Package 'hmer'

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```
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URL https://github.com/andy-iskauskas/hmer,
      https://hmer-package.github.io/website/
Description A set of objects and functions for Bayes Linear emulation and history matching.
      Core functionality includes automated training of emulators to data, diagnostic functions
      to ensure suitability, and a variety of proposal methods for generating 'waves' of points.
      For details on the mathematical background, there are many papers available on the topic
      (see references attached to function help files); for details of the functions in this package,
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analyze_diagnostic

Diagnostic Analysis for Emulators

Description

Produces summary and plots for diagnostics

Usage

```
analyze_diagnostic(
  in_data,
  output_name,
  targets = NULL,
  plt = interactive(),
  cutoff = 3,
  target_viz = NULL,
  ...
)
```

Arguments

in_data	The data to perform the analysis on
output_name	The name of the output emulated
targets	If required or desired, the targets for the system outputs
plt	Whether or not to plot the analysis
cutoff	The implausibility cutoff for diagnostic 'ce'
target_viz	How to show the targets on the diagnostic plots
	Any other parameters to pass to subfunctions

Details

Given diagnostic information (almost certainly provided from get_diagnostic), we can plot the results and highlight the points that are worthy of concern or further consideration. Each diagnostic available has a plot associated with it which can be produced here:

Standardized Error: A histogram of standardized errors. Outliers should be considered, as well as whether very many points have either large or small errors.

Comparison Diagnostics: Error bars around points, corresponding to emulator prediction plus or minus emulator uncertainty. A green line indicates where the emulator and simulator prediction would be in complete agreement: error bars that do not overlap with this line (coloured red) are

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to be considered. Where targets are provided, the colouration is limited only to points where the simulator prediction would be close to the targets.

Classification Error: A point plot comparing emulator implausibility to simulator implausibility, sectioned into regions horizontally and vertically by cutoff. Points that lie in the lower right quadrant (i.e. emulator would reject; simulator would not) should be considered.

This function takes a data.frame that contains the input points, simulator values and, depending on the diagnostic, a set of summary measures. It returns a data.frame of any points that failed the diagnostic.

We may also superimpose the target bounds on the comparison diagnostics, to get a sense of where it is most important that the emulator and simulator agree. The target_viz argument controls this, and has three options: 'interval' (a horizontal interval); 'solid' (a solid grey box whose dimensions match the target region in both vertical and horizontal extent); and 'hatched' (similar to solid, but a semi-transparent box with hatching inside). Any such vizualisation has extent equal to the target plus/mius 4.5 times the target uncertainty. By default, target_viz = NULL, indicating that no superposition is shown.

Value

A data frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826

See Also

```
get_diagnostic
```

```
Other diagnostic functions: classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

behaviour_plot

Output Plotting

Description

A simple diagnostic plot that compares the output values to input values, for each possible combination. If emulators are provided, the emulator predictions are plotted; otherwise the model outputs are plotted.

Usage

```
behaviour_plot(
   ems,
   points,
   model = missing(ems),
   out_names = unique(names(collect_emulators(ems))),
   targets = NULL
)
```

Arguments

ems A	set of	Emulator	objects.
-------	--------	----------	----------

points A set of points at which to evaluate the emulator expectation

model If TRUE, use the model outputs; else use emulator expectation

targets If targets are provided, these are added into the plots.

Details

If emulators are provided, then the points argument is optional: if given then the emulator predictions will correspond to those at the points provided. If no points are provided, 100*d (where d is the number of input parameters) are sampled uniformly from the space and used to predict at.

If no emulators are provided, then points must be provided, along with the names of the outputs to plot; each named output must exist as a column in the points data.frame.

Value

The dependency plots.

See Also

```
Other visualisation tools: diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
behaviour_plot(SIREmulators$ems, model = FALSE)
behaviour_plot(points = SIRSample$training, out_names = names(SIREmulators$ems))
#> Throws a warning
behaviour_plot(SIRMultiWaveEmulators, model = TRUE, targets = SIREmulators$targets)
```

```
bimodal_emulator_from_data
```

Bimodal Emulation

Description

Performs emulation of bimodal outputs and/or systems.

Usage

```
bimodal_emulator_from_data(
   data,
   output_names,
   ranges,
   input_names = names(ranges),
   verbose = interactive(),
   na.rm = FALSE,
   ...
)
```

Arguments

data The data to train emulators on (as in variance_emulator_from_data)

ranges The parameter ranges

input_names The names of the parameters (by default inferred from ranges)

verbose Should status updates be provided?

na.rm Should NA values be removed before training?

... Any other parameters to pass to emulator training

Details

In many stochastic systems, particularly disease models, the outputs exhibit bimodality - a familiar example is where a disease either takes off or dies out. In these cases, it is not sensible to emulate the outputs based on all realisations, and instead we should emulate each mode separately.

This function first tries to identify bimodality. If detected, it determines which of the outputs in the data exhibits the bimodality: to these two separate emulators are trained, one to each mode. The emulators are provided with any data that is relevant to their training; for example, bimodality can exist in some regions of parameter space but not others. Points where bimodality is present have their realisations allocated between the two modes while points where no bimodality exists have their realisations provided to both modes. Targets that do not exhibit bimodality are trained as a normal stochastic output: that is, using the default of variance_emulator_from_data.

The function also estimates the proportion of realisations in each mode for the set of outputs. This value is also emulated as a deterministic emulator and included in the output.

The output of the function is a list, containing three objects: mode1, mode2, and prop. The first two objects have the form produced by variance_emulator_from_data while prop has the form of an emulator_from_data output.

Value

A list (mode1, mode2, prop) of emulator lists and objects.

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Examples

```
# Use the stochastic SIR dataset  SIR\_ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05)) \\ SIR\_names <- c("I10", "I25", "I50", "R10", "R25", "R50") \\ b\_ems <- bimodal\_emulator\_from\_data(SIR\_stochastic\$training, SIR\_names, SIR\_ranges)
```

BirthDeath

Birth-Death Model Results

Description

An RData object containing two data.frames. The first consists of ten parameter sets run through a simple, two-parameter, stochastic birth-death model; five of the points have 500 replicates and the other five have only 5 replicates. The second consists of ten further points, each with ten replicates. The objects are denoted training and validation, representing their expected usage.

Usage

BirthDeath

Format

A list of two data.frames training and validation: each data.frame has the following columns:

lambda Birth rate

mu Death rate

Y The number of people at time t = 15

Details

The initial population for the simulations is 100 people; the model is run until t = 15 to obtain the results to emulate.

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classification_diag Classification Diagnostics

Description

Shorthand function for diagnostic test 'ce'.

Usage

```
classification_diag(
  emulator,
  targets,
  validation,
  cutoff = 3,
  plt = interactive()
)
```

Arguments

emulator The emulator in question
targets The output targets
validation The validation set
cutoff The implausibility cutoff
plt Whether to plot or not

Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), comparison_diag(), get_diagnostic(),
individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

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ostics	
--------	--

Description

Shorthand function for diagnostic test 'cd'.

Usage

```
comparison_diag(emulator, targets, validation, sd = 3, plt = interactive())
```

Arguments

emulator	The emulator in o	question

targets The output targets validation The validation set

sd The range of uncertainty allowed

plt Whether to plot or not

Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), classification_diag(), get_diagnostic(),
individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

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Correlator

Correlation Structure

Description

Creates a correlation structure, with the necessary specifications.

The correlator has three main elements: the type of correlator, the associated hyperparameters, and the nugget term. The nugget term is broadly separate from the other two parameters, being type-independent.

Constructor

Correlator\$new(corr, hp, nug)

Arguments

corr The type of correlation function. This is provided as a string which corresponds exactly with a function - the function should take three arguments x, xp, hp. This gives a correlation function u(x, xp) defined by hyperparameters hp. For a simple example, see exp_sq .

hp The associated hyperparameters needed to define the correlation structure, as a named list. In the case of exp_sq, this is a list of one element, list(theta).

nug The size of the nugget term. In situations where not all variables are active, the main part of u(x) operates only on the active parts, xA. The presence of the nugget term accounts for the fact that points at the same position in the active space need not be at the same position in the full space.

By default, Correlatornew() initialises with corr = exp_sq, hp = list(theta = 0.1), and nug = 0.

Accessor Methods

get_corr(x, xp = NULL, actives = TRUE) Returns the correlation between two points. If xp is NULL, then this is correlation between a set of points and themselves (i.e. 1 on the diagonal). All variables are assumed to be active unless otherwise stated in actives.

get_hyper_p() Returns the list of hyperparameters.

print() Produces a summary of the correlation structure specification.

Object Methods

set_hyper_p(hp, nugget) Modifies the hyperparameter and/or nugget terms. Returns a new Correlator object.

Options for Correlations

The default choice (and that supported by other functions in this package, particularly emulator_from_data) for the correlation structure is exponential-squared, due to the useful properties it possesses. However, one can manually instantiate a Correlator with a different underlying structure. Built-in alternatives are as follows, as well as whether a form exists for its derivative:

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```
matern the Matérn function (derivative exists)

orn_uhl the Ornstein-Uhlenbeck function (no derivative)

rat_quad the rational quadratic function (derivative exists)
```

One more function, gamma_exp, is available but not directly supported by emulator_from_data, for example, due to its very limited suitability to emulating model outputs. However, this can be used as a test case for writing one's own correlation functions and using them with emulator_from_data.

A user-defined correlation function can be provided to the Correlator: the requirements are that the function accept data.matrix objects as its first and second arguments, and accept a named list of hyperparameters as its third argument, and return a matrix of correlations between rows of the data.matrices. If a derivative also exists, it should take the same name as the correlation function with "_d" appended to it, and the directions to differentiate with respect to should come after the hyperparameter argument. For example, the rational quadratic functions have the form

```
rat_quad(x1, x2, hp = list(alpha, theta))
rat_quad_d(x1, x2, hp = list(alpha, theta), dx1, dx2)
```

If defining a custom correlation function, care should be taken with hyperparameter estimation - see emulator_from_data examples for details.

Examples

```
test_corr <- Correlator$new(nug = 0.1)
test_corr
point1 <- data.frame(a = 0.1, b = 0.2, c = 0.3)
point2 <- data.frame(a = 0.15, b = 0.18, c = 0.295)
test_corr$get_corr(point1) #> 1
test_corr$get_corr(point1, point2) #> 0.6717557
test_corr$get_corr(point1, point2, actives = c(TRUE, TRUE, FALSE)) #> 0.6734372

new_corr <- test_corr$set_hyper_p(list(theta = 0.5), nug = 0.01)
new_corr$get_corr(point1, point2) #> 0.9784845
new_corr$get_corr(point1, point2, actives = c(TRUE, TRUE, FALSE)) #> 0.9785824

mat_corr <- Correlator$new('matern', list(nu = 1.5, theta = 0.5))
mat_corr$get_corr(data.frame(a = c(1, 0.9), b = c(4, 4.2)))</pre>
```

diagnostic_wrap

Diagnostic plots for wave outputs

Description

A wrapper function for the set of diagnostic plots for multiple waves.

