Package ‘itdr’

October 13, 2022

Type Package

Title Integral Transformation Methods for SDR in Regression

Version 1.2.0

Depends R(>= 3.5.0)

Imports stats,utils,MASS

Description The routine, itdr(), which allows to estimate the sufficient dimension reduction subspaces, i.e., central mean subspace or central subspace in regression, using Fourier transformation proposed by Zhu and Zeng (2006) <https:doi.org/10.1198/016214506000000140>, convolution transformation proposed by Zeng and Zhu (2010) <https:doi.org/10.1016/j.jmva.2009.08.004> and iterative Hessian transformation methods proposed by Cook and Li (2002) <https:doi.org/10.1214/aos/1021379861>. The predictor variables can be consider to have a multivariate normal distribution or an elliptical contoured distribution. If the distribution of the predictor variables is unknown, then the predictors' distribution can be estimated by the kernel density estimation method. Moreover, each of these routines is supported with a bootstrap procedure to estimate their tuning parameters. That is, wx() estimates the tuning parameter for the predictor variables, wy() estimates the tuning parameter for the response variable, and wh() estimates the bandwidth parameter for the kernel density estimation method. The function invFM() estimates the central subspace using Fourier transform approach for inverse dimension reduction method proposed by Weng and Yin (2018) <https:doi.org/10.1080/10485252.2018.1515432>. The function d.test() estimates the dimension of the central mean subspace using hypothesis under invFM(). Moreover, the dsp() function provides the two distance measures between two subspaces spanned by the columns of two matrices; Vector correlation proposed by Hooper (1959) <https:doi.org/10.2307/1909445>, and Trace correlation proposed by Hotelling (1936) <https:doi.org/10.2307/2333955>.

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R topics documented:

`automobile` *Automobiles data*

Description

This dataset contains the details about automobiles from 1985 Ward's automotive yearbook.

Usage

```
data(automobile)
```

Format

A dataset with 205 observations and 26 attributes.

symboling -3, -2, -1, 0, 1, 2, 3.

normalized-losses continuous from 65 to 256.

make alfa-romero, audi, bmw, chevrolet, dodge, honda,isuzu, jaguar, mazda, mercedes-benz, mercury,mitsubishi, nissan, peugot, plymouth, porsche,renault, saab, subaru, toyota, volkswagen, volvo

fuel-type diesel, gas.

aspiration std, turbo.

num-of-doors four, two.

body-style hardtop, wagon, sedan, hatchback, convertible.

drive-wheels 4wd, fwd, rwd.

engine-location front, rear.

wheel-base continuous from 86.6 120.9.

length continuous from 141.1 to 208.1.

width continuous from 60.3 to 72.3.

height continuous from 47.8 to 59.8.

curb-weight continuous from 1488 to 4066.

engine-type dohc, dohcv, l, ohc, ohcf, ohcv, rotor.

num-of-cylinders eight, five, four, six, three, twelve, two.

engine-size continuous from 61 to 326.

fuel-system 1bbl, 2bbl, 4bbl, idi, mfi, mpfi, spdi, spfi.

bore continuous from 2.54 to 3.94.

stroke continuous from 2.07 to 4.17.

compression-ratio continuous from 7 to 23.

horsepower continuous from 48 to 288.

peak-rpm continuous from 4150 to 6600.

city-mpg continuous from 13 to 49.

highway-mpg continuous from 16 to 54.

price continuous from 5118 to 45400.

Source

https://archive.ics.uci.edu/ml/datasets/automobile

`d.boots` *Bootstrap estimation for dimension (d) of sufficient dimension reduction subspaces.*

Description

d.boots() estimates the dimension of the central mean subspace and the central subspaces in regression.

