# Package 'pcsstools'

October 14, 2022

```
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Title Tools for Regression Using Pre-Computed Summary Statistics
Version 0.1.1
Description Defines functions to describe regression models using only
      pre-computed summary statistics (i.e. means, variances, and covariances)
      in place of individual participant data.
      Possible models include linear models for linear combinations, products,
      and logical combinations of phenotypes.
      Implements methods presented in
      Wolf et al. (2021) <doi:10.1101/2021.03.08.433979>
      Wolf et al. (2020) <doi:10.1142/9789811215636 0063> and
      Gasdaska et al. (2019) <doi:10.1142/9789813279827_0036>.
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       holder of modified 'stats' fragments)
Maintainer Jack Wolf < jackwolf 910@gmail.com>
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```

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anova.pcsslm

ANOVA for linear models fit using PCSS

### Description

Compute an analysis of variance table for one or more linear model fitted using PCSS.

## Usage

Index

```
## S3 method for class 'pcsslm'
anova(object, ...)
## S3 method for class 'pcsslmlist'
anova(object, ..., scale = 0, test = "F")
```

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### **Arguments**

object, ... objects of class pcsslm.

scale numeric. An estimate of the noise variance  $\sigma^2$ . If zero this will be estimated

from the largest model considered.

test a character string specifying the test statistic to be used. Can be one of "F",

"Chisq" or "Cp", with partial matching allowed, or NULL for no test.

#### Value

An object of class "anova" inheriting from class "data.frame".

#### Author(s)

R Core Team and contributors worldwide. Modified by Jack Wolf

approx\_and

Approximate a linear model for a series of logical AND statements

### Description

approx\_and approximates the linear model for the a conjunction of m phenotypes as a function of a set of predictors.

#### Usage

```
approx_and(
  means,
  covs,
  n,
  predictors,
  add_intercept = TRUE,
  verbose = FALSE,
  response_assumption = "binary",
  ...
)
```

### Arguments

means vector of predictor and response means with the last m means being the means

of m binary responses to combine in a logical and statement.

covs a matrix of the covariance of all model predictors and the responses with the

order of rows/columns corresponding to the order of means.

n sample size

predictors list of objects of class predictor corresponding to the order of the predictors in

means.

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add\_intercept logical. Should the linear model add an intercept term?

verbose should output be printed to console?

response\_assumption

character. Either "binary" or "continuous". If "binary", specific calcula-

tions will be done to estimate product means and variances.

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call

terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

approx\_conditional

Approximate the mean of Y conditional on X

#### **Description**

Approximate the mean of Y conditional on X

#### Usage

```
approx_conditional(means, covs, response, n)
```

approx\_mult\_prod 5

#### **Arguments**

means Vector of the mean of X and the mean of Y

covs Matrix of covariances for X and Y

response Character. If "binary" truncates means to interval [0, 1]. If "continuous" does

not restrict.

n Sample size

#### Value

A list of length 2 consisting of 2 functions that give the estimated conditional mean and conditional variance of Y as a function of X

approx\_mult\_prod Approximate ti

Approximate the covariance of a set of predictors and a product of responses

### Description

approx\_mult\_prod recursively estimates the covariances and means of a set of responses. Estimates are approximated using all unique response orderings and aggregated.

### Usage

```
approx_mult_prod(
  means,
  covs,
  n,
  response,
  predictors,
  responses,
  verbose = FALSE
)
```

### **Arguments**

means a vector of predictor and response means with all response means at the end of

the vector.

covs covariance matrix of all predictors and responses with column and row order

corresponding to the order of means.

n sample size (an integer).

response a string. Currently supports "binary" or "continuous".

predictors, responses

lists of objects of class predictor where each entry corresponds to one predic-

tor/response variable.

verbose logical.

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#### Value

A list containing the following elements:

means a vector of the (approximated) means of all predictors and the product of re-

sponses

covs a matrix of (approximated) covariances between all predictors and the product

of responses

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

approx\_or

Approximate a linear model for a series of logical OR statements

#### **Description**

approx\_or approximates the linear model for a disjunction of m phenotypes as a function of a set of predictors.

### Usage