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Usage

```
diagnostic_wrap(
 waves,
  targets,
  output_names = names(targets),
  input_names = names(waves[[1]])[!names(waves[[1]]) %in% names(targets)],
  directory = NULL,
  s.heights = rep(1000, 4),
  s.widths = s.heights,
  include.norm = TRUE,
  include.log = TRUE,
)
```

Arguments

The wave points, as a list of data.frames. waves The output targets. targets output_names The outputs to plot. input_names The inputs to plot. directory The location of files to be saved (if required). s.heights The heights of the saved pngs (if directory is not NULL). The widths of the saved pngs (if directory is not NULL). s.widths Should normalized versions of simulator_plot and wave_dependencies be made? include.norm Should the log-scale version of simulator_plot be made? include.log Optional parameters (eg p_size, l_wid, ...)

Details

. . .

The functions simulator_plot, wave_points, wave_points, and wave_dependencies are called, one after the other, to allow diagnosis of waves of emulation.

The directory option should be used as follows. If the desired location is in fact a folder, it should end in "/"; if instead the structure requires each plot to be saved with a prefix, then it should be provided. For example, directory = "Plots/" in the first event or directory = "Plots/unique-identifier" in the second event.

Value

The set of plots (either into console or saved).

See Also

```
Other visualisation tools: behaviour_plot(), effect_strength(), emulator_plot(), output_plot(),
plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(),
wave_dependencies(), wave_points(), wave_values()
```

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Examples

```
diagnostic_wrap(SIRMultiWaveData, SIREmulators$targets)
diagnostic_wrap(SIRMultiWaveData, SIREmulators$targets,
  input_names = c('aSI', 'aIR'), output_names = c('nI', 'nR'),
  p_size = 0.8, l_wid = 0.8, wave_numbers = 1:3, zero_in = FALSE, surround = TRUE)
```

directional_deriv

Derivative inner product

Description

Find the (uncertainty modified) inner product between the derivative at a point x and a proposed direction v.

Usage

```
directional_deriv(em, x, v, sd = NULL, ...)
```

Arguments

em	The emulator in question
X	The point in input space to evaluate at
V	The direction to assess
sd	How many standard deviations to consider.
	Additional arguments to pass through (eg local.var to the emulator functions)

Details

Given a point x and a direction v, we find the overlap between E[f'(x)] and v. The emulated derivative has uncertainty associated with it: the variance is taken into account using $v^T Var[f'(x)]v$.

If sd == NULL, then only the (normed) overlap between the derivative and the direction vector is returned. Otherwise a pair of values are returned: these are the normed overlap plus or minus sd times the uncertainty.

This function is concerned with ascertaining whether a direction is oriented in the direction of the emulator gradient, subject to the uncertainty around the estimate of the derivative. It allows for a consideration of "emulated gradient descent".

Value

Either a single numeric or a pair of numerics (see description)

Examples

```
directional_deriv(SIREmulators$ems[[1]], SIRSample$validation[1,], c(1,1,1))
```

directional_proposal Emulated Derivative Point Proposal

Description

Proposes a new point by applying 'emulated gradient descent' on an existing point.

Usage

```
directional_proposal(
  ems,
  x,
  targets,
  accept = 2,
  hstart = 1e-04,
  hcutoff = 1e-09,
  iteration.measure = "exp",
  iteration.steps = 100,
  nv = 500
)
```

Arguments

The emulators to evaluate with respect to. ems The original point. targets The list of emulator targets. accept The implausibility below which we allow an output to worsen. hstart The initial step size. hcutoff The minimum allowed step size. iteration.measure Either 'exp' for expectation or 'imp' for implausibility. iteration.steps The number of allowed iterations. The number of directions on the n-sphere to try. nν

Details

Given a point (preferably close to the implausibility boundary) x, we can calculate the emulated gradient at this point for each emulator. If the estimate of the expectation at this point for a given emulator is larger than the target value, then we would like to move in the direction of greatest decrease for this emulator, and conversely for an estimate of the expectation that's smaller than the target value. The combination of this information for every emulator under consideration defines a preferred set of directions of travel from this point.

We may try to find a shared direction which improves (or at least does not worsen) all emulator evaluations. If a point is already well inside the implausibility boundary for a given output (where

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'well inside' is defined by the value of accept), we may allow this output to worsen in order to improve the others.

Provided a shared direction, v, can be identified, we iteratively move in this direction. Define the new proposed point x' = x + h*v, where h is a step-size given by hstart. Compare the summary statistic (either expectational difference or implausibility) to that provided by the original point; if the new point gives improvement, then continue to move in this direction until no further improvement is possible for this step-size. The step-size is reduced (up to a minimum of hcutoff) and the process is repeated. Only finitely many iteration steps are permitted; this can be tuned by supplying a value of iteration.steps.

Value

Either a new proposal point, or the original point if an improvement could not be found.

Examples

```
# Take a point from the SIR system at later waves with low (but >3) implausibility
start_point <- SIRMultiWaveData[[2]][90,1:3]
ems <- SIRMultiWaveEmulators[[3]]
targs <- SIREmulators$targets
# Using expected error as measure
new_point1 <- directional_proposal(ems, start_point, targs, iteration.steps = 50,
nv = 100)
# Using implausibility as measure
new_point2 <- directional_proposal(ems, start_point, targs, iteration.measure = 'imp',
iteration.steps = 50, nv = 100)
all_points <- do.call('rbind.data.frame', list(start_point, new_point1, new_point2))
nth_implausible(ems, all_points, targs)</pre>
```

effect_strength

Find Effect Strength of Active Variables

Description

Collates the linear and quadratic contributions of the active variables to the global emulators' behaviour

Usage

```
effect_strength(
  ems,
  plt = interactive(),
  line.plot = FALSE,
  grid.plot = FALSE,
  labels = TRUE,
  quadratic = TRUE,
  xvar = TRUE
)
```

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Arguments

ems	The Emulator object(s) to be analysed.
plt	Should the results be plotted?

line.plot Should a line plot be produced?

grid.plot Should the effect strengths be plotted as a grid?

labels Whether or not the legend should be included.

quadratic Whether or not quadratic effect strength should be calculated.

xvar Should the inputs be used on the x-axis?

Details

For a set of emulators, it can be useful to see the relative contributions of various parameters to the global part of the emulator (i.e. the regression surface). This function extracts the relevant information from a list of emulator objects.

The parameter quadratic controls whether quadratic effect strength is calculated and plotted (an unnecessary plot if, say, linear emulators have been trained). The remaining options control visual aspects of the plots: line.plot determines whether a line or bar (default) plot should be produced, grid.plot determines whether the results are plotted as a graph or a grid, and labels determines if a legend should be provided with the plot (for large numbers of emulators, it is advisable to set this to FALSE).

Value

A list of data.frames: the first is the linear strength, and the second quadratic.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
effect <- effect_strength(SIREmulators$ems)
effect_line <- effect_strength(SIREmulators$ems, line.plot = TRUE)
effect_grid <- effect_strength(SIREmulators$ems, grid.plot = TRUE)</pre>
```

Emulator

Bayes Linear Emulator

Description

Creates a univariate emulator object.

The structure of the emulator is f(x) = g(x) * beta + u(x), for regression functions g(x), regression coefficients beta, and correlation structure u(x). An emulator can be created with or without data; the preferred method is to create an emulator based on prior specifications in the absence of data, then use that emulator with data to generate a new one (see examples).

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Constructor

```
Emulator$new(basis_f, beta, u, ranges, ...)
```

Arguments

Required:

basis_f A list of basis functions to be used. The constant function function(x) 1 should be provided as the first element.

beta The specification for the regression parameters. This should be provided in the form list(mu, sigma), where mu are the expectations of the coefficients (aligning with the ordering of basis_f) and sigma the corresponding covariance matrix.

u The specifications for the correlation structure. This should be specified in the form list(sigma, corr), where sigma is a single-valued object, and corr is a Correlator object.

ranges A named list of ranges for the input parameters, provided as a named list of length-two numeric vectors.

Optional:

data A data. frame consisting of the data with which to adjust the emulator, consisting of input values for each parameter and the output.

out_name The name of the output variable.

a_vars A logical vector indicating which variables are active for this emulator.

discs Model discrepancies: does not include observational error. Ideally split into list(internal = ..., external = ...).

Internal:

model If a linear model, or otherwise, has been fitted to the data, it lives here.

original_em If the emulator has been adjusted, the unadjusted Emulator object is stored, for use of set_sigma or similar.

multiplier A multiplicative factor to be applied to u_sigma. Typically equal to 1, unless changes have been made by, for example, mult_sigma.

Constructor Details

The constructor must take, as a minimum: a list of vectorised basis functions, whose length is equal to the number of regression coefficients; a correlation structure, which can be non-stationary; and the parameter ranges, used to scale all inputs to the range [-1,1].

The construction of a correlation structure is detailed in the documentation for Correlator.

Accessor Methods

get_exp(x, include_c) Returns the emulator expectation at a point, or at a collection of points. If include_c = FALSE, the contribution made by the correlation structure is not included.

get_cov(x, xp = NULL, full = FALSE, include_c) Returns the covariance between collections of points x and xp. If xp is not supplied, then this is equivalent to get_cov(x, x, ...); if full = TRUE, then the full covariance matrix is calculated - this is FALSE by default due to most built-in

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uses requiring only the diagonal terms, and allows us to take advantage of computational tricks for efficiency.

implausibility(x, z, cutoff = NULL) Returns the implausibility for a collection of points x. The implausibility is the distance between the emulator expectation and a desired output value, weighted by the emulator variance and any external uncertainty. The target, z, should be specified as a named pair list(val, sigma), or a single numeric value. If cutoff = NULL, the output is a numeric I; if cutoff is a numeric value, then the output is boolean corresponding to I <= cutoff.

 $get_exp_d(x, p)$ Returns the expectation of the derivative of the emulated function, E[f'(x)]. Similar in structure to get_exp but for the additional parameter p, which indicates which of the input dimensions the derivative is performed with respect to.

 $get_cov_d(x, p1, xp = NULL, p2 = NULL, ful1 = FALSE)$ Returns the variance of the derivative of the emulated function, Var[f'(x)]. The arguments are similar to that of get_cov , but for the addition of parameters p1 and p2, which indicate the derivative directions. Formally, the output of this function is equivalent to Cov[df/dp1, df/dp2].

