

Package ‘qfa’

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Type Package

Title Quantile-Frequency Analysis (QFA) of Time Series

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Description Quantile-frequency analysis (QFA) of univariate or multivariate time series based on the method of trigonometric quantile regression. See Li, T.-H. (2012) “Quantile periodograms”, *Journal of the American Statistical Association*, 107, 765–776, <[doi:10.1080/01621459.2012.682815](https://doi.org/10.1080/01621459.2012.682815)>; Li, T.-H. (2014) *Time Series with Mixed Spectra*, CRC Press, <[doi:10.1201/b15154](https://doi.org/10.1201/b15154)>; Li, T.-H. (2022) “Quantile Fourier transform, quantile series, and nonparametric estimation of quantile spectra”, <[doi:10.48550/arXiv.2211.05844](https://doi.org/10.48550/arXiv.2211.05844)>.

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License GPL (>= 2)

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qdft	<i>Quantile Discrete Fourier Transform (QDFT)</i>
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Description

This function computes quantile discrete Fourier transform (QDFT) for univariate or multivariate time series.

Usage

```
qdft(y, tau, n.cores = 1, cl = NULL)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of the quantile discrete Fourier transform of y

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y,tau)
# Make a cluster for repeated use
n.cores <- 2
cl <- parallel::makeCluster(n.cores)
parallel::clusterExport(cl, c("tqr.fit"))
doParallel::registerDoParallel(cl)
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y1,tau,n.cores=n.cores,cl=cl)
```

```

y2 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y2,tau,n.cores=n.cores,cl=c1)
parallel::stopCluster(cl)

```

qdft2qacf

Quantile Autocovariance Function (QACF)

Description

This function computes quantile autocovariance function (QACF) from QDFT.

Usage

```
qdft2qacf(y.qdft, return.qser = FALSE)
```

Arguments

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`
`return.qser` if TRUE, return quantile series (QSER) along with QACF

Value

matrix or array of quantile autocovariance function if `return.qser = FALSE` (default), else a list with the following elements:

`qacf` matrix or array of quantile autocovariance function
`qser` matrix or array of quantile series

Examples

```

# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qacf <- qdft2qacf(y.qdft)
plot(c(0:9),qacf[c(1:10),1],type='h',xlab="LAG",ylab="QACF")
qser <- qdft2qacf(y.qdft,return.qser=TRUE)$qser
plot(qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
plot(c(0:9),qacf[1,2,c(1:10),1],type='h',xlab="LAG",ylab="QACF")

```

qdft2qper

Quantile Periodogram and Cross-Periodogram (QPER)

Description

This function computes quantile periodogram/cross-periodogram (QPER) from QDFT.

Usage

```
qdft2qper(y.qdft)
```

Arguments

`y.qdft` matrix or array of QDFT from `qdft()`

Value

matrix or array of quantile periodogram/cross-periodogram

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qper <- qdft2qper(y.qdft)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(qper[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qper <- qdft2qper(y.qdft)
qfa.plot(ff[sel.f],tau,Re(qper[1,1,sel.f,]))
qfa.plot(ff[sel.f],tau,Re(qper[1,2,sel.f,]))
```

qdft2qser

Quantile Series (QSER)

Description

This function computes quantile series (QSER) from QDFT.

Usage

```
qdft2qser(y.qdft)
```

Arguments

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`

Value

matrix or array of quantile series

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qser <- qdft2qser(y.qdft)
plot(qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qser <- qdft2qser(y.qdft)
plot(qser[,1,1],type='l',xlab="TIME",ylab="QSER")
```

qfa.plot

Quantile-Frequency Plot

Description

This function creates an image plot of quantile spectrum.

Usage

```
qfa.plot(
  freq,
  tau,
  qper,
  rg.qper = range(qper),
  rg.tau = range(tau),
  rg.freq = c(0, 0.5),
  color = colorRamps::matlab.like2(1024),
  ylab = "QUANTILE LEVEL",
  xlab = "FREQUENCY",
  tlab = NULL,
  set.par = TRUE,
  legend.plot = TRUE
)
```

Arguments

freq	sequence of frequencies in (0,0.5) at which quantile spectrum is evaluated
tau	sequence of quantile levels in (0,1) at which quantile spectrum is evaluated
qper	real-valued matrix of quantile spectrum evaluated on the freq x tau grid
rg.qper	zlim for qper (default = range(qper))
rg.tau	ylim for tau (default = range(tau))
rg.freq	xlim for freq (default = c(0, 0.5))
color	colors (default = colorRamps::matlab.like2(1024))
ylab	label of y-axis (default = "QUANTILE LEVEL")
xlab	label of x-axis (default = "FREQUENCY")
tlab	title of plot (default = NULL)
set.par	if TRUE, par() is set internally (single image)
legend.plot	if TRUE, legend plot is added

Value

no return value

qkl.divergence *Kullback-Leibler Divergence of Quantile Spectral Estimate*

Description

This function computes Kullback-Leibler divergence (KLD) of quantile spectral estimate.

Usage

```
qkl.divergence(qper, qspec, sel.f = NULL, sel.tau = NULL)
```

Arguments

qper	matrix or array of quantile spectral estimate from, e.g., qspec.lw()
qspec	matrix of array of true quantile spectrum/cross-spectrum (same dimension as qper)
sel.f	index of selected frequencies for computation (default = NULL: all frequencies)
sel.tau	index of selected quantile levels for computation (default = NULL: all quantile levels)

Value

real number of Kullback-Leibler divergence

qper2qcoh	<i>Quantile Coherence Spectrum (QCOH)</i>
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Description

This function computes quantile coherence spectrum (QCOH) from quantile spectrum and cross-spectrum.