```
approx_or(
  means,
  covs,
  n,
  predictors,
  add_intercept = TRUE,
  verbose = FALSE,
  response_assumption = "binary",
  ...
)
```

### **Arguments**

means vector of predictor and response means with the last m means being the means

of m binary responses to combine in a logical OR statement.

covs a matrix of the covariance of all model predictors and the responses with the

order of rows/columns corresponding to the order of means.

n sample size.

predictors list of objects of class predictor corresponding to the order of the predictors in

means.

add\_intercept logical. Should the linear model add an intercept term?

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verbose should output be printed to console?

response\_assumption

character. Either "binary" or "continuous". If "binary", specific calculations of the continuous of t

tions will be done to estimate product means and variances.

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call

terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'.

adj.r.squared the above  $\mathbb{R}^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

#### **Description**

Approximate summary statistics for a product of phenotypes and a set of predictors

#### Usage

```
approx_prod_stats(means, covs, n, response, predictors)
```

### **Arguments**

means Vector of means of predictors and the two phenotypes to be multiplied

covs Covariance matrix of all predictors and the two phenotypes

n Sample size

response character. Either "binary" or "continuous".

predictors a list of elements of class predictor

### Value

A list with the predicted covariance matrix of all predictors and the product and the means of all predictors and the product.

```
approx_response_cov_recursive
```

Approximate the covariance of one response with an arbitrary product of responses.

### **Description**

Approximate the covariance of one response with an arbitrary product of responses.

#### Usage

```
approx_response_cov_recursive(
  ids,
  r_covs,
  r_means,
  n,
  responses,
  response,
  verbose = FALSE
)
```

### **Arguments**

ids Column ids of responses to use. First is taken alone while 2nd to last are to be

multiplied

r\_covs Response covariance matrix r\_means Response means (vector)

n Sample size

responses List of lists with elements of class predictor response Character, Either "binary" or "continuous"

verbose logical

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#### Value

A vector with the approximated covariance, and approximated mean and variance of the product

calculate\_lm

Calculate a linear model using PCSS

### **Description**

calculate\_lm describes the linear model of the last listed variable in means and covs as a function of all other variables in means and covs.

### Usage

```
calculate_lm(
  means,
  covs,
  n,
  add_intercept = FALSE,
  keep_pcss = FALSE,
  terms = NULL
)
```

### **Arguments**

means a vector of means of all model predictors and the response with the last element

the response mean.

covs a matrix of the covariance of all model predictors and the response with the

order of rows/columns corresponding to the order of means.

n sample size

add\_intercept logical. If TRUE adds an intercept to the model. keep\_pcss logical. If TRUE, returns means and covs.

terms terms

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call

terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

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df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'.

adj.r.squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." *Pacific Symposium on Biocomputing*, **24**, 391–402. ISSN 2335-6928, doi: 10.1142/9789813279827\_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

calculate\_lm\_combo

Calculate a linear model for a linear combination of responses

#### **Description**

calculate\_lm\_combo describes the linear model for a linear combination of responses as a function of a set of predictors.

#### Usage

```
calculate_lm_combo(means, covs, n, phi, m = length(phi), add_intercept, ...)
```

#### **Arguments**

means	a vector of means of all model	predictors and the response	with the last m ele-

ments the response means (with order corresponding to the order of weights in

phi).

covs a matrix of the covariance of all model predictors and the responses with the

order of rows/columns corresponding to the order of means.

n sample size.

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phi vector of linear combination weights with one entry per response variable.

m number of responses to combine. Defaults to length(weighs).

add\_intercept logical. If TRUE adds an intercept to the model.

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call

terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'.

adj.r.squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

### References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." *Pacific Symposium on Biocomputing*, **24**, 391–402. ISSN 2335-6928, doi: 10.1142/9789813279827\_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

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check_terms Check that independent and dependent variables are accounted for through PCSS	check_terms	Check that independent and dependent variables are accounted for through PCSS
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### **Description**

Check that independent and dependent variables are accounted for through PCSS

### Usage

```
check_terms(xterms, yterms, pcssterms, pcsstype)
```

#### **Arguments**

xterms, yterms character vector of model's independent variables or variables combined to the

dependent variable

pcssterms character vector of variables with provided PCSS

pcsstype character describing the PCSS being checked. Either "means", "covs", "predictors",

or "responses".

### Value

No return value, called for side effects

## Description

Extract independent variables from a formula

#### Usage