```
print(...) Returns a summary of the emulator specifications.
```

plot(...) A wrapper for emulator_plot for a single Emulator object.

Object Methods

adjust(data, out_name) Performs Bayes Linear Adjustment, given data. The data should contain all input parameters, even inactive ones, and the single output that we wish to emulate. adjust creates a new Emulator object with the adjusted expectation and variance resulting from Bayes Linear adjustment, allowing for the requisite predictions to be made using get_exp and get_cov.

set_sigma(sigma) Modifies the (usually constant) global variance of the correlation structure, Var[u(X)]. If the emulator has been trained, the original emulator is modified and Bayes Linear adjustment is again performed.

mult_sigma(m) Modifies the global variance of the correlation structure via a multiplicative factor. As with set_sigma, this change will chain through any prior emulators if the emulator in question is Bayes Linear adjusted.

set_hyperparams(hp, nugget) Modifies the underlying correlator for u(x). Behaves in a similar way to set_sigma as regards trained emulators. See the Correlator documentation for details of hp and nugget.

References

```
Goldstein & Wooff (2007) <ISBN: 9780470065662>
Craig, Goldstein, Seheult & Smith (1998) <doi:10.1111/1467-9884.00115>
```

Examples

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```
# Individual evaluations of points
# Points should still be declared in a data.frame
emsget_exp(data.frame(a = 0.1, b = 0.1)) #> 0.6
em\$get\_cov(data.frame(a = 0.1, b = 0.1)) #> 9.5
# 4x4 grid of points
sample_points \leftarrow expand.grid(a = seq(-0.5, 0.5, length.out = 4), b = seq(-1, 2, length.out = 4))
em$get_exp(sample_points) # Returns 16 expectations
em$get_cov(sample_points) # Returns 16 variances
sample_points_2 <- expand.grid(a = seq(-0.5, 0.5, length.out = 3),</pre>
                                b = seq(-1, 2, length.out = 4))
em$get_cov(sample_points, xp = sample_points_2, full = TRUE) # Returns a 16x12 matrix of covariances
fake_data <- data.frame(a = runif(10, -0.5, 0.5), b = runif(10, -1, 2))
fake_data$c <- fake_data$a + 2*fake_data$b</pre>
newem <- em$adjust(fake_data, 'c')</pre>
all(round(newem$get_exp(fake_data[,names(ranges)]),5) == round(fake_data$c,5)) #>TRUE
matern_em < -Emulator new(basis_f = c(function(x) 1, function(x) x[[1]], function(x) x[[2]]),
beta = list(mu = c(1, 0.5, 2), sigma = diag(0, nrow = 3)),
u = list(corr = Correlator$new('matern', list(nu = 1.5, theta = 0.4))),
ranges = list(x = c(-1, 1), y = c(0, 3)))
matern_em\$get_exp(data.frame(x = 0.4, y = 2.3))
newem_data <- Emulator$new(basis_functions, beta, u, ranges, data = fake_data)</pre>
all(round(newem$get_exp(fake_data[,names(ranges)]),5)
   == round(newem_data$get_exp(fake_data[,names(ranges)]), 5)) #>TRUE
newem$get_exp_d(sample_points, 'a')
newem$get_cov_d(sample_points, 'b', p2 = 'a')
```

emulator_from_data

Generate Emulators from Data

Description

Given data from simulator runs, generates a set of univariate Emulator objects, one for each output.

Usage

```
emulator_from_data(
  input_data,
  output_names,
  ranges,
  input_names = names(ranges),
  beta,
  u,
  c_lengths,
  funcs,
  deltas,
```

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```
ev,
quadratic = TRUE,
beta.var = FALSE,
adjusted = TRUE,
discrepancies = NULL,
has.hierarchy = FALSE,
verbose = interactive(),
na.rm = FALSE,
check.ranges = FALSE,
corr_name = "exp_sq",
targets = NULL,
...
```

Arguments

input_data Required. A data.frame containing parameter and output values.

output_names Required. A character vector of output names.

ranges A named list of input parameter ranges.

input_names The names of the parameters (if ranges is not provided).

beta A list of regression coefficients for each output.

u A list of Correlator objects for each output.

c_lengths A list of correlation lengths for each output.

funcs A list of regression functions for each output.

deltas Nugget terms for each correlation structure.

ev Estimates of ensemble variability for each output.

quadratic Should a quadratic or linear fit be found?

beta.var Should regression coefficient uncertainty be included?

adjusted Are the raw emulators wanted, or Bayes Linear updated ones?

discrepancies Any internal or external discrepancies in the model.

has.hierarchy For hierarchical emulators, this will be TRUE.

verbose Should status updates be printed?

na.rm If NAs exist in the dataset, should those rows be removed? check.ranges Should the ranges be modified in light of the data provided?

corr_name What correlation function to use. Defaults to exp_sq

targets If provided, outputs are checked for over/underestimation

... Any additional parameters (eg for custom correlation functions)

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Details

Many of the parameters that can be passed to this function are optional: the minimal operating example requires input_data, output_names, and one of ranges or input_names. If ranges is supplied, the input names are generated from that list, data.frame, or data.matrix; if only input_names is specified, then the ranges are assumed to be [-1, 1] for every input.

The ranges can be provided in alternative ways: either as a named list of length-2 numeric vectors (corresponding to the maximum and minimum for each parameter); as a data.frame with 2 columns where each row corresponds to a parameter; or as a data.matrix defined similarly as the data.frame. In the cases where the ranges are provided as a data.frame or a data.matrix, the row.names of the data object must be provided, corresponding to the names of the parameters.

If the minimum information is provided, then an emulator is fitted as follows. The basis functions and associated regression coefficients are generated using step and lm up to a desired order (default 2, determined by quadratic). These regression parameters are assumed to be 'known' unless beta.var = TRUE, in which case the derived parameter variance is taken from the model fit too (and the regression coefficients themselves can be modified by the maximum likelihood estimate performed below).

The correlation function c(x, x') is assumed to be exp_sq and a corresponding Correlator object is created. The hyperparameters of the correlation structure are determined using a combination of maximum likelihood estimation and restriction to a 'sensible' range of values, to avoid the correlation length tending to 0 or very large values. This determines the variance sigma^2, correlation length theta, any other hyperparameters (eg nu for the matern correlation function), and nugget term delta. The hyperparameter priors can be overridden either by directly specifying them using the c_lengths argument, or by supplying ranges to the theta_ranges argument. Examples of this customisation can be found in the examples to this function.

If ev is provided, then the ensemble variability is taken into account in the determination of the nugget term via a two-stage training process.

Some rudimentary data handling functionality is available but should be approached with caution. The na.rm option will strip out rows of the training data that have NA values in them; this of course may leave too few points to train to, and any consistent occurrence of NAs in model data should be investigated. The check.ranges option allows a redefinition of the ranges of the input parameters for emulator training; this is a common practice in later waves in order to maximise the predictive power of the emulators, but should only be used here if one is sure that the training set is representative of (and certainly spanning) the full minimum enclosing hyperrectangle.

Value

A list of Emulator objects.

Examples

```
# Use the SIRSample training dataset as an example. ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05)) out_vars <- c('nS','nI','nR') ems_linear <- emulator_from_data(SIRSample$training, out_vars, ranges, quadratic = FALSE) ems_linear # Printout of the key information.
```

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```
ems_quad <- emulator_from_data(SIRSample$training, out_vars, ranges)</pre>
ems_quad # Now includes quadratic terms (but only where needed)
ems_unadjusted <- emulator_from_data(SIRSample$training, out_vars, ranges, adjusted = FALSE)</pre>
ems_unadjusted # Looks the same as ems_quad, but the emulators are not BL adjusted
# Reproduce the linear case, but with slightly changed beta values
basis_f <- list(</pre>
 \texttt{c(function(x) 1, function(x) x[[1]], function(x) x[[2]], function(x) x[[3]]),}\\
 c(function(x) 1, function(x) x[[1]], function(x) x[[2]]),
 c(function(x) 1, function(x) x[[1]], function(x) x[[2]])
beta_vals <- list(</pre>
 list(mu = c(300, -260, 220, -120)),
 list(mu = c(120, 110, -260)),
 list(mu = c(580, 160, 130))
ems2 <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                            funcs = basis_f, beta = beta_vals)
ems2
# Custom correlation functions
corr_structs <- list(</pre>
  list(sigma = 83, corr = Correlator$new('exp_sq', list(theta = 0.5), nug = 0.1)),
  list(sigma = 95, corr = Correlator$new('exp_sq', list(theta = 0.4), nug = 0.25)),
  list(sigma = 164, corr = Correlator$new('exp_sq', list(theta = 0.2), nug = 0.45))
ems3 <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                            u = corr_structs)
# Using alternative correlation functions and c_lengths
# Allow code to choose hyperparameters
ems_matern <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                                   corr_name = 'matern')
# Providing hyperparameters to the function directly, via c_lengths
matern_hp \leftarrow list(list(theta = 0.8, nu = 1.5), list(theta = 0.6, nu = 2.5),
 list(theta = 1.2, nu = 0.5)
ems_matern2 <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                                    corr_name = 'matern', c_lengths = matern_hp)
\# If only one set of hyperparameters are provided to c_lengths, they are used for all
ems_matern3 <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                                    corr_name = 'matern', c_lengths = matern_hp[[1]])
# "Custom" correlation function with user-specified ranges: gamma exponential
# 'gamma_exp' can be substituted for any correlation function - see Correlator documentation
ems_gamma <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
                                    corr_name = 'gamma_exp',
                              theta_ranges = list(gamma = c(0.01, 2), theta = c(1/3, 2)))
```

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Description

A function for plotting emulator expectations, variances, and implausibilities

Usage

```
emulator_plot(
  ems,
  plot_type = "exp",
  ppd = 30,
  targets = NULL,
  cb = FALSE,
  params = NULL,
  fixed_vals = NULL,
  nth = 1
)
```

Arguments

ems An Emulator object, or a list thereof.

plot_type The statistic to plot (see description or examples).

ppd The number of points per plotting dimension

targets If required, the targets from which to calculate implausibility

cb A boolean representing whether a colourblind-friendly plot is produced.

params Which two input parameters should be plotted?

fixed_vals For fixed input parameters, the values they are held at.

nth If plotting nth maximum implausibility, which level maximum to plot.

