# Package 'stcos'

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Type Package

Title Space-Time Change of Support

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URL https://github.com/holans/ST-COS

#### Description

Spatio-temporal change of support (STCOS) methods are designed for statistical inference on geographic and time domains which differ from those on which the data were observed. In particular, a parsimonious class of STCOS models supporting Gaussian outcomes was introduced by Bradley, Wikle, and Holan <doi:10.1002/sta4.94>. The 'stcos' package contains tools which facilitate use of STCOS models.

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acs\_sf

Shapes and ACS estimates for Boone County, MO.

## Description

An sf object with ACS estimates for:

- Boone County, Missouri
- Table B19013
- Block group level geography
- Years 2013 2017

## Usage

acs5\_2013 acs5\_2014 acs5\_2015 acs5\_2016 acs5\_2017

## Format

sf objects.

An object of class sf (inherits from data.frame) with 87 rows and 9 columns. An object of class sf (inherits from data.frame) with 87 rows and 9 columns. An object of class sf (inherits from data.frame) with 85 rows and 9 columns. An object of class sf (inherits from data.frame) with 87 rows and 9 columns. An object of class sf (inherits from data.frame) with 87 rows and 9 columns.

## Details

Shapefiles were gathered via the tigris package, and ACS estimates were downloaded from the American FactFinder http://factfinder.census.gov. Data were assembled on 2/28/2019. See data-prep-aff.R in the Columbia example code for details.

adjacency\_matrix Sparse adjacency matrix between two sets of areas.

## Description

A convenience function to convert output from sf::st\_touches to a sparse matrix as defined in the Matrix package.

#### Usage

```
adjacency_matrix(dom)
```

## Arguments

dom

An sf object representing a domain of areal units.

## Details

Returns a matrix A whose (i,j)th entry contains a 1 if areal units dom[i,] and dom[j,] are adjacent; 0 otherwise.

## Value

An adjacency matrix

```
data("acs_sf")
dom = acs5_2013[1:4,]
A = adjacency_matrix(dom)
```

areal\_spacetime\_bisquare

Areal Space-Time Bisquare Basis

## Description

Space-Time bisquare basis on areal data.

## Usage

areal\_spacetime\_bisquare(dom, period, knots, w\_s, w\_t, control = NULL)

## Arguments

dom	An sf or sfc object with areas $A_1, \ldots, A_n$ to evaluate.
period	A numeric vector of time periods $v_1, \ldots, v_m$ to evaluate for each area.
knots	Spatio-temporal knots $(c_1, g_1), \ldots, (c_r, g_r)$ for the basis. See "Details".
W_S	Spatial radius for the basis.
w_t	Temporal radius for the basis.
control	A list of control arguments. See "Details".

#### Details

Notes about arguments:

- knots may be provided as either an sf or sfc object, or as a matrix of points.
- If an sf or sfc object is provided for knots, r three-dimensional POINT entries are expected in st\_geometry(knots). Otherwise, knots will be interpreted as an  $r \times 3$  numeric matrix.
- If knots is an sf or sfc object, it is checked to ensure the coordinate system matches dom.

For each area A in the given domain, and time period  $\boldsymbol{v} = (v_1, \dots, v_m)$  compute the basis functions

$$\psi_j^{(m)}(A, \boldsymbol{v}) = \frac{1}{m} \sum_{k=1}^m \frac{1}{|A|} \int_A \psi_j(\boldsymbol{u}, v_k) d\boldsymbol{u},$$

for j = 1, ..., r. Here,  $\varphi_j(u, v)$  represent spacetime\_bisquare basis functions defined at the point level using  $c_j, g_j, w_s$ , and  $w_t$ .