```
extract_predictors(formula = formula())
```

### Arguments

formula an object of class formula.

#### Value

A list with a character vector of all predictors and a logical value indicating whether the model includes an intercept term.

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extract\_response

Extract dependent variables from a formula as a string

### Description

Extract dependent variables from a formula as a string

### Usage

```
extract_response(formula = formula())
```

## **Arguments**

formula

an object of class formula.

#### Value

a character vector of all responses

get\_pcor

Approximate the partial correlation of Y and Z given X

### Description

Approximate the partial correlation of Y and Z given X

### Usage

```
get_pcor(covs, cors = cov2cor(covs))
```

### **Arguments**

covs Covariance matrix of X, Y, and Z.

cors Correlation matrix of X, Y, and Z.

### Value

Approximated partial correlation of the later two terms given the first

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guess\_response

Guess the function that is applied to a set of responses

### **Description**

guess\_response takes a character vector of the dependent variable from a formula object and identifies which function separates the individual variables that make up the response. It then returns the model\_\* function to model the appropriate response using PCSS.

### Usage

```
guess_response(response = character())
```

### **Arguments**

response

character. Output of extract\_response.

#### Value

A character. Either "model\_combo", "model\_product", "model\_or", "model\_and", or "model\_singular".

make\_permutations

List all permutations of a sequence of integers

#### **Description**

Lists all permutations of 1,2,...,m unique up to the first two elements

### Usage

```
make_permutations(m)
```

### **Arguments**

m

number of elements to permute

#### Value

A list of vectors of permutations of 1,2,...,m.

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model_and	Approximate a linear model for a series of logical AND statements using PCSS

#### **Description**

model\_and approximates the linear model for the conjunction of m phenotypes as a function of a set of predictors.

#### Usage

```
model_and(formula, n, means, covs, predictors, ...)
```

### **Arguments**

formula an object of class formula whose dependent variable is a combination of vari-

ables and logical & operators. All model terms must be accounted for in means

and covs.

n sample size.

means named vector of predictor and response means.

covs named matrix of the covariance of all model predictors and the responses.

predictors named list of objects of class predictor.

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

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#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

### **Examples**

```
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
predictors <- list(
    g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
    x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1)))

model_and(
    y4 & y5 ~ g1 + x1,
    means = means, covs = covs, n = n, predictors = predictors
)
summary(lm(y4 & y5 ~ g1 + x1, data = ex_data))</pre>
```

model\_combo

Model a linear combination of a set of phenotypes using PCSS

### Description

model\_combo calculates the linear model for a linear combination of phenotypes as a function of a set of predictors.

#### Usage

```
model_combo(formula, phi, n, means, covs, ...)
```

## Arguments

formula	an object of class formula whose dependent variable is a series of variables joined by + operators. model_combo will treat a principal component score of those variables as the actual dependent variable. All model terms must be accounted for in means and covs.
phi	named vector of linear weights for each variable in the dependent variable in formula.
n	sample size.
means	named vector of predictor and response means.
covs	named matrix of the covariance of all model predictors and the responses.
	additional arguments

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#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." *Pacific Symposium on Biocomputing*, **24**, 391–402. ISSN 2335-6928, doi: 10.1142/9789813279827\_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

#### **Examples**

```
ex_data <- pcsstools_example[c("g1", "x1", "x2", "x3", "y1", "y2", "y3")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
phi <- c("y1" = 1, "y2" = -1, "y3" = 0.5)

model_combo(
    y1 + y2 + y3 ~ g1 + x1 + x2 + x3,
    phi = phi, n = n, means = means, covs = covs
)

summary(lm(y1 - y2 + 0.5 * y3 ~ g1 + x1 + x2 + x3, data = ex_data))</pre>
```

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model_or Approximate a linear model for a series of logical OR state PCSS	statements using
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### **Description**

model\_or approximates the linear model for the a disjunction of m phenotypes as a function of a set of predictors.

#### Usage

```
model_or(formula, n, means, covs, predictors, ...)
```

### **Arguments**

formula an object of class formula whose dependent variable is a combination of vari-

ables and logical | operators. All model terms must be accounted for in means

and covs.

n sample size.

means named vector of predictor and response means.

covs named matrix of the covariance of all model predictors and the responses.

predictors named list of objects of class predictor.

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

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#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

### **Examples**

