Package 'mggd'

Title Multivariate Generalised Gaussian Distribution; Kullback-Leibler
Divergence

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Description Distance between multivariate generalised Gaussian distributions, as pre-
sented by N. Bouhlel and A. Dziri (2019) <doi:10.1109/LSP.2019.2915000>. Manipula-
tion of multivariate generalised Gaussian distributions (methods presented by Gomez, Gomez-
Villegas and Marin (1998) <doi:10.1080/03610929808832115> and Pascal, Bom-
brun, Tourneret and Berthoumieu (2013) <doi:10.1109/TSP.2013.2282909>).

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mggd-package   *Tools for Multivariate Generalized Gaussian Distributions*

Description

This package provides tools for multivariate generalized Gaussian distributions (MGGD):

- Calculation of distances/divergences between multivariate generalized Gaussian distributions:
  - Kullback-Leibler divergence: `kldiv`
- Tools for MGGD:
  - Probability density: `mvdggd`
  - Estimation of the parameters: `estparmvggd`
  - Simulation from a MGGD: `mvrggd`
  - Plot of the density of a MGGD with 2 variables: `plotmvggd`, `contourmvggd`

Author(s)

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References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

F. Pascal, L. Bombrun, J.Y. Tourneret, Y. Berthoumieu. Parameter Estimation For Multivariate Generalized Gaussian Distribution. IEEE Trans. Signal Processing, vol. 61 no. 23, p. 5960-5971, Dec. 2013. doi:10.1109/TSP.2013.2282909

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contourmvggd | *Contour Plot of the Bivariate Generalised Gaussian Density*

Description

Draws the contour plot of the probability density of the generalised Gaussian distribution with 2 variables with mean vector mu, dispersion matrix Sigma and shape parameter beta.

Usage

```
contourmvggd(mu, Sigma, beta,
                xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]),
                zlim = NULL, npt = 30, nx = npt, ny = npt,
                main = "Multivariate generalised Gaussian density",
                sub = NULL, nlevels = 10,
                levels = pretty(zlim, nlevels), tol = 1e-6, ...)
```

Arguments

mu	length 2 numeric vector.
Sigma	symmetric, positive-definite square matrix of order 2. The dispersion matrix.
beta	positive real number. The shape of the first distribution.
xlim, ylim	x-and y- limits.
zlim	z- limits. If NULL, it is the range of the values of the density on the x and y values within xlim and ylim.
npt	number of points for the discretisation.
nx, ny	number of points for the discretisation among the x- and y- axes.
main, sub	main and sub title, as for [title]().
nlevels, levels	
	arguments to be passed to the [contour]() function.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. see [mvdggd]().
...	additional arguments to [plot.window](), [title](), [Axis]() and [box](), typically [graphical parameters]() such as cex.axis.

Value

Returns invisibly the probability density function.

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

plotmvggd: plot of a bivariate generalised Gaussian density.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
beta <- 0.74
contourmvggd(mu, Sigma, beta)
```

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estparmvggd	*Estimation of the Parameters of a Multivariate Generalized Gaussian Distribution*

Description

Estimation of the mean vector, dispersion matrix and shape parameter of a multivariate generalized Gaussian distribution (MGGD).

Usage

```
estparmvggd(x, eps = 1e-6, display = FALSE, plot = display)
```

Arguments

x	numeric matrix or data frame.
eps	numeric. Precision for the estimation of the beta parameter.
display	logical. When TRUE the value of the beta parameter at each iteration is printed.
plot	logical. When TRUE the successive values of the beta parameter are plotted, allowing to visualise its convergence.

Details

The μ parameter is the mean vector of x.

The dispersion matrix Σ and shape parameter: β are computed using the method presented in Pascal et al., using an iterative algorithm.

The precision for the estimation of beta is given by the eps parameter.

Value

A list of 3 elements:

- mu the mean vector.
- Sigma: symmetric positive-definite matrix. The dispersion matrix.
- beta non-negative numeric value. The shape parameter.

with two attributes attr(, "epsilon") (precision of the result) and attr(, "k") (number of iterations).

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

F. Pascal, L. Bombrun, J.Y. Tourneret, Y. Berthoumieu. Parameter Estimation For Multivariate Generalized Gaussian Distribution. IEEE Trans. Signal Processing, vol. 61 no. 23, p. 5960-5971, Dec. 2013. doi: 10.1109/TSP.2013.2282909

See Also

mvdggd: probability density of a MGGD.

mvrggd: random generation from a MGGD.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
x <- mvrggd(100, mu, Sigma, beta)

# Esimation of the parameters
estparmvggd(x)
```

---

kldiv	*Kullback-Leibler Divergence between Centered Multivariate generalized Gaussian Distributions*

Description

Computes the Kullback- Leibler divergence between two random variables distributed according to multivariate generalized Gaussian distributions (MGGD) with zero means.