The basis requires an integration which may be computed using one of two methods. The mc method uses a Monte Carlo approximation

$$\psi_j^{(m)}(A, \boldsymbol{v}) \approx \frac{1}{m} \sum_{k=1}^m \frac{1}{Q} \sum_{q=1}^Q \psi_j(\boldsymbol{u}_q, \boldsymbol{v}_k),$$

based on a random sample of locations  $u_1, \ldots, u_Q$  from a uniform distribution on area A. The rect method uses a simple quadrature approximation

$$\psi_j^{(m)}(A, \boldsymbol{v}) \approx \frac{1}{m} \sum_{k=1}^m \frac{1}{|A|} \sum_{a=1}^{n_x} \sum_{b=1}^{n_y} \psi_j(\boldsymbol{u}_{ab}, v_k) I(\boldsymbol{u}_{ab} \in A) \Delta_x \Delta_y.$$

Here, the bounding box st\_bbox(A) is divided evenly into a grid of  $n_x \times n_y$  rectangles, each of size  $\Delta_x \times \Delta_y$ . Each  $u_{ab} = (u_a, u_b)$  is a point from the (a, b)th rectangle, for  $a = 1, \ldots, n_x$  and  $b = 1, \ldots, n_y$ .

Due to the treatment of  $A_i$  and  $c_j$  as objects in a Euclidean space, this basis is more suitable for coordinates from a map projection than coordinates based on a globe representation.

The control argument is a list which may provide any of the following:

- method specifies computation method: mc or rect. Default is mc.
- mc\_reps is number of repetitions to use for mc. Default is 1000.
- nx is number of x-axis points to use for rect method. Default is 50.
- ny is number of y-axis points to use for rect method. Default is 50.
- report\_period is an integer; print a message with progress each time this many areas are processed. Default is Inf so that message is suppressed.
- verbose is a logical; if TRUE print descriptive messages about the computation. Default is FALSE.
- mc\_sampling\_factor is a positive number; an oversampling factor used to compute blocksize in the rdomain function. I.e., blocksize = ceiling(mc\_sampling\_factor \* mc\_reps). Default is 1.2.

#### Value

A sparse  $n \times r$  matrix whose *i*th row is  $\mathbf{s}_i^{\top} = \left(\psi_1^{(m)}(A_i), \dots, \psi_r^{(m)}(A_i)\right)$ .

## See Also

Other bisquare: areal\_spatial\_bisquare(), spacetime\_bisquare(), spatial\_bisquare()

```
set.seed(1234)
# Create knot points
seq_x = seq(0, 1, length.out = 3)
seq_y = seq(0, 1, length.out = 3)
knots = expand.grid(x = seq_x, y = seq_y, t = seq_t)
knots_sf = st_as_sf(knots, coords = c("x","y","t"), crs = NA, dim = "XYM", agr = "constant")
# Create a simple domain (of rectangles) to evaluate
shape1 = matrix(c(0.0,0.0, 0.5,0.0, 0.5,0.5, 0.0,0.5, 0.0,0.0), ncol=2, byrow=TRUE)
shape2 = shape1 + cbind(rep(0.5,5), rep(0.0,5))
shape3 = shape1 + cbind(rep(0.0,5), rep(0.5,5))
```

```
shape4 = shape1 + cbind(rep(0.5,5), rep(0.5,5))
sfc = st_sfc(
  st_polygon(list(shape1)),
   st_polygon(list(shape2)),
   st_polygon(list(shape3)),
   st_polygon(list(shape4))
)
dom = st_sf(data.frame(geoid = 1:length(sfc), geom = sfc))
rad = 0.5
period = c(0.4, 0.7)
areal_spacetime_bisquare(dom, period, knots, w = rad, w_t = 1)
areal_spacetime_bisquare(dom, period, knots_sf, w_s = rad, w_t = 1)
# Plot the (spatial) knots and the (spatial) domain at which we evaluated
# the basis.
plot(knots[,1], knots[,2], pch = 4, cex = 1.5, col = "red")
plot(dom[,1], col = NA, add = TRUE)
# Draw a circle representing the basis' radius around one of the knot points
tseq = seq(0, 2*pi, length=100)
coords = cbind(rad * cos(tseq) + seq_x[2], rad * sin(tseq) + seq_y[2])
lines(coords, col = "red")
```

areal\_spatial\_bisquare

Areal Spatial Bisquare Basis

## Description

Spatial bisquare basis on areal data.

#### Usage

```
areal_spatial_bisquare(dom, knots, w, control = NULL)
```

## Arguments

dom	An sf or sfc object with areas $A_1, \ldots, A_n$ to evaluate.
knots	Knots $c_1, \ldots, c_r$ for the basis. See "Details".
w	Radius for the basis.
control	A list of control arguments. See "Details".