```
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
predictors <- list(
    g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
    x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1)))
)
model_or(
    y4 | y5 ~ g1 + x1,
    means = means, covs = covs, n = n, predictors = predictors
)
summary(lm(y4 | y5 ~ g1 + x1, data = ex_data))</pre>
```

model\_prcomp

Model the principal component score of a set of phenotypes using PCSS

#### **Description**

model\_prcomp calculates the linear model for the mth principal component score of a set of phenotypes as a function of a set of predictors.

### Usage

```
model_prcomp(
  formula,
  comp = 1,
  n,
  means,
  covs,
  center = FALSE,
  standardize = FALSE,
  ...
)
```

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### Arguments

formula an object of class formula whose dependent variable is a series of variables

joined by + operators. model\_prcomp will treat a principal component score of those variables as the actual dependent variable. All model terms must be

accounted for in means and covs.

comp integer indicating which principal component score to analyze. Must be less

than or equal to the total number of phenotypes.

n sample size.

means named vector of predictor and response means.

covs named matrix of the covariance of all model predictors and the responses.

center logical. Should the dependent variables be centered before principal compo-

nents are calculated?

standardize logical. Should the dependent variables be standardized before principal com-

ponents are calculated?

... additional arguments

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call

terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'.

adj.r.squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

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#### **Examples**

```
ex_data <- pcsstools_example[c("g1", "x1", "x2", "y1", "y2", "y3")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)

model_prcomp(
    y1 + y2 + y3 ~ g1 + x1 + x2,
    comp = 1, n = n, means = means, covs = covs
)</pre>
```

model\_product

Approximate a linear model for a product using PCSS

#### **Description**

model\_product approximates the linear model for the product of m phenotypes as a function of a set of predictors.

### Usage

```
model_product(
  formula,
  n,
  means,
  covs,
  predictors,
  responses = NULL,
  response = "continuous",
  ...
)
```

#### **Arguments**

formula an object of class formula whose dependent variable is a combination of vari-

ables and \* operators. All model terms must be accounted for in means and

covs.

n sample size.

means named vector of predictor and response means.

covs named matrix of the covariance of all model predictors and the responses.

predictors named list of objects of class predictor

responses named list of objects of class predictor corresponding to all terms being mul-

tiplied in the response. Can be left NULL if only multiplying two terms

response character. Describe distribution of all product terms. Either "continuous" or

"binary". If "binary" different approximations of product means and vari-

ances are used.

... additional arguments

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#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used coefficients a px4 matrix with columns for the estimated coefficient, its standard error, tstatistic and corresponding (two-sided) p-value. sigma the square root of the estimated variance of the random error. df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of nonaliased coefficients, the last being the total number of coefficients. fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.  $R^2$ , the 'fraction of variance explained by the model'. r.squared

r.squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r.squared the above  $R^2$  statistic 'adjusted', penalizing for higher p. cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." *bioRxiv*. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

#### **Examples**

```
ex_data <- pcsstools_example[c("g1", "g2", "g3", "x1", "y4", "y5", "y6")]
head(ex_data)
means <- colMeans(ex_data)</pre>
covs <- cov(ex_data)</pre>
n <- nrow(ex_data)</pre>
predictors <- list(</pre>
  g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
  g2 = new_predictor_snp(maf = mean(ex_data$g2) / 2),
  g3 = new_predictor_snp(maf = mean(ex_data$g3) / 2),
  x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
responses <- lapply(means[c("y4", "y5", "y6")], new_predictor_binary)</pre>
model_product(
  y4 * y5 * y6 ~ g1 + g2 + g3 + x1,
  means = means, covs = covs, n = n,
  predictors = predictors, responses = responses, response = "binary"
)
summary(lm(y4 * y5 * y6 ~ g1 + g2 + g3 + x1, data = ex_data))
```

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model_singular	Model an individual phenotype using PCSS	
----------------	--	--

#### **Description**

model\_singular calculates the linear model for a singular phenotype as a function of a set of predictors.

### Usage

```
model_singular(formula, n, means, covs, ...)
```

#### **Arguments**

formula an object of class formula whose dependent variable is only variable. All model

terms must be accounted for in means and covs.

n sample size.

means named vector of predictor and response means.

covs named matrix of the covariance of all model predictors and the responses.

... additional arguments

### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'.

adj.r.squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

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#### References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

### **Examples**

```
ex_data <- pcsstools_example[c("g1", "x1", "y1")]
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)

model_singular(
    y1 ~ g1 + x1,
    n = n, means = means, covs = covs
)
summary(lm(y1 ~ g1 + x1, data = ex_data))</pre>
```

new\_predictor

Create an object of class "predictor"

### Description

Create an object of class "predictor"

### Usage

```
new_predictor(
   f = function() { },
   predictor_type = character(),
   lb,
   ub,
   support
)
```

### **Arguments**

```
f a function that gives the probability mass/distribution function of a random variable.

predictor_type a character describing the random variable. Either "discrete" or "continuous".

