

# Package ‘hdiVAR’

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**Type** Package

**Title** Statistical Inference for Noisy Vector Autoregression

**Version** 1.0.1

**Description** The model is high-dimensional vector autoregression with measurement error, also known as linear gaussian state-space model. Provable sparse expectation-maximization algorithm is provided for the estimation of transition matrix and noise variances. Global and simultaneous testings are implemented for transition matrix with false discovery rate control. For more information, see the accompanying paper: Lyu, X., Kang, J., & Li, L. (2020). "Statistical inference for high-dimensional vector autoregression with measurement error", arXiv preprint <[arXiv:2009.08011](https://arxiv.org/abs/2009.08011)>.

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**License** GPL (>= 2)

**Depends** R (>= 3.1)

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**LazyData** true

**RoxygenNote** 7.1.1

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

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CV_VARMLE	<i>cross-validation for transition matrix update in maximization step</i>
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### Description

Tune the tolerance parameter of generalized Dantzig selector and hard thresholding level via prediction error in test data.

### Usage

```
CV_VARMLE(tol_seq, ht_seq, S0_train, S1_train, Y_test, is_echo = FALSE)
```

### Arguments

tol_seq	vector; grid of tolerance parameter in Dantzig selector for cross-validation.
ht_seq	vector; grid of hard-thresholding levels for transition matrix estimate. To avoid hard thresholding, set ht_seq=0.
S0_train	a p by p matrix; average (over time points in training data) of conditional expectation of $x_t x_t^\top$ on $y_1, \dots, y_T$ and parameter estimates, obtained from expectation step.
S1_train	a p by p matrix; average (over time points in training data) of conditional expectation of $x_t x_{t+1}^\top$ on $y_1, \dots, y_T$ and parameter estimates, obtained from expectation step.
Y_test	a p by T_test matrix; observations of time series in test set.
is_echo	logical; if true, display the information of CV-optimal (tol, ht).

### Value

a list of CV-optimal parameters and test prediction error.

tol_min	CV-optimal tolerance parameter in Dantzig selector.
ht_min	CV-optimal hard thresholding level for the output of Dantzig selector.
test_loss	a matrix of prediction error in test data; columns match tol_seq, and rows match ht_seq.

### Author(s)

Xiang Lyu, Jian Kang, Lexin Li

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Estep	<i>expectation step in sparse expectation-maximization algorithm</i>
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**Description**

Compute conditional expectation and covariance of  $x_t$  given  $y_1, \dots, y_T$  and current parameter estimates of  $A, \sigma_\eta, \sigma_\epsilon$  via kalman filter and smoothing.

**Usage**

```
Estep(Y,A_init,sig_eta_init,sig_epsilon_init,X_init,P_init)
```

**Arguments**

Y	observations of time series, a p by T matrix.
A_init	current estimate of transition matrix $A$ .
sig_eta_init	current estimate of $\sigma_\eta$ .
sig_epsilon_init	current estimate $\sigma_\epsilon$ .
X_init	current estimate of latent $x_1$ at the first iteration, a p-dimensional vector.
P_init	current covariance estimate of latent $x_1$ at the first iteration, a p by p matrix.

**Value**

a list of conditional expectations and covariances for the sequential Maximization step.

EXtT	a p by T matrix of column $E[x_t y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ .
EXtt	a p by p by T tensor of first-two-mode slice $E[x_t x_t^\top   y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ .
EXtt1	a p by p by T-1 matrix of first-two-mode slice $E[x_t x_{t+1}^\top   y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ .

**Author(s)**

Xiang Lyu, Jian Kang, Lexin Li

**Examples**

```
p= 2; Ti=10 # dimension and time
A=diag(1,p) # transition matrix
sig_eta=sig_epsilon=0.2 # error std
Y=array(0,dim=c(p,Ti)) #observation t=1, ..., Ti
X=array(0,dim=c(p,Ti)) #latent t=1, ..., T
Ti_burnin=100 # time for burn-in to stationarity
for (t in 1:(Ti+Ti_burnin)) {
  if (t==1){
    x1=rnorm(p)
```

```

} else if (t<=Ti_burnin) { # burn in
  x1=A%%x1+rnorm(p,mean=0,sd=sig_eta)
} else if (t==(Ti_burnin+1)){ # time series used for learning
  X[,t-Ti_burnin]=x1
  Y[,t-Ti_burnin]=X[,t-Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
} else {
  X[,t- Ti_burnin]=A%%X[,t-1- Ti_burnin]+rnorm(p,mean=0,sd=sig_eta)
  Y[,t- Ti_burnin]=X[,t- Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
}
}
}

Efit=Estep(Y,A,sig_eta,sig_epsilon,x1,diag(1,p))

```

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hdVARtest

*statistical inference for transition matrix in high-dimensional vector autoregression with measurement error*


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### Description

Conduct global and simultaneous testing on the transition matrix.