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## Details

Notes about arguments:

- knots may be provided as either an sf or sfc object, or as a matrix of points.
- If an sf or sfc object is provided for knots, r two-dimensional POINT entries are expected in st\_geometry(knots). Otherwise, knots will be interpreted as an  $r \times 2$  numeric matrix.
- If knots is an sf or sfc object, it is checked to ensure the coordinate system matches dom.

For each area A in the given domain, compute an the basis functions

$$\bar{\varphi}_j(A) = \frac{1}{|A|} \int_A \varphi_j(\boldsymbol{u}) d\boldsymbol{u}$$

for j = 1, ..., r. Here,  $\varphi_j(u)$  represent spatial\_bisquare basis functions defined at the point level using  $c_j$  and w.

The basis requires an integration which may be computed using one of two methods. The mc method uses

$$\bar{\varphi}_j(A) \approx \frac{1}{Q} \sum_{q=1}^Q \varphi_j(\boldsymbol{u}_q),$$

based on a random sample of locations  $u_1, \ldots, u_Q$  from a uniform distribution on area A. The rect method uses a simple quadrature approximation

$$\bar{\varphi}_j(A) \approx \frac{1}{|A|} \sum_{a=1}^{n_x} \sum_{b=1}^{n_y} \varphi_j(\boldsymbol{u}_{ab}) I(\boldsymbol{u}_{ab} \in A) \Delta_x \Delta_y.$$

Here, the bounding box st\_bbox(A) is divided evenly into a grid of  $n_x \times n_y$  rectangles, each of size  $\Delta_x \times \Delta_y$ . Each  $u_{ab} = (u_a, u_b)$  is a point from the (a, b)th rectangle, for  $a = 1, \ldots, n_x$  and  $b = 1, \ldots, n_y$ .

Due to the treatment of  $A_i$  and  $c_j$  as objects in a Euclidean space, this basis is more suitable for coordinates from a map projection than coordinates based on a globe representation.

The control argument is a list which may provide any of the following:

- method specifies computation method: mc or rect. Default is mc.
- mc\_reps is number of repetitions to use for mc. Default is 1000.
- nx is number of x-axis points to use for rect method. Default is 50.
- ny is number of y-axis oints to use for rect method. Default is 50.
- report\_period is an integer; print a message with progress each time this many areas are processed. Default is Inf so that message is suppressed.
- verbose is a logical; if TRUE print descriptive messages about the computation. Default is FALSE.
- mc\_sampling\_factor is a positive number; an oversampling factor used to compute blocksize in the rdomain function. I.e., blocksize = ceiling(mc\_sampling\_factor \* mc\_reps). Default is 1.2.

#### Value

A sparse  $n \times r$  matrix whose *i*th row is  $\boldsymbol{s}_i^{\top} = (\bar{\varphi}_1(A_i), \dots, \bar{\varphi}_r(A_i)).$ 

## See Also

Other bisquare: areal\_spacetime\_bisquare(), spacetime\_bisquare(), spatial\_bisquare()

## Examples

set.seed(1234)

```
# Create knot points
seq_x = seq(0, 1, length.out = 3)
seq_y = seq(0, 1, length.out = 3)
knots = expand.grid(x = seq_x, y = seq_y)
knots_sf = st_as_sf(knots, coords = c("x","y"), crs = NA, agr = "constant")
# Create a simple domain (of rectangles) to evaluate
shape1 = matrix(c(0.0,0.0, 0.5,0.0, 0.5,0.5, 0.0,0.5, 0.0,0.0), ncol=2, byrow=TRUE)
shape2 = shape1 + cbind(rep(0.5,5), rep(0.0,5))
shape3 = shape1 + cbind(rep(0.0,5), rep(0.5,5))
shape4 = shape1 + cbind(rep(0.5,5), rep(0.5,5))
sfc = st_sfc(
   st_polygon(list(shape1)),
   st_polygon(list(shape2)),
   st_polygon(list(shape3)),
   st_polygon(list(shape4))
)
dom = st_sf(data.frame(geoid = 1:length(sfc), geom = sfc))
rad = 0.5
areal_spatial_bisquare(dom, knots, rad)
areal_spatial_bisquare(dom, knots_sf, rad)
# Plot the knots and the points at which we evaluated the basis
plot(knots[,1], knots[,2], pch = 4, cex = 1.5, col = "red")
plot(dom[,1], col = NA, add = TRUE)
# Draw a circle representing the basis' radius around one of the knot points
tseq = seq(0, 2*pi, length=100)
coords = cbind(rad * cos(tseq) + seq_x[2], rad * sin(tseq) + seq_y[2])
lines(coords, col = "red")
```

autocov\_VAR1

*Compute the autocovariance matrix for a VAR(1) process.* 

#### Description

Compute the autocovariance matrix for a VAR(1) process.