1b, ub if predictor_type == "continuous" double giving the lower/upper bound of the pdf f.

support if predictor_type == "discrete" vector of the support of the pmf for f.
```

### Value

```
an object of class "predictor".
```

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#### See Also

```
new_predictor_normal, new_predictor_snp and new_predictor_binary.
```

### **Examples**

```
new_predictor(
   f = function(x0) dnorm(x0, mean = 0, sd = 1),
   predictor_type = "continuous", lb = -Inf, ub = Inf)
```

new\_predictor\_binary Shortcut to create a predictor object for a binary variable

### Description

```
new_predictor_binary calls new_predictor
```

### Usage

```
new_predictor_binary(p)
```

### **Arguments**

р

probability of success (predictor mean)

## Value

```
an object of class "predictor".
```

### **Examples**

```
new_predictor_binary(p = 0.75)
```

### Description

```
{\tt new\_predictor\_normal\ calls\ new\_predictor}
```

### Usage

```
new_predictor_normal(mean, sd)
```

26 new\_predictor\_snp

### **Arguments**

mean predictor mean (double).

sd predictor standard deviation (double)

### Value

```
an object of class "predictor".
```

## **Examples**

```
new_predictor_normal(mean = 10, sd = 1)
```

new\_predictor\_snp

Shortcut to create a predictor object for a SNP's minor allele counts

### Description

```
{\tt new\_predictor\_snp}\ calls\ {\tt new\_predictor}
```

## Usage

```
new_predictor_snp(maf)
```

### Arguments

maf

minor allele frequency

## Value

```
an object of class "predictor".
```

## **Examples**

```
new_predictor_snp(maf = 0.3)
```

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pcsslm	Approximate a linear model using PCSS

Description

pcsslm approximates a linear model of a combination of variables using precomputed summary statistics.

### Usage

```
pcsslm(formula, pcss = list(), ...)
```

### Arguments

formula	an object of class formula whose dependent variable is a combination of variables and logical   operators. All model terms must have appropriate PCSS in pcss.
pcss	a list of precomputed summary statistics. In all cases, this should include n: the sample size, means: a named vector of predictor and response means, and covs: a named covariance matrix including all predictors and responses. See Details for more information.
	additional arguments. See Details for more information.

#### **Details**

pcsslm parses the input formula's dependent variable for functions such as sums (+), products (\*), or logical operators (| and &). It then identifies models the combination of variables using one of model\_combo, model\_product, model\_or, model\_and, or model\_prcomp.

Different precomputed summary statistics are needed inside pcss depending on the function that combines the dependent variable.

- For linear combinations (and principal component analysis), only n, means, and covs are required
- For products and logical combinations, the additional items predictors and responses are required. These are named lists of objects of class predictor generated by new\_predictor, with a predictor object for each independent variable in predictors and each dependent variable in responses. However, if only modeling the product or logical combination of only two variables, responses can be NULL without consequence.

If modeling a principal component score of a set of variables, include the argument comp where comp is an integer indicating which principal component score to analyze. Optional logical arguments center and standardize determine if responses should be centered and standardized before principal components are calculated.

If modeling a linear combination, include the argument phi, a named vector of linear weights for each variable in the dependent variable in formula.

If modeling a product, include the argument response, a character equal to either "continuous" or "binary". If "binary", specialized approximations are performed to estimate means and variances.

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#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n-p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

#### References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10.1101/2021.03.08.433979v1.

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." *Pacific Symposium on Biocomputing*, **25**, 719–730. ISSN 2335-6928, doi: 10.1142/9789811215636\_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." *Pacific Symposium on Biocomputing*, **24**, 391–402. ISSN 2335-6928, doi: 10.1142/9789813279827\_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

### See Also

```
model_combo, model_product, model_or, model_and, and model_prcomp.
```

#### **Examples**