Usage

```
d.boots(y,x,wx=0.1,wy=1,wh=1.5,B=500,Plot=FALSE,space="mean"
                                        ,xdensity="normal",method="FM")
```

Arguments

y	The n-dimensional response vector.
x	The design matrix of the predictors with dimension n-by-p.
wx	(default 0.1). The tuning parameter for the predictor variables.
wy	(default 1). The tuning parameter for the response variable.
wh	(default 1.5). The bandwidth of the kernel density estimation.
B	(default 500). Number of bootstrap samples.
Plot	(default FALSE). If TRUE, then it provides the dimension variability plot.
space	(default “mean”). The defalult is “mean” for the central mean subspace. Other option is “pdf” for estimating the central subspace.
xdensity	(default “normal”). Density function of the predictor variables. If “normal” then predictor variables are coming from a multivariate normal distribution. If “elliptic” then predictors are coming from an elliptical contoured distribution function. If the distribution of the predictor variables is unknown, then use “kernel” to estimate the distribution function using a kernel smoothing method.
method	(default “FM”). The integral transformation method. “FM” for Fourier transformation method (Zhu and Zeng 2006), and “CM” for convolution transformation method (see Zeng and Zhu 2010).

Value

The outputs are a table of average bootstrap distances between two subspaceses for each candidate value of *d* and the estimated value for *d*.

dis_d	A table of average bootstrap distances for each candidate value of *d*.
d.hat	The estimated value for *d*.
plot	Provides the dimension variability plot if *plot=TRUE*.

Examples

```
library(itdr)
# Use dataset available in itdr package
data(automobile)
head(automobile)
automobile.na=na.omit(automobile)
# prepare response and predictor variables
auto_y=log(automobile.na[,26])
auto_xx=automobile.na[,c(10,11,12,13,14,17,19,20,21,22,23,24,25)]
auto_x=scale(auto_xx) # Standardize the predictors
# call to the d.boots() function with required arguments
d_est=d.boots(auto_y,auto_x,Plot=TRUE,space="pdf",xdensity = "normal",method="FM")
auto_d=d_est$d.hat
```

d.test *Testing methods for selecting dimension of the central mean subspace.*

Description

d.test() provides p-values for the hypothesis tests for the dimension of the subpsace based on three test statistics: Cook's, Scaled, and Adjusted test statistics, using Fourier transform approach for inverse dimension reduction method.

Usage

```
d.test(y,x,m)
```

Arguments

y	The n-dimensional response vector.
x	The design matrix of the predictors with dimension n-by-p.
m	An integer specifying the dimension of the central mean reduction subspace to be tested.

Details

The null and alternative hypothesis are

$$H_0 : d = m$$

$$vs$$

$$H_a : d > m$$

Weighted Chi-Square test statistics (Weng and Yin, 2018):

$$\hat{\Lambda} = n \sum_{j=m+1}^{p} \hat{\lambda}_j,$$

where λ_j's are the eigenvalues of $\widehat{\mathbf{V}}$ where $\widehat{\mathbf{V}}$ is defined under *invFM()* function.

Scaled test statistic (Bentler and Xie, 2000):

$$\overline{T}_m = [trace(\hat{\Omega}_n)/p^\star]^{-1} n \sum_{j=m+1}^{p} \hat{\lambda}_j \sim \mathcal{X}^2_{p^\star},$$

where $\hat{\Omega}_n$ is a covariance matrix (Bentler and Xie, 2000), and $p^\star = (p - m)(2t - m)$.

Adjusted test statistic (Bentler and Xie, 2000):

$$\tilde{T}_m = [trace(\hat{\Omega}_n)/d^\star]^{-1} n \sum_{j=m+1}^{p} \hat{\lambda}_j \sim \mathcal{X}^2_{d^\star},$$

where $\hat{\Omega}_n$ is a covariance matrix (Bentler and Xie, 2000), and $d^\star = [trace(\hat{\Omega}_n)]^2/trace(\hat{\Omega}_n^2)$.

Value

The *d.test()* returns a table of p-values for each test.

References

Bentler P. M., and Xie, J. (2000). Corrections to Test Statistics in Principal Hessian Directions. *Statistics and Probability Letters*. 47, 381-389.

Weng J., and Yin X. (2018). Fourier Transform Approach for Inverse Dimension Reduction Method. *Journal of Nonparametric Statistics*. 30, 4, 1029-0311.