## car\_precision

## Usage

autocov\_VAR1(A, Sigma, lag\_max)

### Arguments

A	Coefficient matrix A of the autoregression term.
Sigma	Covariance matrix $\Sigma$ of the errors.
lag_max	maximum number of lags to compute.

## Details

Computes the autocovariance matrix  $\Gamma(h)$  of the *m*-dimensional VAR(1) process

$$Y_t = AY_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \Sigma)$$

For the required computation of  $\Gamma(0)$ , this function solves the  $m^2 \times m^2$  system

$$\operatorname{vec}(\boldsymbol{\Gamma}(0)) = [\boldsymbol{I} - \boldsymbol{A} \otimes \boldsymbol{A}]^{-1}\operatorname{vec}(\boldsymbol{\Sigma}).$$

without directly computing  $m^2 \times m^2$  matrices.

## Value

An array Gamma of dimension  $c(m, m, lag_max + 1)$ , where the slice Gamma[,,h] represents the autocovariance at lag h = 0, 1, ..., lag\_max.

## Examples

```
U = matrix(NA, 3, 3)
U[,1] = c(1, 1, 1) / sqrt(3)
U[,2] = c(1, 0, -1) / sqrt(2)
U[,3] = c(0, 1, -1) / sqrt(2)
B = U %*% diag(c(0.5, 0.2, 0.1)) %*% t(U)
A = (B + t(B)) / 2
Sigma = diag(x = 2, nrow = 3)
autocov_VAR1(A, Sigma, lag_max = 5)
```

car\_precision CAR Precision Matrix

## Description

A convenience function to compute the CAR precision matrix based on a given adjacency matrix.

#### Usage

```
car_precision(A, tau = 1, scale = FALSE)
```

#### Arguments

A	An adjacency matrix.
tau	The CAR dependency parameter $\tau \in [0,1]$ . See "Value". Default: 1.
scale	Whether to scale matrix entries. See "Value". Default: FALSE.

#### Details

Suppose A is an  $n \times n$  adjacency matrix and  $D = \text{Diag}(A1) = \text{Diag}(a_{1+}, \ldots, a_{n+})$ . If scale is FALSE, return the CAR precision matrix

$$\boldsymbol{Q} = \boldsymbol{D} - \tau \boldsymbol{A}.$$

If scale is TRUE, return a scaled version

$$\tilde{\boldsymbol{Q}} = \boldsymbol{D}^{-1} \boldsymbol{Q}.$$

An error is thrown if scale = TRUE and any of  $\{a_{1+}, \ldots, a_{n+}\}$  are equal to 0. Taking  $\tau = 1$  corresponds to the Intrinsic CAR precision matrix.

Typically in a modeling context, the precision matrix will be multiplied by a scaling parameter; e.g., a CAR model for random effects  $\phi$  could be

$$f(\boldsymbol{\phi} \mid \boldsymbol{\alpha}) \propto \boldsymbol{\alpha}^{-q} \exp\left\{-\frac{1}{2\alpha^2} \boldsymbol{\phi}^{\top} \boldsymbol{Q} \boldsymbol{\phi}\right\}.$$

where  $q = \operatorname{rank}(Q)$ .

## Value

CAR precision matrix.

## Examples

```
data("acs_sf")
dom = acs5_2013[1:4,]
A = adjacency_matrix(dom)
Q = car_precision(A)
```

columbia\_neighbs City of Columbia neighborhoods.

## Description

An sf object containing the geometry of four neighborhoods in the City of Columbia, Boone County, Missouri. Based on shapefiles provided by the Office of Information Technology / GIS, City of Columbia, Missouri.

## Covariance Approximation

#### Usage

columbia\_neighbs

### Format

An sf object with 4 features (neighborhoods).