Details

Given a single emulator, or a set of emulators, the emulator statistics can be plotted across a twodimensional slice of the parameter space. Which statistic is plotted is determined by plot_type: options are 'exp', 'var', 'sd', 'imp', and 'nimp', which correspond to expectation, variance, standard deviation, implausibility, and nth-max implausibility.

By default, the slice varies in the first two parameters of the emulators, and all other parameters are taken to be fixed at their mid-range values. This behaviour can be changed with the params and fixed_vals parameters (see examples).

If the statistic is 'exp', 'var' or 'sd', then the minimal set of parameters to pass to this function are ems (which can be a list of emulators or a single one) and plot_type. If the statistic is 'imp' or 'nimp', then the targets must be supplied - it is not necessary to specify the individual target for a single emulator plot. If the statistic is 'nimp', then the level of maximum implausibility can be chosen with the parameter nth.

Implausibility plots are typically coloured from green (low implausibility) to red (high implausibility): a colourblind-friendly option is available and can be turned on by setting cb = TRUE.

The granularity of the plot is controlled by the ppd parameter, determining the number of points per dimension in the grid. For higher detail, at the expense of longer computing time, increase this value. The default is 30.

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Value

A ggplot object, or collection thereof.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
# Reducing ppd to 10 for speed.
emulator_plot(SIREmulators$ems, ppd = 10)
emulator_plot(SIREmulators$ems, ppd = 10)
emulator_plot(SIREmulators$ems, plot_type = 'var', ppd = 10, params = c('aIR', 'aSR'))

emulator_plot(SIREmulators$ems, plot_type = 'imp', ppd = 10,
    targets = SIREmulators$targets,
    fixed_vals = list(aSR = 0.02))
emulator_plot(SIREmulators$ems, plot_type = 'nimp', cb = TRUE,
    targets = SIREmulators$targets, nth = 2, ppd = 10)
```

exp_sq

Exponential squared correlation function

Description

For points x, xp and a correlation length theta, gives the exponent of the squared distance between x and xp, weighted by theta squared.

Usage

```
exp_sq(x, xp, hp)
```

Arguments

X	A data frame of rows corresponding to position vectors
хр	A data frame of rows corresponding to position vectors
hp	The hyperparameter theta (correlation length)

Value

The exponential-squared correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

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Examples

```
exp_sq(data.frame(a=1), data.frame(a=2), list(theta = 0.1))
#> 3.720076e-44
exp_sq(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(theta = 0.2))
#> 3.266131e-13
```

full_wave

Automatic Wave Calculation

Description

Performs a full wave of emulation and history matching, given data.

Usage

```
full_wave(
   data,
   ranges,
   targets,
   old_emulators = NULL,
   prop_train = 0.7,
   cutoff = 3,
   nth = 1,
   verbose = interactive(),
   n_points = nrow(data),
   ...
)
```

Arguments

data	The data to train with.
ranges	The ranges of the input parameters
targets	The output targets to match to.
old_emulators	Any emulators from previous waves.
prop_train	What proportion of the data is used for training.
cutoff	The implausibility cutoff for point generation and diagnostics.
nth	The level of maximum implausibility to consider.
verbose	Should progress be printed to console?
n_points	The number of points to generate from generate_new_runs.
	Any arguments to be passed to emulator_from_data.

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Details

This function uses all of the functionality from the package in a relatively conservative form. The function performs the following steps:

- 1) Split the data into a training set and a validation set, where prop_train indicates what proportion of the data is used to train.
- 2) Perform emulator training using emulator_from_data. If a more involved specification is desired, optional arguments can be passed to emulator_from_data using the . . . argument.
- 3) Perform diagnostics on the trained emulators, removing emulators that do not display acceptable performance. Global emulator variance may also be modified to ensure that none of the emulators demonstrate misclassification errors (from classification_diag).
- 4) Ordering the remaining emulators from most restrictive to least restrictive on the dataset provided at this wave. Some point generation mechanisms terminate early if a point is ruled out by a single emulator, so the ordering ensures this happens earlier rather than later.
- 5) Generate the new points using the default method of generate_new_runs, using the normal procedure (for details, see the description for generate_new_runs). By default, it generates the same number of points as it was provided to train and validate on.

If the parameter old_emulators is provided, this should be a list of emulators used at all previous waves - for example if full_wave is used to do a second wave of history matching, then old_emulators would contain the list of first-wave emulators.

The function returns a list of two objects: emulators corresponding to this wave's emulators, and points corresponding to the new proposed points. The points can then be put into the simulator to generate runs for a subsequent wave.

Value

A list of two objects: points and emulators

Examples

```
ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05)) default <- full_wave(do.call('rbind.data.frame', SIRSample), ranges, SIREmulators$targets) non_quad <- full_wave(do.call('rbind.data.frame', SIRSample), ranges, SIREmulators$targets, quadratic = FALSE) second <- full_wave(SIRMultiWaveData[[2]], ranges, SIREmulators$targets, old_emulators = SIRMultiWaveEmulators[[1]])
```

gamma_exp

Gamma-exponential correlation function

Description

For points x, xp, and a pair of hyperparameters gamma and theta, gives the gamma-exponential correlation between the two points.

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Usage

```
gamma_exp(x, xp, hp)
```

Arguments

A data.frame of rows corresponding to position vectors
 A data.frame of rows corresponding to position vectors
 The hyperparameters theta (correlation length) and gamma (exponent), as a named list

Details

The gamma-exponential correlation function, for d = |x-x'|, is given by $\exp(-(d/\theta)^{\gamma})$. Gamma must be between 0 (exclusive) and 2 (inclusive).

Value

The gamma-exponential correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

generate_new_runs

Generate Proposal Points

Description

Given a set of trained emulators, this finds the next set of points that will be informative for a subsequent wave of emulation.

Usage

```
generate_new_runs(
  ems,
  n_points,
  z,
  method = "default",
  cutoff = 3,
  nth = NULL,
```

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```
plausible_set,
verbose = interactive(),
cluster = FALSE,
resample = 1,
seek = 0,
c_tol = 0.5,
i_tol = 0.01,
to_file = NULL,
imp_func = function(ems, x, z, ...) nth_implausible(ems, x, z, ...),
...
)
```

Arguments

ems	A list of Emulator objects, trained on previous design points.
n_points	The desired number of points to propose for the next wave.
z	The targets to match to.
method	The method(s) to use.
cutoff	The implausibility cutoff(s) to compare outputs to.
nth	A parameter to be passed to the n argument of nth_implausible.
plausible_set	An optional set of known non-implausible points (for eg line sampling).
verbose	Should progress statements be printed to the console?
cluster	Should emulator clustering be considered in the LHS generation?
resample	Number of times to resample using line and/or importance sampling.
seek	How many 'good' points to search for
c_tol	The tolerance with which to determine that best implausibility has been reached.
i_tol	The tolerance on final desired implausibility
to_file	The filename to write to sequentially during proposal. Default is NULL (no writing)
imp_func	The implausibility measure to use to determine point acceptance.
• • •	Any parameters to pass to individual sampling functions, eg distro for importance sampling.

Details

If the method is '1hs', a Latin hypercube is generated and non-implausible points from this design are retained. If enough points are accepted, the points outputted are chosen using either a maximin or V-optimality criteria (chosen by measure.method).

The methods 'line' and 'importance' both require a predetermined set of non-implausible points s_points; if they are not provided then lhs sampling is performed first.

The method 'line' performs line sampling for boundary detection. Given a set of non-implausible points, rays are drawn between pairs of points (selected so as to maximise the distance between them), and more points are sampled along these lines. Points are kept if they lie near a boundary of the non-implausible space.

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The method 'importance' performs importance sampling, using a mixture distribution of multivariate normal or uniform spherical proposals around the current non-implausible points (determined by distro). The optimal standard deviation or radius is found using a burn-in phase, before the full set of points is generated.

The method 'slice' performs slice sampling. Given one known non-implausible point, it attempts to find a minimum enclosing hyperrectangle for the non-implausible region around this point and samples uniformly from this, shrinking the hyperrectangle as appropriate. This method is also called if LH sampling has only generated one point (since any later methods require at least two points to be useful).

The method 'optical' uses optical depth sampling: given a set of known non-implausible points, the an approximation of the one-dimensional marginal distributions in each parameter direction can be determined. From these derived marginals, points are sampled.

For any sampling strategy, the parameters ems and z must be provided. All of these methods depend on a means of assessing point suitability (henceforth referred to as 'implausibility'). By default, this is uses nth-maximum implausibility as provided by nth_implausible: a user-defined method can be substituted for this by supplying the function call to imp_func. As a minimum, the function should take four arguments: the emulators, the points, and the targets, and a . . . argument to allow compatibility with the standard behaviour.

The option seek determines how many points should be chosen that have a higher probability of matching targets, as opposed to not missing targets. Due to the danger of such an approach in terms of obtaining a representative space-filling design over the space, this value should not be too high: a rough guide is that it should be no larger than 10% of the desired number of points. The default is seek = 0. If seek is in the range [0, 1], it is assumed to represent a proportion of the total number of points that should be sought using this method; otherwise, it is assumed to be the number of points that are desired.

The default behaviour is as follows. A set of initial points are generated from an LHD; line sampling is performed to find the boundaries; and finally this collection of points is augmented to the desired number of points by importance sampling using uniform spherical proposals.

In regions where the non-implausible space (at the given cutoff) is very hard to find, the function will start at a higher implausibility where it can find a space-filling design; using this as a starting point any other methods are performed. From this new proposal, a subset of lower-implausibility points are selected. This process iterates until either the desired implausibility has been reached or the process has reached a barrier to further reductions in implausibility. The argument c_tol is used to determine if the improvement in implausibility is small enough to justify stopping the process; by default this is 0.1. The process will also stop if it has produced points 'close to' the desired implausibility: the level of closeness is defined using the i_tol argument.

These methods may not work, or may work slowly, if the target space is very small compared to the current not-yet-ruled-out space, or it may miss small disconnected regions of parameter space.

Value

A data frame containing the set of new points to run the model at.