```
## Principal Component Analysis
ex_data <- pcsstools_example[c("g1", "x1", "y1", "y2", "y3")]
pcss <- list(
  means = colMeans(ex_data),
  covs = cov(ex_data),
  n = nrow(ex_data)</pre>
```

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```
)
pcsslm(y1 + y2 + y3 \sim g1 + x1, pcss = pcss, comp = 1)
## Linear combination of variables
ex_data <- pcsstools_example[c("g1", "g2", "y1", "y2")]</pre>
pcss <- list(</pre>
 means = colMeans(ex_data),
 covs = cov(ex_data),
 n = nrow(ex_data)
)
pcsslm(y1 + y2 \sim g1 + g2, pcss = pcss, phi = c(1, -1))
summary(lm(y1 - y2 \sim g1 + g2, data = ex_data))
## Product of variables
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5", "y6")]</pre>
pcss <- list(</pre>
 means = colMeans(ex_data),
  covs = cov(ex_data),
  n = nrow(ex_data),
  predictors = list(
    g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
    x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
  responses = lapply(
    colMeans(ex_data)[3:length(colMeans(ex_data))],
    new_predictor_binary
  )
)
pcsslm(y4 * y5 * y6 ~ g1 + x1, pcss = pcss, response = "binary")
summary(lm(y4 * y5 * y6 ~ g1 + x1, data = ex_data))
## Disjunct (OR statement) of variables
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]</pre>
pcss <- list(</pre>
  means = colMeans(ex_data),
  covs = cov(ex_data),
 n = nrow(ex_data),
  predictors = list(
    g1 = new\_predictor\_snp(maf = mean(ex\_data$g1) / 2),
    x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
  )
pcsslm(y4 | y5 \sim g1 + x1, pcss = pcss)
summary(lm(y4 | y5 \sim g1 + x1, data = ex_data))
```

30 pcsstools\_example

pcsstools	pcsstools: Tools for Regression Using Pre-Computed Summary Statis- tics

### Description

The posstools package provides functions to describe various regression models using only precomputed summary statistics (PCSS) from genome-wide association studies (GWASs) and PCSS repositories.

#### **Details**

The main function of interest is pcsslm, which can be used to approximate linear models for various combinations of phenotypes using PCSS.

pcsstools\_example Simulated example data

### **Description**

A dataset containing simulated genetic data with 3 SNPs, 3 continuous covariates, and 6 continuous phenotypes.

### Usage

```
pcsstools_example
```

### **Format**

A data frame with 1000 rows and 12 columns:

g1,g2,g3 Minor allele counts at three sites

x1,x2,x3 Continuous covariates

y1,y2,y3 Continuous phenotypes

y4,y5,y6 Binary phenotypes

print.pcsslm 31

print.pcsslm Print an object of class pcsslm
--

#### **Description**

Prints a linear model fit through pre-computed summary statistics

### Usage

```
## $3 method for class 'pcsslm'
print(
    X,
    digits = max(3L, getOption("digits") - 3L),
    symbolic.cor = x$symbolic.cor,
    signif.stars = getOption("show.signif.stars"),
    ...
)
```

#### **Arguments**

x an object of class "pcsslm"
digits the number of significant digits to use when printing.
symbolic.cor logical. If TRUE, print the correlations in a symbolic form (see symnum) rather than as numbers.
signif.stars logical. If TRUE, 'significance stars' are printed for each coefficient.
... further arguments passed to or from other methods.

#### Value

an object of class "pcsslm".

An object of class "pcsslm" is a list containing at least the following components:

call the matched call terms the terms object used

coefficients a px4 matrix with columns for the estimated coefficient, its standard error, t-

statistic and corresponding (two-sided) p-value.

sigma the square root of the estimated variance of the random error.

df degrees of freedom, a 3-vector p, n - p, p\*, the first being the number of non-

aliased coefficients, the last being the total number of coefficients.

fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator

degrees of freedom.

r. squared  $R^2$ , the 'fraction of variance explained by the model'. adj.r. squared the above  $R^2$  statistic 'adjusted', penalizing for higher p.

cov.unscaled a pxp matrix of (unscaled) covariances of the coef[j], j = 1, ...p.

Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares

Error (SSE), and Sum of Squares Total (SST).

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## Author(s)

R Core Team and contributors worldwide. Modified by Jack Wolf

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