

# Package ‘kantorovich’

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**Type** Package

**Title** Kantorovich Distance Between Probability Measures

**Version** 3.0.1

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**Description** Computes the Kantorovich distance between two probability measures on a finite set. The Kantorovich distance is also known as the Monge-Kantorovich distance or the first Wasserstein distance.

**License** GPL-3

**Suggests** testthat, knitr, rmarkdown

**Imports** rcdd, gmp, methods, lpSolve, Rglpk, slam, CVXR

**Depends** R (>= 2.5.3)

**SystemRequirements** GMP (<https://gmplib.org/>)

**RoxxygenNote** 7.1.1

**Encoding** UTF-8

**VignetteBuilder** knitr

**URL** <https://github.com/stla/kantorovich>

**BugReports** <https://github.com/stla/kantorovich/issues>

**NeedsCompilation** no

**Repository** CRAN

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kantorovich-package    *Kantorovich Distance Between Probability Measures*

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**Description**

Computes the Kantorovich distance between two probability measures on a finite set.

To learn more, start with the vignettes: `browseVignettes(package="kantorovich")`.

If you encounter a bug, or if you have a suggestion to improve the package, please file an issue on the github repo <https://github.com/stla/kantorovich>.

**Details**

Package:	<code>kantorovich</code>
Type:	Package
Version:	2.0.0
Date:	2016-05-25
License:	GPL-2

**Author(s)**

Stéphane Laurent

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*edistances*                  *Extremal distances*

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**Description**

Compute the distances at the extreme joinings.

**Usage**

`edistances(mu, nu, dist = NULL, ...)`

**Arguments**

<code>mu</code>	(row margins) probability measure in numeric or bigq/character mode
<code>nu</code>	(column margins) probability measure in numeric or bigq/character mode
<code>dist</code>	function or matrix, the distance to be minimized on average. If <code>NULL</code> , the 0-1 distance is used.
<code>...</code>	arguments passed to <code>dist</code>

**Value**

A list with two components: the extreme joinings in a list and the distances in a vector.

**Note**

This function, called by `kantorovich`, is rather for internal purpose.

ejoinings

*Extreme joinings***Description**

Return extreme joinings between `mu` and `nu`.

**Usage**

```
ejoinings(mu, nu, zeros = FALSE)
```

**Arguments**

<code>mu</code>	(row margins) probability measure in numeric or bigq/character mode
<code>nu</code>	(column margins) probability measure in numeric or bigq/character mode
<code>zeros</code>	logical; in case when <code>mu</code> and <code>nu</code> have different lengths, set <code>FALSE</code> to remove lines or columns full of zeros

**Value**

A list containing the extreme joinings (matrices).

**Examples**

```
mu <- nu <- c(0.5, 0.5)
ejoinings(mu, nu)
# use exact arithmetic
library(gmp)
mu <- nu <- as.bigq(c(0.5,0.5))
ejoinings(mu, nu)
# different lengths example
mu <- setNames(as.bigq(c(1,2,4), 7), c("a", "b", "c"))
```

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```
nu <- setNames(as.bigq(c(3,1), 4), c("b", "c"))
ejoinings(mu, nu)
```

---

**kantorovich***Kantorovich distance***Description**

Compute the Kantorovich distance between two probability measures on a finite set.

**Usage**

```
kantorovich(mu, nu, dist = NULL, details = FALSE, ...)
```

**Arguments**

<code>mu</code>	(row margins) probability measure in numeric or bigq/character mode
<code>nu</code>	(column margins) probability measure in numeric or bigq/character mode
<code>dist</code>	function or matrix, the distance to be minimized on average; if <code>NULL</code> , the 0-1 distance is used.
<code>details</code>	prints the joinings achieving the Kantorovich distance and returns them in the "joinings" attribute of the output
<code>...</code>	arguments passed to <code>dist</code> (only if it is a function)

**Details**

The function firstly computes all the extreme joinings of `mu` and `nu`, then evaluates the average distance for each of them, and then returns the minimal one.

**Value**

The Kantorovich distance between `mu` and `nu`.

**Examples**