Examples

A simple example that uses a number of the native and ... parameter options

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```
pts <- generate_new_runs(SIREmulators$ems, 100, SIREmulators$targets,
measure.method = 'maximin', distro = 'sphere', resample = 0)
pts_optical <- generate_new_runs(SIREmulators$ems, 100, SIREmulators$targets,
method = c('optical'))
pts_slice <- generate_new_runs(SIREmulators$ems, 100, SIREmulators$targets,
method = c('slice'))
pts_no_importance <- generate_new_runs(SIREmulators$ems, 100, SIREmulators$targets,
method = c('line'))</pre>
```

get_diagnostic

Diagnostic Tests for Emulators

Description

Given an emulator, return a diagnostic measure.

Usage

```
get_diagnostic(
  emulator,
  targets = NULL,
  validation = NULL,
  which_diag = "cd",
  stdev = 3,
  cleaned = NULL,
  warn = TRUE,
  kfold = NULL,
  ...
)
```

Arguments

emulator	An object of class Emulator
targets	If desired, the target values for the output(s) of the system
validation	If provided, the emulator is tested against the outputs of these points
which_diag	Which diagnostic measure to use (choosing from cd, ce, se above)
stdev	For 'cd', a measure of the allowed distance from prediction and reality
cleaned	Internal for stochastic emulators
warn	Should a warning be shown if ce is chosen and no targets provided?
kfold	Mainly internal: pre-computed k-fold diagnostic results for output
	Any other parameters to be passed through to subfunctions.

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Details

An emulator's suitability can be checked in a number of ways. This function combines all current diagnostics available in the package, returning a context-dependent data.frame containing the results.

Comparison Diagnostics (cd): Given a set of points, the emulator expectation and variance are calculated. This gives a predictive range for the input point, according to the emulator. We compare this against the actual value given by the simulator: points whose emulator prediction is further away from the simulator prediction are to be investigated. This 'distance' is given by stdev, and an emulator prediction correspondingly should not be further away from the simulator value than stdev*uncertainty.

Classification Error (ce): Given a set of targets, the emulator can determine implausibility of a point with respect to the relevant target, accepting or rejecting it as appropriate. We can define a similar 'implausibility' function for the simulator: the combination of the two rejection schemes gives four classifications of points. Any point where the emulator would reject the point but the simulator would not should be investigated.

Standardized Error (se): The known value at a point, combined with the emulator expectation and uncertainty, can be combined to provide a standardized error for a point. This error should not be too large, in general. but the diagnostic is more useful when looking at a collection of such measures, where systematic bias or over/underconfidence can be seen.

Which of the diagnostics is performed can be controlled by the which_diag argument. If performing classification error diagnostics, a set of targets must be provided; for all diagnostics, a validation (or holdout) set can be provided. If no such set is given, then the emulator diagnostics are performed with respect to its training points, using k-fold cross-validation.

Value

A data frame consisting of the input points, output values, and diagnostic measures.

See Also

```
validation_diagnostics
```

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
# Use the simple SIR model via SIREmulators
get_diagnostic(SIREmulators$ems$nS, validation = SIRSample$validation)
# Classification error fails without the set of targets
get_diagnostic(SIREmulators$ems$nI, SIREmulators$targets, SIRSample$validation, 'ce')
# No validation set: k-fold cross-validation will be used.
get_diagnostic(SIREmulators$ems$nR, which_diag = 'se')
```

32 HierarchicalEmulator

HierarchicalEmulator Hierarchical Bayes Linear Emulator

Description

Creates a univariate emulator with hierarchical structure.

This object does not differ extensively from the standard Emulator object, so most of the functionality will not be listed here: the main difference is that it allows for the variance structure of the emulator to be modified by a higher order object. The typical usage is to create a variance emulator, whose predictions inform the behaviour of a mean emulator with regard to a stochastic process.

Constructor

```
HierarchicalEmulator$new(basis_f, beta, u, ranges, ...)
```

Arguments

For details of shared arguments, see Emulator.

s_diag The function that modifies the structure of the Bayes Linear adjustment.

samples A numeric vector that indicates how many replicates each of the training points has.

em_type Whether the emulator is emulating a mean surface or a variance surface.

Constructor Details

See Emulator: the constructor structure is the same save for the new arguments discussed above.

Accessor Methods

 $get_exp(x, samps = NULL)$ Similar in form to the normal Emulator method; the samps argument allows the estimation of summary statistics derived from multiple realisations.

 $get_cov(x, xp = NULL, full = FALSE, samps = NULL)$ Differences here are in line with those described in get_exp .

Object Methods

Identical to those of Emulator: the one internal difference is that adjust returns a Hierarchical Emulator rather than a standard one.

References

Goldstein & Vernon (2016), in preparation

Examples

```
h_em <- variance_emulator_from_data(BirthDeath$training, c('Y'), list(lambda = c(0, 0.08), mu = c(0.04, 0.13))) names(h_em) #> c("expectation', 'variance')
```

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idemc

IDEMC Point Generation

Description

Performs Implausibility-driven Evolutionary Monte Carlo

Usage

```
idemc(
   ems,
   N,
   targets,
   cutoff = 3,
   s = max(500, ceiling(N/5)),
   sn = s,
   p = 0.4,
   thin = 1,
   pm = 0.9,
   w = 0.8,
   M = 10,
   detailed = FALSE,
   verbose = interactive()
)
```

Arguments

ems	The emulators to evaluate implausibility on
N	The desired number of points to generate
targets	The target values for the outputs
cutoff	The desired final implausibility cutoff
s	The number of points to generate at each intermediate burn-in stage
sn	The number of points to generate in the final burn-in stage
р	The proportion of space that should remain in a move along the ladder
thin	The thinning factor: a factor T means that N^*T points are generated to get N
pm	The probability that mutation is chosen in an IDEMC step
W	The probability of local random walk moves in the mutation step
М	The number of mutations to perform in an IDEMC step, if chosen
detailed	If TRUE, points from every ladder rung are returned
verbose	Should information about burn-in be displayed during the process?

34 idemc

Details

This method for generating points is focused on finding non-implausible regions that are either extremely small relative to the initial space, are have interesting structure (particularly disconnected structure) that would potentially be overlooked by more standard point generation methods. The method is robust but computationally intensive, and should not be used as a default - for more standard methods of finding points, see generate_new_runs and the methods therein.

The IDEMC method operates on an 'implausibility ladder', in the spirit of annealing methods. Each 'rung' of the ladder is characterised by within-cluster and overall variance. The stages performed in one step of the evolutionary algorithm are as follows:

Mutation: A point is modified using a process akin to a random walk step. The parameters that determine the walk can be a global step (determined by the second-order quantities of the entire rung) or a within-cluster step (where the point's cluster is determined and the second-order quantities are drawn from that particular cluster). Local, within-cluster, moves are chosen with probability w. The move is retained if the new point satisfies the constraints of its rung.

Crossover: Points are reorganised in descending order of how active their variables are in the emulated outputs, and two different rungs are selected. The points are 'mixed' using a one-point crossover: given a randomly selected index k and two points x1, x2, the new points are y1 = (x11, x12, ..., x1k, x2(k+1), ... x2n) and similarly for y2. The move is retained if both new points satisfy the constraints on their respective rungs. Choices of rung where the first is a later (more restrictive) rung are favoured.

Exchange: Two adjacent rungs are chosen and their points swapped wholesale. The move is retained if the point coming from the less restrictive rung satisfies the constraints of the rung it moves to.

At a given step, only one of mutation or crossover is performed: the probability of performing mutation is given by pm. If mutation is chosen, M such moves are performed on each rung; if crossover is chosen, then (n+1)/2 such moves are performed across the n rungs. Exchange is always performed and n+1 such moves are performed.

The choice of 'implausibility ladder' and clusters has a large bearing on the results. This function performs a 'burn-in' to determine a reasonable ladder by starting with a uniform sample across the whole space and defining the next rung by demanding that a percentage (determined by p) of the original points satisfy the constraint of this new rung. The IDEMC process is performed on these two rungs to generate s points, from which the process is repeated. Once the desired implausibility has been reached, sn steps of the algorithm are performed on all rungs to determine final clusters.

Value

Either a list of points (for each rung), or a single data.frame from the last rung.

References

Vernon & Williamson (2013) <arXiv:1309.3520>

See Also

generate_new_runs for more standard point generation methods

individual_errors 35

Examples

```
idemc_res <- idemc(SIREmulators$ems, 500, SIREmulators$targets, s = 250, p = 0.3)</pre>
```

individual_errors

Predictive Error Plots

Description

Plots the predictive error with respect to a variety of quantities.

Usage

```
individual_errors(
  em,
  validation,
  errtype = "normal",
  xtype = "index",
  plottype = "normal")
```

Arguments

em The emulator to perform diagnostics on validation The validation set of points with output(s) errtype The type of individual error to be plotted.

xtype The value to plot against

plottype Whether to plot a standard or Q-Q plot.

Details

The choice of errors to plot is controlled by errtype, and can be one of four things: normal, corresponding to the regular standardised errors; eigen, corresponding to the errors after reordering given by the eigendecomposition of the emulator covariance matrix; chol, similarly deriving errors after Cholesky decomposition; and cholpivot, deriving the errors after pivoted Cholesky decomposition.

What the errors are plotted with respect to is controlled by xtype. The options are index, which plots them in their order in the validation set; em, which plots errors with respect to the emulator prediction at that point; and any named parameter of the model, which plots with respect to the values of that parameter.

Finally, the plot type is controlled by plottype: this can be one of normal, which plots the errors; or qq, which produces a Q-Q plot of the errors.

The default output is to plot the standardised errors (with no decomposition) against the ordering in the validation set; i.e. errtype = "normal", xtype = "index", plottype = "normal".

36 matern

Some combinations are not permitted, as the output would not be meaningful. Errors arising from an eigendecomposition cannot be plotted against either emulator prediction or a particular parameter (due to the transformation induced by the eigendecomposition); Q-Q plots are not plotted for a non-decomposed set of errors, as the correlation between errors makes it much harder to interpret.

Value

The relevant plot.