Examples

```
library(itdr)
data(PDB)
colnames(PDB)=NULL
p=15
df=PDB[,c(79,73,77,103,112,115,124,130,132,145,149,151,153,155,167,169)]
dff=as.matrix(df)
planingdb=dff[complete.cases(dff),]
y=planingdb[,1]
x=planingdb[,c(2:(p+1))]
x=x+0.5
xt=cbind(x[,1]^(.33),x[,2]^(.33),x[,3]^(.57),x[,4]^(.33),x[,5]^(.4),
x[,6]^(.5),x[,7]^(.33),x[,8]^(.16),x[,9]^(.27),x[,10]^(.5),
x[,11]^(.5),x[,12]^(.33),x[,13]^(.06),x[,14]^(.15),x[,15]^(.1))
m=1
W=sapply(1,rnorm)
d.test(y,x,m)
```

---

dsp *Distance between two subspaces.*

Description

dsp() returns the distance between two subspaces, which are spanned by the columns of two matrices.

Usage

```
dsp(A, B)
```

Arguments

A	A matrix with dimension p-by-d.
B	A matrix with dimension p-by-d.

Details

Let $\mathbf{A}$ and $\mathbf{B}$ be two full rank matrices of size $p \times q$. Suppose $\mathcal{S}(\mathbf{A})$ and $\mathcal{S}(\mathbf{B})$ are the column subspaces of matrices $\mathbf{A}$ and $\mathbf{B}$, respectively. And, let λ_i 's with $1 \geq \lambda_1^2 \geq \lambda_2^2 \geq, \cdots, \lambda_p^2 \geq 0$, are the eigenvalues of the matrix $\mathbf{B}^T\mathbf{A}\mathbf{A}^T\mathbf{B}$.

(Trace correlation, Hotelling, 1936)

$$\gamma = \sqrt{\frac{1}{p}\sum_{i=1}^{p}\lambda_i^2}$$

(Vector correlation, Hooper, 1959)

$$\theta = \sqrt{\prod_{i=1}^{p}\lambda_i^2}$$

Value

Outputs are the following scale values.

`r`	One mines the trace correlation. That is, $r = 1 - \gamma$
`q`	One mines the vector correlation. That is, $q = 1 - \theta$

References

Hooper J. (1959). Simultaneous Equations and Canonical Correlation Theory. *Econometrica* 27, 245-256.

Hotelling H. (1936). Relations Between Two Sets of Variates. *Biometrika* 28, 321-377.

`invFM` *Fourier transform approach on inverse dimention reduction method.*

Description

invFM() estimates the basis vector for the central subspace in regression.

Usage

```
invFM(x,y,d,w,x_scale = TRUE)
```

Arguments

`x`	The design matrix of the predictor with dimension n-by-p.
`y`	The n-dimensional response vector.
`d`	An integer specifying the dimension of the sufficient dimension reduction subspace.
`w`	A vector of choices for *w*.
`x_scale`	(default TRUE). It scales the predictor variables.

Details

Let $(\mathbf{y}_i, \mathbf{x}_i), i = 1, \cdots, n$, be a random sample, and assume that the dimension of $S_{E(\mathbf{Z}|\mathbf{Y})}$ is known to be d. Then, for a random finite sequence of $\boldsymbol{\omega}_j \in R^p$, $j = 1, \cdots, t$ compute $\widehat{\boldsymbol{\psi}}(\boldsymbol{\omega}_j)$ as follows. For more details see Weng and Yin (2018).

$$\widehat{\boldsymbol{\psi}}(\boldsymbol{\omega}_j) = n^{-1} \sum_{k=1}^{n} \exp(i\boldsymbol{\omega}_j^T \mathbf{y}_k)\widehat{\mathbf{Z}}_k, j = 1, \cdots, t,$$