```
mu <- c(1/7, 2/7, 4/7)
nu <- c(1/4, 1/4, 1/2)
kantorovich(mu, nu)
library(gmp)
mu <- as.bigq(c(1,2,4), 7)
nu <- as.bigq(c(1,1,1), c(4,4,2))
kantorovich(mu, nu)
mu <- c("1/7", "2/7", "4/7")
nu <- c("1/4", "1/4", "1/2")
kantorovich(mu, nu, details=TRUE)
```

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kantorovich_CVX	<i>Computes Kantorovich distance with CVX</i>
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## Description

Kantorovich distance using the CVXR package

## Usage

```
kantorovich_CVX(
  mu,
  nu,
  dist = NULL,
  solution = FALSE,
  stop_if_fail = TRUE,
  solver = "ECOS",
  ...
)
```

## Arguments

<code>mu</code>	(row margins) probability measure in numeric mode
<code>nu</code>	(column margins) probability measure in numeric mode
<code>dist</code>	matrix, the distance to be minimized on average; if <code>NULL</code> , the 0-1 distance is used.
<code>solution</code>	logical; if <code>TRUE</code> the solution is returned in the <code>"solution"</code> attributes of the output
<code>stop_if_fail</code>	logical; if <code>TRUE</code> , an error is returned in the case when no solution is found; if <code>FALSE</code> , the output of <code>psolve</code> is returned with a warning
<code>solver</code>	the CVX solver, passed to <code>psolve</code>
<code>...</code>	other arguments passed to <code>psolve</code>

## Examples

```
mu <- c(1/7, 2/7, 4/7)
nu <- c(1/4, 1/4, 1/2)
kantorovich_CVX(mu, nu)
```

`kantorovich_glpk`      *Computes Kantorovich distance with GLPK*

## Description

Kantorovich distance using the Rglpk package

## Usage

```
kantorovich_glpk(
  mu,
  nu,
  dist = NULL,
  solution = FALSE,
  stop_if_fail = TRUE,
  ...
)
```

## Arguments

<code>mu</code>	(row margins) probability measure in numeric mode
<code>nu</code>	(column margins) probability measure in numeric mode
<code>dist</code>	matrix, the distance to be minimized on average; if <code>NULL</code> , the 0-1 distance is used.
<code>solution</code>	logical; if <code>TRUE</code> the solution is returned in the "solution" attributes of the output
<code>stop_if_fail</code>	logical; if <code>TRUE</code> , an error is returned in the case when no solution is found; if <code>FALSE</code> , the output of <code>Rglpk_solve_LP</code> is returned with a warning
<code>...</code>	arguments passed to <code>Rglpk_solve_LP</code>

## Examples

```
mu <- c(1/7,2/7,4/7)
nu <- c(1/4,1/4,1/2)
kantorovich_glpk(mu, nu)
```

<code>kantorovich_lp</code>	<i>Computes Kantorovich distance with lp_solve</i>
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### Description

Kantorovich distance using the lpSolve package

### Usage

```
kantorovich_lp(mu, nu, dist = NULL, solution = FALSE, lp.object = FALSE, ...)
```

### Arguments

<code>mu</code>	(row margins) probability measure in numeric mode
<code>nu</code>	(column margins) probability measure in numeric mode
<code>dist</code>	matrix, the distance to be minimized on average; if NULL, the 0-1 distance is used.
<code>solution</code>	logical, to use only if <code>lp.object=FALSE</code> ; if TRUE the solution is returned in the "solution" attributes of the output
<code>lp.object</code>	logical, if FALSE, the output is the Kantorovich distance; if TRUE, the output is a <a href="#">lp.object</a>
...	arguments passed to <a href="#">lp</a>

### Examples

```
mu <- c(1/7,2/7,4/7)
nu <- c(1/4,1/4,1/2)
kantorovich_lp(mu, nu)
```

<code>names.bigq</code>	<i>Names for bigq vectors</i>
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### Description

Names for bigq vectors

### Usage

```
## S3 method for class 'bigq'
names(x)
```

### Arguments

<code>x</code>	a bigq vector
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**Value**

the names of x

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