References

```
Bastos & O'Hagan (2009) <doi:10.1198/TECH.2009.08019>
```

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
i1 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation)
i2 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "chol", "em")
i3 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "eigen", plottype = "qq")
i4 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "cholpivot", xtype = "aSI")</pre>
```

matern

Matern correlation function

Description

For points x, xp, and a pair of hyperparameters nu and theta, gives the Matern correlation between the two points.

Usage

```
matern(x, xp, hp)
```

Arguments

X	A data.frame of rows corresponding to position vectors
xp	A data frame of rows corresponding to position vectors
hp	The hyperparameters nu (smoothness) and theta (correlation length), as a named list
	1151

Details

At present, only half-integer arguments for nu are supported.

nth_implausible 37

Value

The Matern correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

nth_implausible

nth Maximum Implausibility

Description

Computes the nth-maximum implausibility of points relative to a set of emulators.

Usage

```
nth_implausible(
  ems,
  x,
  z,
  n = NULL,
  max_imp = Inf,
  cutoff = NULL,
  sequential = FALSE,
  get_raw = FALSE,
  ordered = FALSE
)
```

Arguments

ems	A set of Emulator objects or nested sets thereof (see description)
X	An input point, or data. frame of points.
Z	The target values, in the usual form or nested thereof.
n	The implausibility level to return.
max_imp	A maximum implausibility to return (often used with plotting)
cutoff	A numeric value, or vector of such, representing allowed implausibility
sequential	Should the emulators be evaluated sequentially?
get_raw	Boolean - determines whether nth-implausibility should be applied.
ordered	If FALSE, emulators are ordered according to restrictiveness.

38 nth_implausible

Details

For a collection of emulators, we often combine the implausibility measures for a given set of observations. The maximum implausibility of a point, given a set of univariate emulators and an associated collection of target values, is the largest implausibility of the collected set of implausibilities. The 2nd maximum is the maximum of the set without the largest value, and so on. By default, maximum implausibility will be considered when there are fewer than 10 targets to match to; otherwise second-maximum implausibility is considered.

If sequential = TRUE and a specific cutoff has been provided, then the emulators' implausibility will be evaluated one emulator at a time. If a point is judged non-implausible by more than n emulators, FALSE is returned without evaluating any more. Due to R efficiencies, this is more efficient than the 'evaluate all' method once more than around 10 emulators are considered.

This function also deals with variance emulators and bimodal emulators, working in a nested fashion. If targets are provided for both the expectation and variance as a list, then given ems = list(expectation = ..., variance = ...) the implausibility is calculated with respect to both sets of emulators, maximising as relevant. If targets are provided in the 'normal' fashion, then only the mean emulators are used. The bimodal case is similar; given a set of emulators list(mode1 = list(expectation = ..., variance = ...), ...) then each mode has implausibility evaluated separately. The results from the two modes are combined via piecewise minimisation.

Value

Either the nth maximum implausibilities, or booleans (if cutoff is given).

```
# A single point
nth_implausible(SIREmulators$ems, data.frame(aSI = 0.4, aIR = 0.25, aSR = 0.025),
SIREmulators$targets)
# A data.frame of points
grid <- expand.grid(</pre>
 aSI = seq(0.1, 0.8, length.out = 4),
 aIR = seq(0, 0.5, length.out = 4),
aSR = seq(0, 0.05, length.out = 4)
)
# Vector of numerics
i1 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets)</pre>
# Vector of booleans (same as i1 <= 3)</pre>
i2 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets, cutoff = 3)</pre>
# Throws a warning as n > no. of targets
i3 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets, n = 4)</pre>
# Vector of booleans (note different output to i2)
i4 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets,</pre>
 cutoff = c(4, 2.5, 2)
# Variance Emulators
v_ems <- variance_emulator_from_data(BirthDeath$training, c('Y'),</pre>
list(lambda = c(0, 0.08), mu = c(0.04, 0.13)))
v_{targs} = list(expectation = list(Y = c(90, 110)), variance = list(Y = c(55, 95)))
nth_implausible(v_ems, unique(BirthDeath$validation[,1:2]), v_targs)
## If there is a mismatch between emulators and targets, expectation is assumed
```

orn_uhl 39

```
nth_implausible(v_ems$expectation, unique(BirthDeath$validation[,1:2]), v_targs)
nth_implausible(v_ems, unique(BirthDeath$validation[,1:2]), v_targs$expectation)
```

orn_uhl

Ornstein-Uhlenbeck correlation function

Description

For points x, xp, and a hyperparameter theta, gives the Ornstein-Uhlenbeck correlation between the two points.

Usage

```
orn_uhl(x, xp, hp)
```

Arguments

X	A data frame of rows corresponding to position vectors
хр	A data frame of rows corresponding to position vectors
hp	The hyperparameter theta (correlation length) in a named list

Details

This correlation function can be seen as a specific case of the Matern correlation function when nu = 1/2.

Value

The Ornstein-Uhlenbeck correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

```
orn_uhl(data.frame(a=1), data.frame(a=2), list(theta = 0.1))
#> 4.539993e-05
orn_uhl(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(theta = 0.2))
#> 0.00469197
orn_uhl(data.frame(a=1,b=1,c=1), data.frame(a=1.2,b=0.9,c=0.6), list(theta = 0.2)) ==
matern(data.frame(a=1,b=1,c=1), data.frame(a=1.2,b=0.9,c=0.6), list(theta = 0.2, nu = 0.5)) #> TRUE
```

40 output_plot

output_plot Emulator Expectation Against Target Outputs	output_plot	Emulator Expectation Against Target Outputs	
---	-------------	---	--

Description

Plots emulator expectation across the parameter space, with comparison to the corresponding target values (with appropriate uncertainty).

Usage

```
output_plot(ems, targets, points = NULL, npoints = 1000)
```

Arguments

ems The Emulator objects.

targets A named list of observations, given in the usual form.

points A list of points at which the emulators should be evaluated.

npoints If no points are provided, the number of input points to evaluate at.

Details

If a points data.frame is not provided, then points are sampled uniformly from the input region. Otherwise, the provided points are used: for example, if a representative sample of the current NROY space is available.

Value

A ggplot object

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

```
output_plot(SIREmulators$ems, SIREmulators$targets)
output_plot(SIREmulators$ems, SIREmulators$targets, points = SIRSample$training)
```

plot_actives 41

tives Active variable plotting
active variable plotting

Description

For a set of emulators, demonstrate which variables are active.

Usage

```
plot_actives(ems, output_names = NULL, input_names = NULL)
```

Arguments

ems The list of emulators to consider

output_names The names of the outputs to include in the plot, if not all input_names The names of the inputs to include in the plot, if not all

Details

Each emulator has a list of 'active' variables; those which contribute in an appreciable way to its regression surface. It can be instructive to examine the differences in active variables for a give collection of emulators. The plot here produces an nxp grid for n emulators in p inputs; a square is blacked out if that variable does not contribute to that output.

Both the outputs and inputs can be restricted to collections of interest, if desired, with the optional output_names and input_names parameters.

Value

A ggplot object corresponding to the plot

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

```
plot_actives(SIREmulators$ems)
# Remove the nR output and aIR input from the plot
plot_actives(SIREmulators$ems, c('nS', 'nI'), c('aSI', 'aSR'))
# Note that we can equally restrict the emulator list...
plot_actives(SIREmulators$ems[c('nS', 'nI')], input_names = c('aSI', 'aSR'))
```

42 plot_lattice

plot_lattice	Plot Lattice of Emulator Implausibilities	
--------------	---	--

Description

Plots a set of projections of the full-dimensional input space.

Usage

```
plot_lattice(ems, targets, ppd = 20, cb = FALSE, cutoff = 3, maxpoints = 50000)
```

Arguments

ems The Emulator objects in question. targets The corresponding target values.

ppd The number of points to sample per dimension.

cb Whether or not a colourblind-friendly plot should be produced.

cutoff The cutoff value for non-implausible points.

maxpoints The limit on the number of points to be evaluated.

Details

The plots are:

One dimensional optical depth plots (diagonal);

Two dimensional optical depth plots (lower triangle);

Two dimensional minimum implausibility plots (upper triangle).

The optical depth is calculated as follows. A set of points is constructed across the full d-dimensional parameter space, and implausibility is calculated at each point. The points are collected into groups based on their placement in a projection to a one- or two-dimensional slice of the parameter space. For each group, the proportion of non-implausible points is calculated, and this value in [0,1] is plotted. The minimum implausibility plots are similar, but with minimum implausibility calculated rather than proportion of non-implausible points.

The maxpoints argument is used as a cutoff for if a regular ppd grid would result in a very large number of points. If this is the case, then maxpoints points are sampled uniformly from the region instead of regularly spacing them.

Value

A ggplot object

References

Bower, Goldstein & Vernon (2010) <doi:10.1214/10-BA524>

plot_wrap 43

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
plot_lattice(SIREmulators$ems, SIREmulators$targets, ppd = 10)
plot_lattice(SIREmulators$ems$nS, SIREmulators$targets)
```

plot_wrap

Plot proposed points

Description

A wrapper around R's base plot to show proposed points

Usage

```
plot_wrap(points, ranges = NULL, p_size = 0.5)
```

Arguments

points The points to plot ranges The parameter ranges

p_size The size of the plotted points (passed to cex)

Details

Given a set of points proposed from emulators at a given wave, it's often useful to look at how they are spread and where in parameter space they tend to lie relative to the original ranges of the parameters. This function provides pairs plots of the parameters, with the bounds of the plots calculated with respect to the parameter ranges provided.

Value

The corresponding pairs plot

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

44 rat_quad

Examples

plot_wrap(SIRSample\$training[,1:3], SIREmulators\$ems[[1]]\$ranges)

problem_data

Data for an interesting emulation problem

Description

An RData object consisting of four objects: a data.frame data of 208 points, a set targets of 19 targets for outputs, a set ranges of 21 ranges for inputs, and a data.frame extra of 26 additional points. This dataset is used to demonstrate some of the subtleties of emulation in the vignettes, where data transformations can be useful and careful attention should be paid to emulation at early waves.

Usage

problem_data

Format

A list of objects:

data The training data of 'space-filling' runs

targets The output targets to match to

ranges The input ranges over which the system is valid

extra A set of 'extra' points, generated around a known point of best fit.

rat_quad

Rational Quadratic correlation function

Description

For points x, xp, and a pair of hyperparameters alpha and theta, gives the rational quadratic correlation between the two points.

Usage

```
rat_quad(x, xp, hp)
```

Arguments

X	A data frame of rows corresponding to position vectors
хр	A data frame of rows corresponding to position vectors

hp The hyperparameters alpha (exponent and scale) and theta (correlation length)

residual_diag 45

Details

This correlation function, for d = |x-x'|, has the form $(1 + d^2/(2\alpha\theta^2))^{-\alpha}$, and can be seen as a superposition of exponential-squared correlation functions.

Value

The rational quadratic correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

```
rat_quad(data.frame(a=1), data.frame(a=2), list(alpha = 1.5, theta = 0.1))
#> 0.004970797
rat_quad(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(alpha = 1.5, theta = 0.2))
#> 0.02904466
```

residual_diag

Emulator Regression Residuals

Description

Plots the emulator residuals.

Usage

```
residual_diag(emulator, histogram = FALSE, ...)
```

Arguments

emulator The emulator to consider.

histogram Should a histogram or a scatter plot be shown? Default: FALSE

... Any additional arguments (used internally)

Details

An emulator is composed of two parts: a global regression surface, and a local correlation structure. It can sometimes be informative to examine the residuals of the regression surface on the training set, to determine the extent to which the regression surface is being 'corrected for' by the correlation structure.

