.org/abs/1610.04960
- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2019) *Group SLOPE adaptive selection of groups of predictors*. Journal of the American Statistical Association 114 (525): 419–33.

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```
# [,1] [,2] [,3] [,4] [,5] [,6]
# lambda.max 2.023449 1.844234 1.730818 1.645615 1.576359 1.517427
# lambda.mean 1.880540 1.723559 1.626517 1.554561 1.496603 1.447609
# lambda.corrected 1.880540 1.729811 1.637290 1.568971 1.514028 1.467551
```

Description

Obtain predictions from a grpSLOPE model on new data

Usage

```
## S3 method for class 'grpSLOPE'
predict(object, newdata, ...)
```

Arguments

object A grpSLOPE object

newdata Predictor variables arranged in a matrix

... Potentially further arguments passed to and from methods

Details

Note that newdata must have the same shape, and must contain the same predictor variables as columns in the same order as the design matrix X that was used for the grpSLOPE model fit.

```
set.seed(1)
A <- matrix(rnorm(100^2), 100, 100)
grp <- rep(rep(1:20), each = 5)
b <- c(rep(1, 20), rep(0, 80))
y <- A %*% b + rnorm(10)
result <- grpSLOPE(X = A, y = y, group = grp, fdr = 0.1)
newdata <- matrix(rnorm(800), 8, 100)
# group SLOPE predictions:
predict(result, newdata)
# [1] -5.283385 -6.313938 -3.173068  1.901488  9.796677 -0.144516 -0.611164 -5.167620
# true mean values:
as.vector(newdata %*% b)
# [1] -5.0937160 -6.5814111 -3.5776124  2.7877449 11.0668777  1.0253236 -0.4261076 -4.8622940</pre>
```

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proxGroupSortedL1	Prox for group SLOPE
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Description

Evaluate the proximal mapping for the group SLOPE problem.

Usage

```
proxGroupSortedL1(y, group, lambda, ...)
```

Arguments

у	The response vector
group	Either a vector or an object of class groupID (e.g., as produced by getGroupID), which is describing the grouping structure. If it is a vector, then it should contain a group id for each predictor variable.
lambda	A decreasing sequence of regularization parameters λ
	Options passed to prox_sorted_L1

Details

proxGroupSortedL1 evaluates the proximal mapping of the group SLOPE problem by reducing it to the prox for the (regular) SLOPE and then applying the fast prox algorithm for the Sorted L1 norm.

References

```
M. Bogdan, E. van den Berg, C. Sabatti, W. Su, E. Candes (2015), SLOPE – Adaptive variable selection via convex optimization, https://arxiv.org/abs/1407.3824
```

```
grp <- c(0,0,0,1,1,0,2,1,0,2)
proxGroupSortedL1(y = 1:10, group = grp, lambda = c(10, 9, 8))
# [1] 0.2032270 0.4064540 0.6096810 0.8771198 1.0963997 1.2193620 1.3338960
# [8] 1.7542395 1.8290430 1.9055657</pre>
```

each

 $\verb"proximalGradientSolverGroupSLOPE"$

Proximal gradient method for Group SLOPE

Description

Compute the coefficient estimates for the Group SLOPE problem.

Usage

```
proximalGradientSolverGroupSLOPE(
   y,
   A,
   group,
   wt,
   lambda,
   max.iter = 10000,
   verbose = FALSE,
   dual.gap.tol = 1e-06,
   infeas.tol = 1e-06,
   x.init = NULL,
   ...
)
```

Arguments

the response vector
the model matrix
A vector describing the grouping structure. It should contain a group id for predictor variable.
A vector of weights (per coefficient)
A decreasing sequence of regularization parameters λ
Maximal number of iterations to carry out
A logical specifying whether to print output or not
The tolerance used in the stopping criteria for the duality gap
The tolerance used in the stopping criteria for the infeasibility
An optional initial value for the iterative algorithm
Options passed to prox_sorted_L1

Details

proximalGradientSolverGroupSLOPE computes the coefficient estimates for the Group SLOPE model. The employed optimization algorithm is FISTA with backtracking Lipschitz search.

Value

A list with the entries:

```
x Solution (n-by-1 matrix)
```

status Convergence status: 1 if optimal, 2 if iteration limit reached

L Approximation of the Lipschitz constant (step size)

iter Iterations of the proximal gradient method

L.iter Total number of iterations spent in Lipschitz search

References

- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2016) *Group SLOPE adaptive selection of groups of predictors*, https://arxiv.org/abs/1610.04960
- D. Brzyski, A. Gossmann, W. Su, and M. Bogdan (2019) *Group SLOPE adaptive selection of groups of predictors*. Journal of the American Statistical Association 114 (525): 419–33. doi:10.1080/01621459.2017.1411269
- A. Gossmann, S. Cao, Y.-P. Wang (2015) *Identification of Significant Genetic Variants via SLOPE,* and Its Extension to Group SLOPE. In Proceedings of ACM BCB 2015. doi:10.1145/2808719.2808743