where $\widehat{\mathbf{Z}}_j = \boldsymbol{\Sigma}_x^{-1/2}(\mathbf{x}_i - \overline{\mathbf{x}})$. Now, let $\mathbf{a}(\boldsymbol{\omega}_j) = Real(\widehat{\boldsymbol{\psi}}(\boldsymbol{\omega}_j))$, and $\mathbf{b}(\boldsymbol{\omega}_j) = Image(\widehat{\boldsymbol{\psi}}(\boldsymbol{\omega}_j))$. Then, $\widehat{\boldsymbol{\Psi}} = (\mathbf{a}(\boldsymbol{\omega}_1), \mathbf{b}(\boldsymbol{\omega}_1), \cdots, \mathbf{a}(\boldsymbol{\omega}_t), \mathbf{b}(\boldsymbol{\omega}_t))$, for some $t > 0$, and the population kernel matrix is $\widehat{\mathbf{V}} = \widehat{\boldsymbol{\Psi}}\widehat{\boldsymbol{\Psi}}^T$. Finally, use the d-leading eigenvectors of $\widehat{\mathbf{V}}$ as an estimate for the central subspace.

Remark: We use *w* instead of $\boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_t$ in the *invFM()* function.

Value

invFM returns the following objects

`beta`	The estimated p by d matrix, whose columns forme a basis of the central subspace.
`eigenvalue`	Eigenvalues of $\widehat{\boldsymbol{V}}$.
`psi`	Estimation for $\widehat{\boldsymbol{\Psi}}$.

References

Weng J. and Yin X. (2018). Fourier Transform Approach for Inverse Dimension Reduction Method. *Journal of Nonparametric Statistics*. 30, 4, 1029-0311.

Examples

```
library(itdr)
library(stats)
data(PDB)
colnames(PDB)=NULL
p=15
df=PDB[,c(79,73,77,103,112,115,124,130,132,145,149,151,153,155,167,169)]
dff=as.matrix(df)
planingdb=dff[complete.cases(dff),]
y=planingdb[,1]
x=planingdb[,c(2:(p+1))]
x=x+0.5
xt=cbind(x[,1]^(.33),x[,2]^(.33),x[,3]^(.57),x[,4]^(.33),
x[,5]^(.4),x[,6]^(.5),x[,7]^(.33),x[,8]^(.16),x[,9]^(.27),x[,10]^(.5),
x[,11]^(.5),x[,12]^(.33),x[,13]^(.06),x[,14]^(.15),x[,15]^(.1))
W=sapply(50,rnorm)
d=1
betahat <-invFM(xt,y,d,W,FALSE)$beta
betahat
```

`itdr` *Integral transformation Methods of Estimating Sufficient Dimension Reduction Subspaces in Regression.*

Description

itdr() function computes a basis for sufficient dimension reduction subspaces in regression.

Usage

```
itdr(y,x,d,wx=0.1,wy=1,wh=1.5,space="mean",xdensity="normal",method="FM")
```

Arguments

y	The n-dimensional response vector.
x	The design matrix of the predictors with dimension n-by-p.
d	An integer specifying the dimension of the sufficient dimension reduction subspace.
wx	(default 0.1). The tuning parameter for the predictor variables.
wy	(default 1). The tuning parameter for the response variable.
wh	(default 1.5). The bandwidth of the kernel density estimation function.
space	(default “mean”). The default is “mean” for the central mean subspace. Other option is “pdf” for estimating the central subspace.
xdensity	(default “normal”). Density function of the predictor variables. If “normal” then predictor variables are coming from a multivariate normal distribution. If “elliptic” then predictors are coming from an elliptical contoured distribution. If the distribution of the predictor variables is unknown, then use “kernel” to estimate the distribution function using a kernel smoothing method.
method	(default “FM”). The integral transformation method. “FM” is for the Fourier trans-formation method (Zhu and Zeng 2006), “CM” for convolution transformation method (Zeng and Zhu 2010), and “iht” for the iterative Hessian transformation method (Cook and Li 2002).