Covariance Approximation

Best Approximation to Covariance Structure

#### Description

Compute the best positive approximant for use in the STCOS model, under several prespecified covariance structures.

## Usage

```
cov_approx_randwalk(Delta, S)
```

cov\_approx\_blockdiag(Delta, S)

#### Arguments

Delta	Covariance $(n \times n)$ for observations within a time point for the process whose
	variance we wish to approximate.
S	Design matrix $(N \times r)$ of basis functions evaluated on the fine-level process over
	T = N/n time points.

## Details

Let  $\Sigma$  be an  $N \times N$  symmetric and positive-definite covariance matrix and S be an  $N \times r$  matrix with rank r. The objective is to compute a matrix K which minimizes the Frobenius norm

$$\|\Sigma - SCS^{\top}\|_{\mathrm{F}},$$

over symmetric positive-definite matrices C. The solution is given by

$$\boldsymbol{K} = (\boldsymbol{S}^{\top}\boldsymbol{S})^{-1}\boldsymbol{S}^{\top}\boldsymbol{\Sigma}\boldsymbol{S}(\boldsymbol{S}^{\top}\boldsymbol{S})^{-1}.$$

In the STCOS model, S represents the design matrix from a basis function computed from a finelevel support having n areas, using T time steps. Therefore N = nT represents the dimension of covariance for the fine-level support.

We provide functions to handle some possible structures for target covariance matrices of the form

$$\boldsymbol{\Sigma} = \left( \begin{array}{ccc} \boldsymbol{\Gamma}(1,1) & \cdots & \boldsymbol{\Gamma}(1,T) \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Gamma}(T,1) & \cdots & \boldsymbol{\Gamma}(T,T) \end{array} \right),$$

where each  $\Gamma(s, t)$  is an  $n \times n$  matrix.

- cov\_approx\_randwalk assumes  $\Sigma$  is based on the autocovariance function of a random walk

$$Y_{t+1} = Y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \Delta).$$

so that

$$\mathbf{\Gamma}(s,t) = \min(s,t)\mathbf{\Delta}.$$

- cov\_approx\_blockdiag assumes  $\Sigma$  is based on

$$Y_{t+1} = Y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \Delta).$$

which are independent across t, so that

$$\Gamma(s,t) = I(s=t)\Delta,$$

The block structure is used to reduce the computational burden, as N may be large.

DIC

Deviance Information Criterion

## Description

Generic function to calculate Deviance Information Criterion (DIC) for a given model object.

## Usage

DIC(object, ...)

## Arguments

object	A fitted model object.
	Additional arguments.

## Value

A numeric value of the DIC.

gibbs\_stcos

## Description

Gibbs Sampler for STCOS Model

## Usage

```
gibbs_stcos(
 z,
  v,
 Η,
  S,
 Kinv,
 R,
  report_period = R + 1,
  burn = 0,
  thin = 1,
  init = NULL,
  fixed = NULL,
  hyper = NULL
)
## S3 method for class 'stcos_gibbs'
logLik(object, ...)
## S3 method for class 'stcos_gibbs'
DIC(object, ...)
## S3 method for class 'stcos_gibbs'
print(x, ...)
## S3 method for class 'stcos_gibbs'
fitted(object, H, S, ...)
## S3 method for class 'stcos_gibbs'
predict(object, H, S, ...)
```

## Arguments

z	Vector which represents the outcome; assumed to be direct estimates from the
	survey.
v	Vector which represents direct variance estimates from the survey.
Н	Matrix of overlaps between source and fine-level supports.
S	Design matrix for basis decomposition.

Kinv	The precision matrix $oldsymbol{K}^{-1}$ of the random coefficient $oldsymbol{\eta}$
R	Number of MCMC reps.
report_period	Gibbs sampler will report progress each time this many iterations are completed.
burn	Number of the R draws to discard at the beginning of the chain.
thin	After burn-in period, save one out of every thin draws.
init	A list containing the following initial values for the MCMC: sig2mu, sig2xi, sig2K, muB, eta, xi. Any values which are not specified are set to arbitrary choices.
fixed	A list specifying which parameters to keep fixed in the MCMC. This can nor- mally be left blank. If elements sig2mu, sig2xi, or sig2K are specified they should be boolean, where TRUE means fixed (i.e. not drawn). If elements muB, eta, or xi are specified, they should each be a vector of indicies; the specified indices are to be treated as fixed (i.e. not drawn).
hyper	A list containing the following hyperparameter values: a_sig2mu, a_sig2K, a_sig2xi, b_sig2mu, b_sig2K, b_sig2xi. Any hyperparameters which are not specified are set to a default value of 2.
object	A result from gibbs_stcos.
	Additional arguments.
x	A result from gibbs_stcos.