### Usage

```

hdVARtest(
  Y,
  A_est,
  sig2_eta,
  sig2_epsilon,
  global_H0 = NULL,
  global_idx = NULL,
  simul_H0 = NULL,
  simul_idx = NULL,
  FDR_levels = 0.05,
  grid_num = 2000
)

```

### Arguments

Y	observations of time series, a p by T matrix.
A_est	a p by p matrix of transition matrix <i>A</i> estimate.
sig2_eta	scalar; estimate of propagation error variance $\sigma_\eta^2$ .
sig2_epsilon	scalar; estimate of measurement error variance $\sigma_\epsilon^2$ .
global_H0	a p by p matrix of global null hypothesis for transition matrix <i>A</i> . If global_H0=NULL, global testing will not be conducted, and global_idx will not be used.

global_idx	a p by p boolean matrix. The TRUE/nonzero entry indicates the entry of interest in global hypothesis testing. If global_idx=NULL, all p*p entries are included in global testing.
simul_H0	a p by p matrix of simultaneous null hypothesis for transition matrix A. If simul_H0=NULL, simultaneous testing will not be conducted, and (simul_idx, FDR_levels, grid_num) will not be used.
simul_idx	a p by p boolean matrix. The TRUE/nonzero entry indicates the entry of interest in simultaneous hypothesis testing. If simul_idx=NULL, all p*p entries are included in simultaneous testing.
FDR_levels	a vector of FDR control levels
grid_num	scalar; the number of grids for cutoff search in FDR control.

**Value**

a list of testing results and gaussian test statistic matrices.

pvalue	scalar; p-value of global testing. Exist if global_H0 is not NULL.
global_test_stat	a p by p matrix of gaussian test statistic for global null hypothesis. Exist if global_H0 is not NULL.
simul_test_stat	a p by p matrix of gaussian test statistic for simultaneous null hypothesis. Exist if simul_H0 is not NULL.
FDR_levels	a vector of FDR control levels. The same as input argument FDR_levels.
crt	a vector of critical values for rejecting entries in simultaneous hypothesis under corresponding FDR control levels.
selected	a three-way tensor. The first two modes are p by p, and the third mode is for FDR control levels. Nonzero entries indicate the entries of interest.

**Author(s)**

Xiang Lyu, Jian Kang, Lexin Li

**Examples**

```

p= 3; Ti=200 # dimension and time
A=diag(1,p) # transition matrix
sig_eta=sig_epsilon=0.2 # error std
Y=array(0,dim=c(p,Ti)) #observation t=1, ..., Ti
X=array(0,dim=c(p,Ti)) #latent t=1, ..., T
Ti_burnin=300 # time for burn-in to stationarity
for (t in 1:(Ti+Ti_burnin)) {
  if (t==1){
    x1=rnorm(p)
  } else if (t<=Ti_burnin) { # burn in
    x1=A%%x1+rnorm(p,mean=0,sd=sig_eta)
  } else if (t==(Ti_burnin+1)){ # time series used for learning
    X[,t-Ti_burnin]=x1
    Y[,t-Ti_burnin]=X[,t-Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  } else {
    X[,t- Ti_burnin]=A%%X[,t-1- Ti_burnin]+rnorm(p,mean=0,sd=sig_eta)
    Y[,t- Ti_burnin]=X[,t- Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  }
}

```

```
# null hypotheses are true
hdVARtest(Y,A,sig_eta^2,sig_epsilon^2,global_H0=A,global_idx=NULL,
          simul_H0=A,simul_idx=NULL,FDR_levels=c(0.05,0.1))

# null hypotheses are false
hdVARtest(Y,A,sig_eta^2,sig_epsilon^2,global_H0=matrix(0,p,p),global_idx=NULL,
          simul_H0=matrix(0,p,p),simul_idx=NULL,FDR_levels=c(0.05,0.1))
```

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kalman	<i>kalman filtering and smoothing for vector autoregression with measurement error</i>
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### Description

kalman filtering and smoothing for vector autoregression with measurement error

### Usage

```
kalman(Y,A,sig_eta,sig_epsilon,X_init=NULL,P_init=NULL)
```

### Arguments

Y	observations of time series, a p by T matrix.
A	current estimate of transition matrix.
sig_eta	current estimate of $\sigma_\eta$ .
sig_epsilon	current estimate $\sigma_\epsilon$ .
X_init	initial estimate of latent $x_1$ at the first iteration, a p-dimensional vector.
P_init	initial covariance estimate of latent $x_1$ at the first iteration, a p by p matrix.