Details

Let m(**x**)=E[y|**X=x**]. Then, integral transformation of gradient of the mean function m(**x**) is defined as

$$\boldsymbol{\psi}(\boldsymbol{\omega}) = \int \frac{\partial}{\partial \mathbf{x}} m(\mathbf{x}) W(\mathbf{x}, \boldsymbol{\omega}) f(\mathbf{x}) d\mathbf{x},$$

where $W(\mathbf{x}, \boldsymbol{\omega})$ is said to be a non degenerate kernel function. Set $W(\mathbf{x}, \boldsymbol{\omega}) = \exp(i\boldsymbol{\omega}^T\mathbf{x})$ for Fourier transformation (FM) method and $W(\mathbf{x}, \boldsymbol{\omega}) = H(\mathbf{x}-\boldsymbol{\omega}) = (2\pi\sigma_w^2)^{-p/2}\exp(-(\mathbf{x}-\boldsymbol{\omega})^T(\mathbf{x}-\boldsymbol{\omega})/(2\sigma_w^2))$ for convolution transformation (CM) method where $W(\mathbf{x}, \boldsymbol{\omega})$ is an absolutely integrable function. The candidate matrix to estimate the central mean subspace (CMS),

$$\mathbf{M}_{CMS} = \int \boldsymbol{\psi}(\boldsymbol{\omega}) \boldsymbol{\psi}(\boldsymbol{\omega})^T K(\boldsymbol{\omega}) d\boldsymbol{\omega},$$

where $K(\boldsymbol{\omega}) = (2\pi\sigma_w^2)^{-p/2} \exp\left(-||\boldsymbol{\omega}||/2\sigma_w^2\right)$ under 'FM', and $K(\boldsymbol{\omega}) = 1$ under 'CM'. Here, σ_w^2 is a tuning parameter and it refers as "tuning parameter for the predictor variables" and denoted by 'wx' in all functions.

Let $\{T_v(y) = H(y, v),\ for\ \ y, v \in \mathcal{R}\}$ be the family of transformations for the response variable. That is, $v \in \mathcal{R}$, the mean response of $T_v(y)$ is $m(\boldsymbol{\omega}, v) = E[H(y, v)|\mathbf{X} = \mathbf{x}]$. Then, integral transformation for the gradient of $m(\boldsymbol{\omega}, v)$ is defined as

$$\boldsymbol{\psi}(\boldsymbol{\omega}, v) = \int \frac{\partial}{\partial \mathbf{x}} m(\mathbf{x}, v) W(\mathbf{x}, \boldsymbol{\omega}) f(\mathbf{x}) d\mathbf{x},$$

where $W(\mathbf{x}, \boldsymbol{\omega})$ is the define as above. Then, the candidate matrix for the central subspace (CS) is defined as

$$\mathbf{M}_{CS} = \int H(y_1, v) H(y_2, v) dv \int \boldsymbol{\psi}(\boldsymbol{\omega}) \bar{\boldsymbol{\psi}}(\boldsymbol{\omega})^T K(\boldsymbol{\omega}) d\boldsymbol{\omega},$$

where $K(\boldsymbol{\omega})$ is the same as above, and $H(y, v) = (2\pi\sigma_t^2)^{-1/2} \exp(v^2/(2\sigma_t^2))$ under 'FM', and $H(y, v) = (2\pi\sigma_t^2)^{-1/2} \exp((y - v)^2/(2\sigma_t^2))$ under 'CM'. Here σ_t^2 is a tuning parameter and it refers as the "tuning parameter for the response variable" and is denote by 'wy' in all functions.

Remark: There is only one tuning parameter in the candidate matrix for the estimate of the CMS, and there are two tuning parameters in the candidate matrix for the estimate of the CS.

Value

The outputs are a p-by-d matrix and a p-by-p matrix defined as follows.

`eta_hat`	The estimated p by d matrix, whose coloumns form a basis of the CMS/CS.
`M`	The estimated p by p candidate matrix.

References

Cook R. D., and Li, B., (2002). Dimension Reduction for Conditional Mean in Regression. *The Annals of Statistics*. 30, 455-474.

Zeng P. and Zhu Y. (2010). An Integral Transform Method for Estimating the Central Mean and Central Subspaces. *Journal of Multivariate Analysis*. 101, 1, 271–290.

Zhu Y. and Zeng P. (2006). Fourier Methods for Estimating the Central Subspace and Central Mean Subspace in Regression. *Journal of the American Statistical Association*. 101, 476, 1638–1651.