Usage

```
kldiv(sigma1, beta1, sigma2, beta2, eps = 1e-06)
```

Arguments

sigma1	symmetric, positive-definite matrix. The dispersion matrix of the first distribution.
beta1	positive real number. The shape parameter of the first distribution.
sigma2	symmetric, positive-definite matrix. The dispersion matrix of the second distribution.
beta2	positive real number. The shape parameter of the second distribution.
eps	numeric. Precision for the computation of the Lauricella function (see [lauricella]). Default: 1e-06.

Details

Given X_1, a random vector of R^p distributed according to the MGGD with parameters $(0, \Sigma_1, \beta_1)$ and X_2, a random vector of R^p distributed according to the MGGD with parameters $(0, \Sigma_2, \beta_2)$.

The Kullback-Leibler divergence between X_1 and X_2 is given by:

$$KL(X_1||X_2) = ln\left(\frac{\beta_1|\Sigma_1|^{-1/2}\Gamma\left(\frac{p}{2\beta_2}\right)}{\beta_2|\Sigma_2|^{-1/2}\Gamma\left(\frac{p}{2\beta_1}\right)}\right) + \frac{p}{2}\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)ln2 - \frac{p}{2\beta_2} + 2^{\frac{\beta_2}{\beta_1}-1}\frac{\Gamma\left(\frac{\beta_2}{\beta_1}+\frac{p}{\beta_1}\right)}{\Gamma\left(\frac{p}{2\beta_1}\right)}\lambda_p^{\beta_2}$$

$$\times F_D^{(p-1)}\left(-\beta_1; \frac{1}{2}, \ldots, \frac{1}{2}; \frac{p}{2}; 1 - \frac{\lambda_{p-1}}{\lambda_p}, \ldots, 1 - \frac{\lambda_1}{\lambda_p}\right)$$

where $\lambda_1 < \ldots < \lambda_{p-1} < \lambda_p$ are the eigenvalues of the matrix $\Sigma_1\Sigma_2^{-1}$.

This computation uses the [lauricella] function.

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the result of the Lauricella function) and `attr(, "k")` (number of iterations).

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

Examples

```
beta1 <- 0.74
beta2 <- 0.55
Sigma1 <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.2, 0.3, 0.5, 0.1, 0.2, 0.1, 0.7), nrow = 3)

# Kullback-Leibler divergence
kl12 <- kldiv(Sigma1, beta1, Sigma2, beta2)
kl21 <- kldiv(Sigma2, beta2, Sigma1, beta1)
```

```
print(kl12)
print(kl21)

# Distance (symmetrized Kullback-Leibler divergence)
kldist <- as.numeric(kl12) + as.numeric(kl21)
print(kldist)
```

---

lauricella   *Lauricella Function*

Description

Computes the Lauricella function.

Usage

```
lauricella(a, b, g, x, eps = 1e-06)
```

Arguments

a	numeric.
b	numeric vector.
g	numeric.
x	numeric vector. x must have the same length as b.
eps	numeric. Precision for the nested sums (default 1e-06).

Details

If n is the length of the b and x vectors, the Lauricella function is given by:

$$F_D^{(n)}\left(a; b_1, ..., b_n; g; x_1, ..., x_n\right) = \sum_{m_1 \geq 0} \cdots \sum_{m_n \geq 0} \frac{(a)_{m_1 + ... + m_n} (b_1)_{m_1} ... (b_n)_{m_n}}{(g)_{m_1 + ... + m_n}} \frac{x_1^{m_1}}{m_1!} \cdots \frac{x_1^{m_n}}{m_n!}$$

where $(x)_p$ is the Pochhammer symbol (see pochhammer).

If $|x_i| < 1, i = 1, \ldots, n$, this sum converges. Otherwise there is an error.

The eps argument gives the required precision for its computation. It is the attr(, "epsilon") attribute of the returned value.

Sometimes, the convergence is too slow and the required precision cannot be reached. If this happens, the attr(, "epsilon") attribute is the precision that was really reached.