Value

A set of residuals, standardised by the regression surface residual standard error.

46 simulator_plot

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
residual_diag(SIREmulators$ems$nS)
residual_diag(SIREmulators$ems$nI, TRUE)
```

simulator_plot

Plot simulator outputs for multiple waves

Description

Plots the simulator results for points at successive waves.

Usage

```
simulator_plot(
  wave_points,
  z,
  zero_in = TRUE,
  palette = NULL,
  wave_numbers = seq(ifelse(zero_in, 0, 1), length(wave_points) - ifelse(zero_in, 1, 0)),
  normalize = FALSE,
  logscale = FALSE,
  barcol = "#444444",
  ...
)
```

Arguments

The set of wave points, as a list of data.frames wave_points The set of target values for each output zero_in Is wave zero included? Default: TRUE If a larger palette is required, it should be supplied here. palette wave_numbers Which waves to plot. If not supplied, all waves are plotted. normalize If true, plotting is done with rescaled target bounds. logscale If true, targets are log-scaled before plotting. barcol The colour of the target error bars/bounds Optional parameters (not to be used directly)

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Details

The values plotted are the outputs from the simulator; the points passed to it are the points suggested by that wave of emulators. By default, wave 0 is included. A colour scheme is chosen outright for all invocations of this function: it is a 10-colour palette. If more waves are required, then an alternative palette should be selected.

Value

A ggplot object.

See Also

Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()

Examples

```
simulator_plot(SIRMultiWaveData, SIREmulators$targets)
simulator_plot(SIRMultiWaveData[2:4], SIREmulators$targets,
zero_in = FALSE, wave_numbers = c(1,3))
```

SIREmulators

Sample Emulators

Description

An RData object containing three trained emulators, and the associated targets, for the SIR example. The emulators have been trained on the SIRSample training dataset using methods documented in this package.

Usage

SIREmulators

Format

A list containing two objects:

ems The trained Emulator objects.

targets The targets to match to, as a named list.

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SIRImplausibility

Sample Implausibility Data

Description

A dataset containing 1000 points from the region bounded by [0.1, 0.8], [0, 0.5], [0, 0.05] for aSI, aIR and aSR respectively. Implausibility has been calculated (for emulators trained on the SIRSample training dataset) for each of the outputs nS, nI, nR, and the maximum implausibility is included. The target values used in calculating implausibility were:

```
nS between 324 and 358nI mean 143 (sigma 7.15)nR between 490 and 542
```

Usage

SIRImplausibility

Format

A data frame with 1000 rows and 7 variables:

aSI Infection: transition rate from S to I **aIR** Recovery: transition rate from I to R

aSR Immunisation: transition rate from S to R

nS Implausibility for nSnI Implausibility for nInR Implausibility for nRI Maximum implausibility

SIRMultiWaveData

Sample Multi-wave Results

Description

An rda object containing four data.frames: an initial set of points also provided in SIRSample, and the 90 points generated at each of three subsequent waves. The trained emulators are provided in SIRMultiWaveEmulators.

Usage

SIRMultiWaveData

SIRMultiWaveEmulators 49

Format

A list of data.frame objects:

Wave 0 The initial points used in other examples

Wave 1 Points generated from the wave 1 emulators

Wave 2 Points generated from the wave 2 emulators

Wave 3 Points generated from the wave 3 emulators

SIRMultiWaveEmulators Sample Multi-wave Emulators

Description

An rda object containing three waves of emulators applied to SIR model (described in SIRSample). The corresponding points (both training and validation) are stored in SIRMultiWaveData.

Usage

SIRMultiWaveEmulators

Format

A list containing Emulator objects:

Wave 1 Emulators trained on Wave 0, generating wave 1 points

Wave 2 Emulators trained on the results of the above wave 1 points

Wave 3 Emulators trained on the results of the above wave 2 points

SIRSample

Sample SIR data

Description

A small dataset containing points generated from a simple deterministic SIR model. The model contains three input parameters, and generates three output parameters. The initial populations are 950 susceptible (S), 50 infected (I), and 0 recovered (R). The final values are taken at time t=10.

Usage

SIRSample

SIR_stochastic

Format

A list of two data frames. The first has 30 rows and 6 variables, the second 60 rows and 6 variables. The structure is the same in both cases:

aSI Infection: transition rate from S to I
aIR Recovery: transition rate from I to R
aSR Immunisation: transition rate from S to R
nS Final number of S
nI Final number of I
nR Final number of R

Details

The model operates using simple differential equations, where

S' = aSR*R - aSI*S*R/(S+I+R) I' = aSI*S*R/(S+I+R) - aIR*I R' = aIR*I - aSR*R.

SIR_stochastic

Stochastic SIR Data

Description

An RData object consisting of two data.frames (in a similar fashion to BirthDeath). The first consists of 30 points in the parameter space (aSI, aIR, aSR), each of which has been inputted into the Gillespie algorithm for the stochastic version of the model used in GillespieSIR (but with changed starting conditions) 100 times. The second has similar form but for 20 unique points, each with 100 repetitions.

Usage

SIR_stochastic

Format

A list of two data.frames training and validation: each has the following columns:

aSI Infection rate

aIR Recovery rate

aSR Waning immunity rate

I10 (25, 50) The number of infected people at t = 10 (25, 50)

R10 (25, 50) The number of recovered people at t = 10 (25, 50)

space_removal 51

Details

The outputs observed are the numbers of infected (I) and recovered (R) people at time points t = 10, 25, 50. All outputs display some level of bimodality. The initial conditions to generate the runs had S(0)=995, I(0)=5, R(0)=0.

space_removal

Percentage of Space Removed

Description

For a wave of emulators, estimates the proportion of space removed at this wave.

Usage

```
space_removal(ems, targets, points = NULL, cutoff = 3, individual = TRUE)
```

Arguments

ems The emulators to compute over, as a list

targets The output target values points The points to test against

cutoff The cutoff value for implausibility

individual If true, gives emulator-by-emulator results; otherwise works with maximum im-

plausibility

Details

Given a collection of emulators corresponding to a wave, we can look at an estimate of the proportion of points from previous waves that will be accepted at this wave, either on an emulator-by-emulator basis (to see which outputs are most restrictive) or as an all-wave determination.

Naturally, such a statement will be an estimate of the restriction on the full space (which will become more unreliable for higher dimensions), but it can give an order-of-magnitude statement, or useful comparators between different emulators in a wave.

If no points are provided, the training points for the emulators are used. For best results, a good number of points should be given: typically one should consider using as many points as one knows to be in the NROY space (including any validation points, if accessible).

Value

A numeric corresponding to the proportions of points accepted.

See Also

space_removed for a visualisation of the space removal.

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Examples

```
space_removal(SIREmulators$ems, SIREmulators$targets,
  rbind(SIRSample$training, SIRSample$validation))
space_removal(SIREmulators$ems, SIREmulators$targets,
  rbind(SIRSample$training, SIRSample$validation), individual = FALSE)
```

space_removed

Space Removal Diagnostics

Description

Finds the proportion of space removed as a function of implausibility cut-off and of one of structural discrepancy, emulator variance, or correlation hyperparameter(s).

Usage

```
space_removed(
   ems,
   targets,
   ppd = 10,
   u_mod = seq(0.8, 1.2, by = 0.1),
   intervals = seq(0, 10, length.out = 200),
   modified = "obs",
   maxpoints = 50000
)
```

Arguments

ems The Emulator objects.

targets The corresponding targets to match to.

The number of points per input dimension to sample at.

u_mod The proportional values by which to inflate/deflate the relevant statistic.

intervals The interval values of the implausibility cutoff at which to evaluate.

modified The statistic to modify: obs, disc, var or hp (see above)

maxpoints The maximum number of points to evaluate at

Details

The reduction in space is found by evaluating a p^d regular grid, where p is chosen by ppd and d is the dimension of the input space. Larger values of p will give a more accurate reflection of the space removed, at a corresponding computational cost. For the purpose of quick-and-dirty diagnostics, ppd = 5 is sufficient: the default is 10.

The parameter modified can be one of three strings: 'obs' corresponding to observation uncertainty; 'disc' corresponding to internal and external discrepancy (as given in Emulator\$disc);

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'var' corresponding to global emulator variance (as given by Emulator\$u_sigma), and 'hp' corresponding to the hyperparameters of the emulator correlation structure. In the first case, the implausibilities are recalculated for each inflation value; in the other two cases the emulators are retrained. For this reason, the 'var' and 'hp' options are computationally more intensive. The default is 'obs'.

The inflationary/deflationary values are chosen by u_mod: the default is to take 80%, 90%, 100%, 110%, and 120% of the original value as the variation. The proportion of points deemed non-implausible is checked at a set of implausibility cutoffs defined by intervals, and a plot is returned showing the relevant data.

Value

A ggplot object

See Also

```
space_removal for a numeric representation of space removed.
```

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
space_removed(SIREmulators$ems, SIREmulators$targets, ppd = 5)
space_removed(SIREmulators$ems$nS, SIREmulators$targets,
ppd = 5, u_mod = seq(0.75, 1.25, by = 0.25), intervals = seq(2, 6, by = 0.1))
```

standard_errors

Standardized Error Diagnostics

Description

Shorthand function for diagnostic test 'se'.

Usage

```
standard_errors(
  emulator,
  targets = NULL,
  validation = NULL,
  plt = interactive()
)
```

Arguments

emulator The emulator in question targets The output targets validation The validation set Whether to plot or not

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Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(),
get_diagnostic(), individual_errors(), residual_diag(), summary_diag(), validation_diagnostics()
```

subset_emulators

Subsetting for Bimodal/Variance Emulators

Description

Takes a collection of bimodal or stochastic emulators and subsets by output name.

Usage

```
subset_emulators(emulators, output_names)
```

Arguments

emulators A set of emulators, often in nested form output_names The names of the desired outputs

Details

It can be useful to consider only a subset of outputs. In the normal case, this can be easily achieved; however, when the emulators are in a nested structure such as that provided by variance_emulator_from_data or bimodal_emulator_from_data, it can be more involved. This function allows the easy selecting of emulators by name, returning a subset of them in the same form as the original object.

This function is compatible with 'standard' emulators; that is, those in a simple list, equivalent to subsetting over the collection of output names of the emulators that exist in output_names.

Value

An object of the same form as 'emulators'.

summary_diag 55

|--|

Description

Generates measures for emulator quality

Usage

```
summary_diag(emulator, validation, verbose = interactive())
```

Arguments

emulator The emulator to test

validation The validation set, consisting of points and output(s)

verbose Should statistics be printed out?

Details

A couple of summary statistics can be generated for emulators, based on their prediction errors on a validation set. This function produces the test statistic for a comparison to a relevant chi-squared distribution, and the similar test statistic for an F-distribution. In both cases, the expectation and standard deviation of the underlying distribution are also provided.

The output of this function is a logical vector stating whether the derived value lies within 3-sigma of the expected value. In systems where errors are expected to be correlated, higher weight should be given to the Mahalanobis measure than the chi-squared measure. Any anomalous results can be investigated in more depth using the individual_errors function.

Value

Whether the observed value lies within 3-sigma of the expected value.

References

```
Bastos & O'Hagan (2009) <doi:10.1198/TECH.2009.08019>
```

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), validation_diagnostics()
```

```
summary_diag(SIREmulators$ems$nR, SIRSample$validation)
```

```
validation_diagnostics
```

Emulator Diagnostics

Description

Performs the standard set of validation diagnostics on emulators.