Examples

```
library(itdr)
data(automobile)
head(automobile)
automobile.na=na.omit(automobile)
wx=.14; wy=.9;wh=1.5;d=2;p=13
df=cbind(automobile[,c(26,10,11,12,13,14,17,19,20,21,22,23,24,25)])
dff=as.matrix(df)
automobi=dff[complete.cases(dff),]
y=automobi[,1]
x=automobi[,c(2:14)]
xt=scale(x)
```

```
fit.F_CMS=itdr(y,xt,d,wx,wy,wh,space="pdf",xdensity = "normal",method="FM")
round(fit.F_CMS$eta_hat,2)
```

PDB *Planning database published in year 2015.*

Description

Planning Database (PDB) contains selected 2010 Census and selected 2009-2013, 5-years American Community Survey (ACS) estimates.

Usage

```
data(PDB)
```

Format

A dataset with 816 observations and 344 attributes.

Source

https://www.census.gov/data/datasets/2015/adrm/research/2015-planning-database.html

---

Recumbent *Recumbent cows*

Description

For unknown reasons, pregnant dairy cows can become recumbent-they lie down-either shortly before or after calving. This condition can be serious, and frequently leads to death of the cow. Clark, Henderson, Hoggard, Ellison and Young (1987) analyze data collected at the Ruakura (N.Z.) Animal Health Laboratory on a sample of recumbent cows.

Usage

```
data(Recumbent)
```

Format

A dataset with 9 columns and 435 rows.

Source

Clark, R. G., Henderson, H. V., Hoggard, G. K. Ellison, R. S. and Young, B. J. (1987). The abiltiy of biochemical and haematolgical tests to predict recovery in periparturient recumbent cows. NZ Veterinary Journal, 35, 126-133.

wh — *Bootstrap estimation for the bandwidth of the Gaussian kernel density estimation.*

Description

wh() estimates the bandwidth of the Gaussian kernel density estimation function if the distribution of the predictor variables is unknown.

Usage

```
wh(y,x,d,wx=0.1,wy=1,wh_seq=seq(0.1,3,by=.1),B=500,space="mean",
                                        method="FM")
```

Arguments

`y`	The n-dimensional response vector.
`x`	The design matrix of the predictors with dimension n-by-p.
`d`	An integer specifying the dimension of the sufficient dimension reduction subspace.
`wx`	(default 0.1). The tuning parameter for the predictor variables.
`wy`	(default 1). The tuning parameter for the response variable.
`wh_seq`	(default 0.1,0.2,...,3). A sequence of candidate bandwidth for the kernel smoothing method.
`B`	(default 500). Number of bootstrap samples.
`space`	(default “mean”). The defalult is “mean” for the central mean subspace. Other option is “pdf” for estimating the central subspace.
`method`	(default “FM”). The integral transformation method. “FM”for Fourier transformation method (Zhu and Zeng 2006), and “CM”for convolution transformation method (Zeng and Zhu 2010).

Details

The kernel density estimation of $f_{\mathbf{X}}(\mathbf{x})$ at a fixed point $\mathbf{x}_0$ is defined as

$$\widehat{f}_{\mathbf{x}_0}(\mathbf{x}_0) = (nh^p)^{-1} \sum_{\ell=1}^{n} G\left(\frac{\mathbf{x}_0 - \mathbf{x}_\ell}{h}\right),$$

where $G(\cdot)$ is a Gaussian kernel function and ‘h’ is the bandwidth of the kernel function. We denote this parameter as ‘wh‘ in all functions.

Value

The outputs are a table of average bootstrap distances between two subspaces for each candidate value of the bandwidth and the estimated value for the bandwith.

`dis_h`	A table of average boostrap distances for each candidate value of the bandwidth.
`h.hat`	The estimated bandwidth parameter for the Gaussian kernel function.

References

Zeng P. and Zhu Y. (2010). An Integral Transform Method for Estimating the Central Mean and Central Subspaces. *Journal of Multivariate Analysis*. 101, 1, 271–290.

Zhu Y. and Zeng P. (2006). Fourier Methods for Estimating the Central Subspace and Central Mean Subspace in Regression. *Journal of the American Statistical Association*. 101, 476, 1638–1651.