Value

A numeric value: the value of the Lauricella function, with two attributes attr(, "epsilon") (precision of the result) and attr(, "k") (number of iterations).

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

N. Bouhlel, A. Dziri, Jullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

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mvdggd	*Density of a Multivariate Generalized Gaussian Distribution*

Description

Density of the multivariate (p variables) generalized Gaussian distribution (MGGD) with mean vector mu, dispersion matrix Sigma and shape parameter beta.

Usage

```
mvdggd(x, mu, Sigma, beta, tol = 1e-6)
```

Arguments

x	length p numeric vector.
mu	length p numeric vector. The mean vector.
Sigma	symmetric, positive-definite square matrix of order p. The dispersion matrix.
beta	positive real number. The shape of the distribution.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate generalized Gaussian distribution is given by:

$$f(x|\mu, \Sigma, \beta) = \frac{\Gamma\left(\frac{p}{2}\right)}{\pi^{\frac{p}{2}}\Gamma\left(\frac{p}{2\beta}\right)2^{\frac{p}{2\beta}}}\frac{\beta}{|\Sigma|^{\frac{1}{2}}}e^{-\frac{1}{2}\left((x-\mu)^T\Sigma^{-1}(x-\mu)\right)^{\beta}}$$

Value

The value of the density.

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

mvrggd: random generation from a MGGD.

estparmvggd: estimation of the parameters of a MGGD.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
mvdggd(c(0, 1, 4), mu, Sigma, beta)
mvdggd(c(1, 2, 3), mu, Sigma, beta)
```

---

mvrggd   *Simulate from a Multivariate Generalized Gaussian Distribution*

Description

Produces one or more samples from a multivariate (p variables) generalized Gaussian distribution (MGGD).

Usage

```
mvrggd(n = 1 , mu, Sigma, beta, tol = 1e-6)
```

Arguments

n	integer. Number of observations.
mu	length p numeric vector. The mean vector.
Sigma	symmetric, positive-definite square matrix of order p. The dispersion matrix.
beta	positive real number. The shape of the distribution.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

A sample from a centered MGGD with dispersion matrix Σ and shape parameter β can be generated using:

$X = \tau \Sigma^{1/2} u$

where u is a random vector uniformly distributed on the unit sphere and τ is such that $\tau^{2\beta}$ is generated from a distribution $\Gamma(\frac{p}{2\beta}, 2)$.

This property is used to generate a sample from a MGGD.

Value

A matrix with p columns and n rows.

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

mvdggd: probability density of a MGGD..

estparmvggd: estimation of the parameters of a MGGD.

Examples

```
mu <- c(0, 0, 0)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
mvrggd(100, mu, Sigma, beta)
```

---

plotmvggd   *Plot of the Bivariate Generalised Gaussian Density*

Description

Plots the probability density of the generalised Gaussian distribution with 2 variables with mean vector mu, dispersion matrix Sigma and shape parameter beta.

Usage

```
plotmvggd(mu, Sigma, beta, xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                  ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]), n = 101,
                  xvals = NULL, yvals = NULL, xlab = "x", ylab = "y",
                  zlab = "f(x,y)", col = "gray", tol = 1e-6, ...)
```

Arguments

mu	length 2 numeric vector.
Sigma	symmetric, positive-definite square matrix of order 2. The dispersion matrix.
beta	positive real number. The shape of the first distribution.
xlim, ylim	x-and y- limits.
n	A one or two element vector giving the number of steps in the x and y grid, passed to plot3d.function.
xvals, yvals	The values at which to evaluate x and y. If used, xlim and/or ylim are ignored.

xlab, ylab, zlab

> The axis labels.

col   The color to use for the plot. See `plot3d.function`.

tol   tolerance (relative to largest variance) for numerical lack of positive-definiteness
                in Sigma, for the estimation of the density. see `mvdggd`.

...   Additional arguments to pass to `plot3d.function`.

Value

Returns invisibly the probability density function.

Author(s)

Nizar Bouhlel, Pierre Santagostini

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

`contourmvggd`: contour plot of a bivariate generalised Gaussian density.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
beta <- 0.74
plotmvggd(mu, Sigma, beta)
```

---

pochhammer   *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

x   numeric.

n   positive integer.

Details

The Pochhammer symbol is given by: $(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Nizar Bouhlel, Pierre Santagostini

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

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