### Value

a list of conditional expectations and covariances of  $x_t$ 's.

### Author(s)

Xiang Lyu, Jian Kang, Lexin Li

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Mstep	<i>maximization step of sparse expectation-maximization algorithm for updating error standard deviations</i>
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**Description**

Update  $\sigma_\eta, \sigma_\epsilon$  based on estimate of  $A$  and conditional expectation and covariance from expectation step.

**Usage**

```
Mstep(Y,A,EXtT,EXtt,EXtt1,is_MLE=FALSE)
```

**Arguments**

Y	observations of time series, a p by T matrix.
A	current estimate of transition matrix $A$ . If <code>is_MLE=TRUE</code> , use naive MLE of transition matrix, by conditional expectation and covariance from expectation step, to update error standard deviations.
EXtT	a p by T matrix of column $E[x_t y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ from expectation step.
EXtt	a p by p by T tensor of first-two-mode slice $E[x_t x_t^\top   y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ from expectation step.
EXtt1	a p by p by T-1 matrix of first-two-mode slice $E[x_t x_{t+1}^\top   y_1, \dots, y_T, \hat{A}, \hat{\sigma}_\eta, \hat{\sigma}_\epsilon]$ from expectation step.
is_MLE	logic; if true, use naive MLE of transition matrix, by conditional expectation and covariance from expectation step, to update error variances. Otherwise, use input argument A.

**Value**

a list of estimates of error standard deviations.

sig_eta	estimate of $\sigma_\eta$ .
sig_epsilon	estimate of $\sigma_\epsilon$ .
A	naive MLE of transition matrix $A$ by conditional expectation and covariance from expectation step. Exist if is_MLE=TRUE.

**Author(s)**

Xiang Lyu, Jian Kang, Lexin Li

**Examples**

```
p= 2; Ti=10 # dimension and time
A=diag(1,p) # transition matrix
sig_eta=sig_epsilon=0.2 # error std
```

```

Y=array(0,dim=c(p,Ti)) #observation t=1, ..., Ti
X=array(0,dim=c(p,Ti)) #latent t=1, ..., T
Ti_burnin=100 # time for burn-in to stationarity
for (t in 1:(Ti+Ti_burnin)) {
  if (t==1){
    x1=rnorm(p)
  } else if (t<=Ti_burnin) { # burn in
    x1=A%%x1+rnorm(p,mean=0,sd=sig_eta)
  } else if (t==(Ti_burnin+1)){ # time series used for learning
    X[,t-Ti_burnin]=x1
    Y[,t-Ti_burnin]=X[,t-Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  } else {
    X[,t- Ti_burnin]=A%%X[,t-1- Ti_burnin]+rnorm(p,mean=0,sd=sig_eta)
    Y[,t- Ti_burnin]=X[,t- Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  }
}

# expectation step
Efit=Estep(Y,A,sig_eta,sig_epsilon,x1,diag(1,p))
EXtT=Efit[["EXtT"]]
EXtt=Efit[["EXtt"]]
EXtt1=Efit[["EXtt1"]]
# maximization step for error standard deviations
Mfit=Mstep(Y,A,EXtT,EXtt,EXtt1)

```

sEM

*sparse expectation-maximization algorithm for high-dimensional vector autoregression with measurement error*

## Description

Alternating between expectation step (by kalman filter and smoothing) and maximization step (by generalized Dantzig selector for transtion matrix) to estimate transtion matrix and error variances.