```
set.seed(1)
A <- matrix(runif(100, 0, 1), 10, 10)
grp \leftarrow c(0, 0, 1, 1, 2, 2, 2, 2, 2, 3)
wt <-c(2, 2, 2, 2, 5, 5, 5, 5, 5, 1)
   <- c(0, 0, 5, 1, 0, 0, 0, 1, 0, 3)
   <- A %*% x
lam <- 0.1 * (10:7)
result <- proximalGradientSolverGroupSLOPE(y=y, A=A, group=grp, wt=wt, lambda=lam, verbose=FALSE)
result$x
            [,1]
# [1,] 0.000000
# [2,] 0.000000
# [3,] 3.856005
# [4,] 2.080736
# [5,] 0.000000
# [6,] 0.000000
# [7,] 0.000000
# [8,] 0.000000
# [9,] 0.000000
# [10,] 3.512833
```

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prox_sorted_L1

Prox for sorted L1 norm

Description

Compute the prox for the sorted L1 norm. That is, given a vector x and a decreasing vector λ , compute the unique value of y minimizing

$$\frac{1}{2}||x-y||_2^2 + \sum_{i=1}^n \lambda_i |x|_{(i)}.$$

At present, two methods for computing the sorted L1 prox are supported. By default, we use a fast custom C implementation. Since SLOPE can be viewed as an isotonic regression problem, the prox can also be computed using the isotone package. This option is provided primarily for testing.

Usage

```
prox_sorted_L1(x, lambda, method = c("c", "isotone"))
```

Arguments

x input vector

lambda vector of λ 's, sorted in decreasing order

method underlying prox implementation, either 'c' or 'isotone' (see Details)

Details

This function has been adapted (with only cosmetic changes) from the R package SLOPE version 0.1.3, due to this function being deprecated and defunct in SLOPE versions which are newer than 0.1.3.

sigma

Extract (estimated) noise level

Description

Extract the noise level of the grpSLOPE model.

Usage

```
## S3 method for class 'grpSLOPE'
sigma(object, ...)
```

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Arguments

object A grpSLOPE object
... Potentially further arguments passed to and from methods

Details

This basically obtains object\$sigma. For R (>= 3.3.0) sigma is an S3 method with the default method coming from the stats package.

Examples

```
set.seed(1)
A <- matrix(rnorm(100^2), 100, 100)
grp <- rep(rep(1:20), each = 5)
b <- c(rep(1, 20), rep(0, 80))
y <- A %*% b + rnorm(10)
# model with unknown noise level
result <- grpSLOPE(X = A, y = y, group = grp, fdr = 0.1)
sigma(result)
# [1] 0.6505558
# model with known noise level
result <- grpSLOPE(X = A, y = y, group = grp, fdr = 0.1, sigma = 1)
sigma(result)
# [1] 1</pre>
```

SLOPE_solver

Sorted L1 solver

Description

Solves the sorted L1 penalized regression problem: given a matrix A, a vector b, and a decreasing vector λ , find the vector x minimizing

$$\frac{1}{2}||Ax - b||_2^2 + \sum_{i=1}^p \lambda_i |x|_{(i)}.$$

This optimization problem is convex and is solved using an accelerated proximal gradient descent method.

Usage

```
SLOPE_solver(
   A,
   b,
   lambda,
   initial = NULL,
   prox = prox_sorted_L1,
```

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```
max_iter = 10000,
  grad_iter = 20,
  opt_iter = 1,
  tol_infeas = 1e-06,
  tol_rel_gap = 1e-06)
```

Arguments

A an n-by-p matrix b vector of length n

lambda p, sorted in decreasing order

initial initial guess for x

prox function that computes the sorted L1 prox

max_i ter maximum number of iterations in the gradient descent

grad_iter number of iterations between gradient updates
opt_iter number of iterations between checks for optimality

tol_infeas tolerance for infeasibility

tol_rel_gap tolerance for relative gap between primal and dual problems

Details

This function has been adapted (with only cosmetic changes) from the R package SLOPE version 0.1.3, due to this function being deprecated and defunct in SLOPE versions which are newer than 0.1.3.

Value

An object of class SLOPE_solver.result. This object is a list containing at least the following components:

x solution vector x

optimal logical: whether the solution is optimal

iter number of iterations

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