## Details

Fits the model

$$\begin{split} \boldsymbol{Z} &= \boldsymbol{H}\boldsymbol{\mu}_B + \boldsymbol{S}\boldsymbol{\eta} + \boldsymbol{\xi} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathrm{N}(0, \boldsymbol{V}), \\ \boldsymbol{\eta} &\sim \mathrm{N}(\boldsymbol{0}, \sigma_K^2 \boldsymbol{K}), \quad \boldsymbol{\xi} \sim \mathrm{N}(0, \sigma_{\xi}^2 \boldsymbol{I}), \\ \boldsymbol{\mu}_B &\sim \mathrm{N}(\boldsymbol{0}, \sigma_{\mu}^2 \boldsymbol{I}), \quad \sigma_{\mu}^2 \sim \mathrm{IG}(a_{\mu}, b_{\mu}), \\ \sigma_K^2 &\sim \mathrm{IG}(a_K, b_K), \quad \sigma_{\xi}^2 \sim \mathrm{IG}(a_{\xi}, b_{\xi}), \end{split}$$

using a Gibbs sampler with closed-form draws.

Helper functions produce the following outputs:

- logLik computes the log-likelihood for each saved draw.
- DIC computes the Deviance information criterion for each saved draw.
- print displays a summary of the draws.
- fitted computes the mean  $E(Y_i)$  for each observation i = 1, ..., n, for each saved draw.
- predict draws  $Y_i$  for each observation i = 1, ..., n, using the parameter values for each saved Gibbs sampler draw.

#### Value

gibbs\_stcos returns an stcos object which contains draws from the sampler. Helper functions take this object as an input and produce various outputs (see details).

## licols

## Examples

licols licols

## Description

Extract a linearly independent set of columns of a matrix.

## Usage

licols(X, tol = 1e-10, quiet = FALSE)

## Arguments

Х	A matrix.
tol	A tolerance for rank estimation. Default is 1e-10.
quiet	logical; if FALSE, print a warning about computation time if X is large.

## Details

An R version of a Matlab licols function given in this MathWorks forum post.

## Value

Xsub contains the extracted columns of X and idx contains the indices (into X) of those columns. The elapsed time is stored in elapsed.sec.

```
x = 0:19 %% 3 + 1
Z = model.matrix(~ as.factor(x) - 1)
X = cbind(1, Z)
licols(X)
```

mle\_stcos

## Description

MLE for STCOS Model

## Usage

```
mle_stcos(
    z,
    v,
    H,
    S,
    K,
    init = NULL,
    optim_control = list(),
    optim_method = "L-BFGS-B"
)
```

## Arguments

Z	Vector which represents the outcome; assumed to be direct estimates from the survey.
V	Vector which represents direct variance estimates from the survey. The diagonal of the matrix $V$ described in the details.
Н	Matrix of overlaps between source and fine-level supports.
S	Design matrix for basis decomposition.
К	Variance of the random coefficient $\eta$
init	A list containing the initial values in the MCMC for sig2xi and sig2K. If not specified, we select an arbitrary initial value.
optim_control	This is passed as the control argument to optim. Note that the value fnscale is ignored if specified.
optim_method	Method to be used for likelihood maximization by optim. Default is L-BFGS-B.