Usage

```
qper2qcoh(qspec, k = 1, kk = 2)
```

Arguments

qspec	array of quantile spectrum and cross-spectrum
k	index of first series (default = 1)
kk	index of second series (default = 2)

Value

matrix of quantile coherence evaluated at Fourier frequencies in (0,0.5)

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qcoh <- qper2qcoh(qper.lw,k=1,kk=2)
qfa.plot(ff[sel.f],tau,Re(qcoh))
```

qsmooth.qdft	<i>Quantile Smoothing of Quantile Discrete Fourier Transform</i>
--------------	--

Description

This function computes quantile-smoothed version of quantile discrete Fourier transform (QDFT).

Usage

```
qsmooth.qdft(
  y.qdft,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

<code>y.qdft</code>	matrix or array of QDFT from <code>qdft()</code>
<code>method</code>	smoothing method: "gamm" for <code>mgcv::gamm()</code> , "sp" for <code>stats::smooth.spline()</code>
<code>spar</code>	smoothing parameter in <code>smooth.spline()</code> (default = "GCV")
<code>n.cores</code>	number of cores for parallel computing (default = 1)
<code>cl</code>	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of quantile-smoothed QDFT

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qdft <- qsmooth.qdft(y.qdft,method="sp",spar=0.9)
qacf <- qdft2qacf(y.qdft)
qper.qslw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.qslw[1,1,sel.f,]))
```

qsmooth.qper

Quantile Smoothing of Quantile Periodogram or Spectral Estimate

Description

This function computes quantile-smoothed version of quantile periodogram/cross-periodogram (QPER) or other quantile spectral estimate.

Usage

```
qsmooth.qper(
  qper,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

qper	matrix or array of quantile periodogram/cross-periodogram or spectral estimate
method	smoothing method: "gamm" for <code>mgcv::gamm()</code> , "sp" for <code>stats::smooth.spline()</code>
spar	smoothing parameter in <code>smooth.spline()</code> (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of quantile-smoothed quantile spectral estimate

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.lw[1,1,sel.f,]))
qper.lwqs <- qsmooth.qper(qper.lw,method="sp",spar=0.9)
qfa.plot(ff[sel.f],tau,Re(qper.lwqs[1,1,sel.f,]))
```

qspec.lw

Lag-Window Estimator of Quantile Spectrum and Cross-Spectrum (QSPEC)

Description

This function computes lag-window (LW) estimate of quantile spectrum/cross-spectrum (QSPEC) from QACF.

Usage

```
qspec.lw(qacf, M = NULL)
```

Arguments

qacf matrix or array of QACF from `qdf t2qacf()`
M bandwidth parameter of lag window (default = NULL: quantile periodogram)

Value

A list with the following elements:

spec matrix or array of LW estimate
lw lag-window sequence

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(qper.lw[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.lw[1,2,sel.f,]))
```

sqdft

Spline Quantile Discrete Fourier Transform (SQDFT)

Description

This function computes spline quantile discrete Fourier transform (SQDFT) for univariate or multivariate time series.

Usage

```
sqdft(y, tau, c0 = 0.02, d = 4, weighted = FALSE, n.cores = 1, cl = NULL)
```

Arguments

y vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
tau sequence of quantile levels in (0,1)
c0 penalty parameter

d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of the spline quantile discrete Fourier transform of y

Examples

```

y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sqdft <- sqdft(y,tau,c0=0.02,d=4)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qacf <- qdft2qacf(y.sqdft)
qper.sqr1w <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.sqr1w[sel.f,]))

```

sqr.fit

Spline Quantile Regression (SQR)

Description

This function computes the spline quantile regression (SQR) solution given response vector and design matrix. It uses the code `rqfnb.f` in the "quantreg" package with the permission of Dr. R. Koenker.

Usage

```
sqr.fit(y, X, tau, c0, d = 1, weighted = FALSE, mthreads = FALSE)
```

Arguments

y	response vector
X	design matrix (nrow(X) = length(y))
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
mthreads	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)

Value

A list with the following elements:

coefficients	matrix of regression coefficients
nit	number of iterations

tqr.fit	<i>Trigonometric Quantile Regression (TQR)</i>
---------	--

Description

This function computes trigonometric quantile regression (TQR) for univariate time series at a single frequency.

Usage

```
tqr.fit(y, f0, tau, prepared = TRUE)
```

Arguments

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when $f0 = 0.5$

Value

object of `rq()` (coefficients in `$coef`)

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
plot(tau,fit$coef[1,],type='o',pch=0.75,xlab='QUANTILE LEVEL',ylab='TQR COEF')
```

 tsqr.fit

Trigonometric Spline Quantile Regression (TSQR)

Description

This function computes trigonometric spline quantile regression (TSQR) for univariate time series at a single frequency.

Usage

```
tsqr.fit(y, f0, tau, c0, d = 1, weighted = FALSE, prepared = TRUE)
```

Arguments

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

Value

object of `sqr.fit()` (coefficients in `$coef`)

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
fit.sqr <- tsqr.fit(y,f0=0.1,tau=tau,c0=0.02,d=4)
plot(tau,fit$coef[1,],type='p',xlab='QUANTILE LEVEL',ylab='TQR COEF')
lines(tau,fit.sqr$coef[1,],type='l')
```

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