`wx` *Bootstrap estimation for the tuning parameter for the predictor variables.*

Description

wx() estimates the turning parameter for the predictors which required in both 'FM' and 'CM' methods.

Usage

```
wx(y,x,d,wx_seq=seq(0.1,5,by=.1),wy=1,wh=1.5,B=500,space="mean",
                                                xdensity="normal",method="FM")
```

Arguments

y	The n-dimensional response vector.
x	The design matrix of the predictors with dimension n-by-p.
d	An integer specifying the dimension of the sufficient dimension reduction subspace.
wx_seq	(default 0.1,0.2,...,0.5). A sequence of candidiate tuning parameter for the predictor variables.
wy	(default 1). The tuning parameter for the response variable.
wh	(default 1.5). The bandwidth of the kernel density estimation function.
B	(default 500). Number of boostrap samples.
space	(default "mean"). The defalult is "mean" for the central mean subspace. Other option is "pdf" for estimating the central subspace.
xdensity	(default "normal"). Density function of the predictor variables. If "normal" then predictor variables are coming from a multivariate normal distribution. If "elliptic" then predictors are coming from an elliptical contoured distribution. If the distribution of the predictor variables is unknown, then use "kernel" to estimate the distribution function using kernel smoothing method.
method	(default "FM"). The integral transformation method. "FM" for Fourier transformation method (Zhu and Zeng 2006), and "CM" for convolution transformation method (Zeng and Zhu 2010).

Value

The outputs are a table of average boostrap distances between two subspaces for each candidate value of wx and estimated value of wx.

`dis_wx`	A table of average boostrap distances for each candidate value of wx.
`wx.hat`	The estimated value for tunning parameter σ_w^2.

References

Zeng P. and Zhu Y. (2010). An Integral Transform Method for Estimating the Central Mean and Central Subspaces. *Journal of Multivariate Analysis*. 101, 1, 271–290.

Zhu Y. and Zeng P. (2006). Fourier Methods for Estimating the Central Subspace and Central Mean Subspace in Regression. *Journal of the American Statistical Association*. 101, 476, 1638–1651.

`wy` *Bootstrap estimation for the tuning parameter of the response variable.*

Description

wy() estimates the turning parameter for the response variable which required in ‘FM‘ and ‘CM‘ methods only when estimating the central subspace.

Usage

```
wy(y,x,d,wx=0.1,wy_seq=seq(0.1,1,by=0.1),wh=1.5,B=500,
                                   xdensity="normal",method="FM")
```

Arguments

`y`	The n-dimensional response vector.
`x`	The design matrix of the predictors with dimension n-by-p.
`d`	An integer specifying the dimension of the sufficient dimension reduction subspace.
`wx`	(default 0.1). The tuning parameter for the predictor variables.
`wy_seq`	(default 0.1,0.2,...,1). A sequence of the candidate tuning parameter for the response.
`wh`	(default 1.5). The bandwidth of the kernel density estimation function.
`B`	(default 500). Number of bootstrap samples.
`xdensity`	(default “normal”). Density function of the predictor variables. If “normal” then predictor variables are coming from a multivariate normal distribution function. If “elliptic” then predictors are coming from an elliptical contoured distribution function. If the distribution of the predictor variables is unknown, then use “kernel“ to estimate the distribution function using kernel smoothing method.
`method`	(default “FM”). The integral transformation method. “FM” for Fourier transformation method (Zhu and Zeng 2006), and “CM” for convolution transformation method (Zeng and Zhu 2010).

Value

The outputs are a table of average bootstrap distances between two subspaceses for each candidate value of wy and estimate value for wy.

`dis_wy`	A table of average bootstrap distances for each candidate value of wy.
`wy.hat`	The estimated value for tunning parameter σ_t^2.

References

Zeng P. and Zhu Y. (2010). An Integral Transform Method for Estimating the Central Mean and Central Subspaces. *Journal of Multivariate Analysis*. 101, 1, 271–290.

Zhu Y. and Zeng P. (2006). Fourier Methods for Estimating the Central Subspace and Central Mean Subspace in Regression. *Journal of the American Statistical Association*. 101, 476, 1638–1651.

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