Usage

```
validation_diagnostics(
  emulators,
  targets = NULL,
  validation = NULL,
  which_diag = c("cd", "ce", "se"),
  analyze = TRUE,
  diagnose = "expectation",
  ...
)
```

Arguments

emulators A list of Emulator objects.

targets The list of observations for the outputs

validation The validation set, containing all inputs and outputs.

which_diag Which diagnostics should be performed (see description)

analyze Should plotting and/or failing points be returned?

diagnose For bimodal systems, should the expectation or variance be considered?

Any additional parameters to pass to the diagnostics (eg sd, cutoff, ...)

Details

All the diagnostics here can be performed with or without a validation (or 'holdout') set of data. The presence of a set of targets is optional for some checks but mandatory for others: the appropriate warnings will be given in the event that some checks cannot be applied.

The current options for diagnostics (with the codes for which_diag) are:

Standardised Errors (se)

Comparison Diagnostics (cd)

Classification Errors (ce)

All of the above (all)

For details of each of the tests, see the help file for get_diagnostic.

validation_pairs 57

Value

A data.frame containing points that failed one or more diagnostic tests.

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), summary_diag()
```

Examples

```
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation)
# data.frame of failed points (empty) and a 3x3 set of plots
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation,
    c('ce','cd'))
# empty data.frame and a 3x2 set of plots
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation,
    cutoff = 2, sd = 2)
# k-fold (with k = 3)
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, k = 3)
```

validation_pairs

Validation Set Diagnostics and Implausibility

Description

Creates pairs plots on the set of validation points of diagnostic suitability and implausibility.

Usage

```
validation_pairs(ems, points, targets, ranges, nth = 1, cb = FALSE)
```

Arguments

ems

	<u></u>
points	The set of validation points to plot.
targets	The set of targets to match to.
ranges	If provided, this gives the plotting region (see above).
nth	The level of maximum implausibility to plot.
cb	Whether or not the colour scheme should be colourblind friendly.

The Emulator object(s).

Details

The plots are organised as follows:

- a) Emulated versus simulated output (lower diagonal). This is similar in spirit to comparison_diag: the plotted points are their location in the input space and the points are coloured by the emulator prediction's deviation from the simulator value.
- b) Implausibility (upper diagonal). The points are again plotted based on their location in input space, but their colouration is now based on the implausibility of the point.

If ranges is provided, then the plotting region is created relative to these ranges. This can be useful if on later waves of a history match and the plotting is to be done relative to the original input space, rather than the (reduced) parameter space upon which the emulators have been trained.

Value

A ggplot object.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
validation_pairs(SIREmulators$ems, SIRSample$validation, SIREmulators$targets)
wider_ranges <- purrr::map(SIREmulators$ems[[1]]$ranges, ~.*c(-2, 2))
validation_pairs(SIREmulators$ems, SIRSample$validation,
    SIREmulators$targets, ranges = wider_ranges, cb = TRUE)</pre>
```

```
variance_emulator_from_data
```

Variance Emulator Creation

Description

Trains hierarchical emulators to stochastic systems

Usage

```
variance_emulator_from_data(
  input_data,
  output_names,
  ranges,
  input_names = names(ranges),
  verbose = interactive(),
  na.rm = FALSE,
  ...
)
```

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Arguments

input_data All model runs at all points.

output_names The observation names.

ranges A named list of parameter ranges

input_names The names of the parameters (if ranges is not provided).

verbose Should status updates be printed to console?
na.rm Should NA values be removed before training?

... Optional parameters that can be passed to link{emulator_from_data}.

Details

For stochastic systems, one may emulate the variance as well as the function itself. This is particularly true if one expects the variance to be very different in different areas of the parameter space (for example, in an epidemic model). This function performs the requisite two-stage Bayes Linear update.

All observations are required (including replicates at points) - this function collects them into the required chunks and calculates the summary statistics as required.

All other parameters passed to this function are equivalent to those in emulators are the Bayes Linear adjusted forms.

Value

A list of lists: one for the variance emulators and one for the function emulators.

References

Goldstein & Vernon (2016) in preparation

Examples

```
# A simple example using the BirthDeath dataset v_ems <- variance_emulator_from_data(BirthDeath$training, c("Y"), list(lambda = c(0, 0.08), mu = c(0.04, 0.13)), c_lengths = c(0.75))
```

wave_dependencies

Multiple Wave Inputs vs Outputs

Description

Given multiple waves of points, produce input-output plots for each pair.

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Usage

```
wave_dependencies(
  waves,
  targets,
  output_names = names(targets),
  input_names = names(waves[[1]])[!names(waves[[1]]) %in% names(targets)],
  p_size = 1.5,
  l_wid = 1.5,
  normalize = FALSE,
  zero_in = TRUE,
  wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 1, 0)),
  ...
)
```

Arguments

waves The list of data.frame objects, one for each set of outputs at that wave.

targets The target values of the outputs.

output_names The outputs to plot, if not all are wanted.

input_names The inputs to plot, if not all are wanted.

p_size Control for the point size on the plots: smaller is better for many plots.

1_wid Control for line width of superimposed targets.

normalize If true, plotting is done with target bounds equal size.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot

... Optional parameters (not to be used directly)

Details

It can be useful to consider what the dependencies between the input values and output values are, to investigate the suitability of the chosen input ranges (i.e. if widening an input range could result in the targets being matchable). This function provides those plots.

For each output-input pair, a points plot is produced with the input value on the x-axis and the output value on the y-axis. The target bounds are superimposed as horizontal lines. The points themselves are coloured by which wave of history matching they came from.

These can show dependencies between specific outputs and inputs and, if points are clustering at the far left or right edge of a plot, can give an indication that the input ranges are unsuitable for matching the target.

Value

A grid of ggplot objects.

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See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_points(), wave_values()
```

Examples

```
wave_dependencies(SIRMultiWaveData, SIREmulators$targets, l_wid = 0.8, p_size = 0.8)
wave_dependencies(SIRMultiWaveData, SIREmulators$targets, c('nS', 'nI'), c('aIR', 'aSI'))

# For many plots, it may be helpful to manually modify the font size
wave_dependencies(SIRMultiWaveData, SIREmulators$targets) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

wave_points

Multiple Wave Point Plotting

Description

Given multiple waves of points, produces pairs plots

Usage

```
wave_points(
  waves,
  input_names,
  surround = FALSE,
  p_size = 1.5,
  zero_in = TRUE,
  wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 1, 0)),
  ...
)
```

Arguments

waves The list of data.frames, one for each set of points at that wave.

input_names The input names to be plotted.

surround If true, points are surrounded by black boundaries.

p_size The size of the points. Smaller values are better for high-dimensional spaces.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot

Optional parameters (not to be used directly)

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Details

Subsequent waves are overlaid on the same pairs plots, to determine the evolution of the non-implausible region. One-dimensional density plots are also created on the diagonal.

Value

A ggplot object

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_values()
```

Examples

```
wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR'))
wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR'), TRUE, 0.8)
# For many plots, it may be helpful to manually modify the font size
wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR')) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

wave_values

Multiple Wave Output Plotting

Description

Given multiple waves of points, produces pairs plots of the outputs.

Usage

```
wave_values(
   waves,
   targets,
   output_names = names(targets),
   ems = NULL,
   surround = FALSE,
   restrict = FALSE,
   p_size = 1.5,
   l_wid = 1.5,
   zero_in = TRUE,
   wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 0, 1)),
   which_wave = ifelse(zero_in, 0, 1),
   upper_scale = 1,
   ...
)
```

wave_values 63

Arguments

waves The list of data frames, one for each set of outputs at that wave.

targets The output targets.
output_names The outputs to plot.

ems If provided, plots the emulator expectations and 3-standard deviations.

surround As in wave_points.

restrict Should the plotting automatically restrict to failing target windows?

p_size As in wave_points.

1_wid The width of the lines that create the target boxes.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot.

which_wave Scaling for lower plots (see description)

upper_scale Scaling for upper plots (ibid)

... Optional parameters (not to be used directly)

Details

This function operates in a similar fashion to wave_points - the main difference is that the output values are plotted. Consequently, the set of targets is required to overlay the region of interest onto the plot.

To ensure that the wave numbers provided in the legend match, one should provide waves as a list of data.frames with the earliest wave at the start of the list.

The parameters which_wave and upper_scale control the level of 'zoom' on each of the lower-triangular and upper-triangular plots, respectively. For the lower plots, which_wave determines which of the provided waves is to be used to determine the output ranges to plot with respect to: generally, higher which_wave values result in a more zoomed-in plot. For the upper plots, upper_scale determines the plot window via a multiple of the target bounds: higher values result in a more zoomed-out plot. If not provided, these default to which_wave=0 (or 1 if no wave 0 is given) and upper_scale = 1. If the value provided to which_wave does not correspond to a provided wave (or one explicitly not included in wave_numbers), it defaults to the closest available wave to the value of which_wave.

If ems is provided, it should follow the same structure as waves: at the very least, it should contain all emulators trained over the course of the waves. The emulator predictions for a target are made by the emulator for that target whose ranges are the smallest such that contain the point.

Value

A ggplot object.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points()
```

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```
wave_values(SIRMultiWaveData, SIREmulators$targets, surround = TRUE, p_size = 1)
wave_values(SIRMultiWaveData, SIREmulators$targets, c('nS', 'nI'), l_wid = 0.8)
wave_values(SIRMultiWaveData, SIREmulators$targets, l_wid = 0.8,
wave_numbers = c(0, 1, 3), which_wave = 2, upper_scale = 1.5)
# For many plots, it may be helpful to manually modify the font size
wave_values(SIRMultiWaveData, SIREmulators$targets) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

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