## Usage

```

sEM(
  Y,
  A_init,
  sig2_eta_init,
  sig2_epsilon_init,
  Ti_train,
  Ti_gap,
  tol_seq,
  ht_seq = 0,
  is_cv = TRUE,

```



```

    thres = 0.001,
    count_vanish = 1,
    n_em = NULL,
    is_echo = FALSE,
    is_sparse = TRUE
)

```

### Arguments

<code>Y</code>	observations of time series, a $p$ by $T$ matrix.
<code>A_init</code>	a $p$ by $p$ matrix as initial value of transition matrix $A$ estimate.
<code>sig2_eta_init</code>	scalar; initial value of propagation error variance $\sigma_\eta^2$ estimate in latent signal process.
<code>sig2_epsilon_init</code>	scalar; initial value of measurement error variance $\sigma_\epsilon^2$ estimate in observation process.
<code>Ti_train</code>	scalar; the number of time points in training data in cross-validation.
<code>Ti_gap</code>	scalar; the number of time points between test data and train data in cross-validation.
<code>tol_seq</code>	vector; grid of tolerance parameter in Dantzig selector for cross-validation. If <code>is_cv=FALSE</code> , use the first element.
<code>ht_seq</code>	vector; grid of hard-thresholding levels for transition matrix estimate. If <code>is_cv=FALSE</code> , use the first element. To avoid hard thresholding, set <code>ht_seq=0</code> .
<code>is_cv</code>	logical; if true, run cross-validation to tune Dantzig selector tolerance parameter each sparse EM iteration.
<code>thres</code>	scalar; if the difference between updates of two consecutive iterations is less than <code>thres</code> , record one hit. The algorithm is terminated due to vanishing updates if hit times accumulate up to <code>count_vanish</code> . If <code>thres=NULL</code> , the algorithm will not be terminated due to vanishing updates, but too many iterations instead.
<code>count_vanish</code>	scalar; if the difference between updates of two consecutive iterations is less than <code>thres</code> up to <code>count_vanish</code> times, the algorithm is terminated due to vanishing updates.
<code>n_em</code>	scalar; the maximal allowed number of EM iterations, otherwise the algorithm is terminated due to too many iterations. If <code>n_em=NULL</code> , the algorithm will not be terminated due to too many iterations, but vanishing updates instead.
<code>is_echo</code>	logical; if true, display the information of CV-optimal ( <code>tol</code> , <code>ht</code> ) each iteration, and of algorithm termination.
<code>is_sparse</code>	logical; if false, use standard EM algorithm.

### Value

a list of parameter estimates.

<code>A_est</code>	estimate of transition matrix $A$ .
<code>sig2_eta_hat</code>	estimate of propagation error variance $\sigma_\eta^2$ .

sig2\_epsilon\_hat estimate of measurement error variance  $\sigma_\epsilon^2$ .  
 iter\_err the difference between updates of two consecutive iterations.  
 iter\_err\_ratio the difference ratio (over the previous estimate) between updates of two consecutive iterations.

### Author(s)

Xiang Lyu, Jian Kang, Lexin Li

### Examples

```

p= 3; Ti=20 # dimension and time
A=diag(1,p) # transition matrix
sig_eta=sig_epsilon=0.2 # error std
Y=array(0,dim=c(p,Ti)) #observation t=1, ..., Ti
X=array(0,dim=c(p,Ti)) #latent t=1, ..., T
Ti_burnin=30 # time for burn-in to stationarity
for (t in 1:(Ti+Ti_burnin)) {
  if (t==1){
    x1=rnorm(p)
  } else if (t<=Ti_burnin) { # burn in
    x1=A%%x1+rnorm(p,mean=0,sd=sig_eta)
  } else if (t==(Ti_burnin+1)){ # time series used for learning
    X[,t-Ti_burnin]=x1
    Y[,t-Ti_burnin]=X[,t-Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  } else {
    X[,t- Ti_burnin]=A%%X[,t-1- Ti_burnin]+rnorm(p,mean=0,sd=sig_eta)
    Y[,t- Ti_burnin]=X[,t- Ti_burnin]+rnorm(p,mean=0,sd=sig_epsilon)
  }
}

sEM_fit=sEM(Y,diag(0.5,p),0.1,0.1,Ti*0.5,Ti*0.2,c(0.01,0.1))

```

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VARMLE

*generalized Dantzig selector for transition matrix update in maximization step*

---

### Description

Sparse estimation of transition matrix in vector autoregression given conditional autocovariance matrices.

### Usage

VARMLE(S0, S1, tol)

**Arguments**

- S0 a p by p matrix; average (over time points) of conditional expectation of  $x_t x_t^\top$  on  $y_1, \dots, y_T$  and parameter estimates, obtained from expectation step.
- S1 a p by p matrix; average (over time points) of conditional expectation of  $x_t x_{t+1}^\top$  on  $y_1, \dots, y_T$  and parameter estimates, obtained from expectation step.
- tol tolerance parameter in Dantzig selector.

**Value**

Sparse estimate of transition matrix by Dantzig selector.

**Author(s)**

Xiang Lyu, Jian Kang, Lexin Li

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