## Details

Maximize the likelihood of the STCOS model

$$f(\boldsymbol{z} \mid \boldsymbol{\mu}_B, \sigma_K^2, \sigma_{\boldsymbol{\xi}}^2) = \mathbf{N}(\boldsymbol{z} \mid \boldsymbol{H}\boldsymbol{\mu}_B, \boldsymbol{\Delta}), \quad \boldsymbol{\Delta} = \sigma_{\boldsymbol{\xi}}^2 \boldsymbol{I} + \boldsymbol{V} + \sigma_K^2 \boldsymbol{S} \boldsymbol{K} \boldsymbol{S}^\top,$$

by numerical maximization of the profile likelihood

$$\ell(\sigma_K^2, \sigma_\xi^2) = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log|\mathbf{\Delta}| - \frac{1}{2}(\mathbf{z} - \mathbf{H}\hat{\boldsymbol{\mu}}_B)^{\top}\mathbf{\Delta}^{-1}(\mathbf{z} - \mathbf{H}\hat{\boldsymbol{\mu}}_B)$$

using  $\hat{\boldsymbol{\mu}}_B = (\boldsymbol{H}^{\top} \boldsymbol{\Delta}^{-1} \boldsymbol{H})^{-1} \boldsymbol{H}^{\top} \boldsymbol{\Delta}^{-1} \boldsymbol{z}.$ 

## overlap\_matrix

## Value

A list containing maximum likelihood estimates.

#### Examples

```
## Not run:
demo = prepare_stcos_demo()
mle_out = mle_stcos(demo$z, demo$v, demo$S, demo$H, demo$K)
sig2K_hat = mle_out$sig2K_hat
sig2xi_hat = mle_out$sig2xi_hat
mu_hat = mle_out$mu_hat
```

```
## End(Not run)
```

overlap\_matrix *Matrix of overlaps between two sets of areas.* 

## Description

A convenience function to convert output from sf::st\_intersection to a sparse matrix as defined in the Matrix package.

## Usage

overlap\_matrix(dom1, dom2, proportion = TRUE)

## Arguments

dom1	An sf object representing a domain of areal units.
dom2	An sf object representing a domain of areal units.
proportion	Logical; if TRUE, normalize so that rows sum to 1. Otherwise areas are returned.

## Details

Returns a matrix H whose (i,j)th entry represent the area of the overlap between areal units dom1[i,] and dom2[j,].

## Value

An matrix of overlaps.

```
data("acs_sf")
dom1 = acs5_2013[1:10,]
dom2 = acs5_2016[1:10,]
H1 = overlap_matrix(dom1, dom2)
H2 = overlap_matrix(dom1, dom2, proportion = FALSE)
```

prepare\_stcos\_demo Prepare Demo Data for STCOS Model

## Description

Create demo data based on ACS example, making a few simple model choices. The purpose of this function is to facilitate examples in other functions. Uses functions in the package to create model terms from shapefiles.

## Usage

```
prepare_stcos_demo(num_knots_sp = 200, basis_mc_reps = 200, eigval_prop = 0.65)
```

## Arguments

num_knots_sp	Number of spatial knots to use in areal space-time basis.
basis_mc_reps	Number of monte carlo reps to use in areal space-time basis.
eigval_prop	Proportion of variability to keep in dimension reduction of basis expansions.

## Value

A list containing the following:

- z direct estimates.
- v direct variance estimates.
- H overlap matrix.
- S design matrix of basis expansion.
- K covariance matrix of the random effect.

```
## Not run:
out = prepare_stcos_demo()
## End(Not run)
```

## Description

An alternative to sf::st\_sample which draws uniformly distributed points using a simple acceptreject method.

## Usage

rdomain(n, dom, blocksize = n, itmax = Inf)

## Arguments

n	Number of points desired in the final sample.
dom	An sf object representing a domain of areal units.
blocksize	Number of candidate points to draw on each pass of accept-reject sampling (see details). Defaults to n.
itmax	Maximum number of accept-reject samples to attempt. Defaults to Inf.

#### Details

Draws a sample of blocksize points uniformly from a bounding box on dom, and accepts only the points which belong to dom. This yields a uniform sample on dom. The process is repeated until n accepted draws are obtained, or until it has been attempted itmax times. If itmax iterations are reached without accepting n draws, an error is thrown.

This seems to be an order of magnitude faster than the current implementation of st\_sample, although the latter can accomplish the same objective and is more general. The improved performance is worthwhile when used in the areal basis functions, which sample repeatedly from the domain.

Performance will degrade when areal units have small area relative to their bounding box, as many candidate points may need to be discarded. For example, this will occur if dom contains a set of small scattered islands in an ocean. In this case, it would be more efficient to sample from each island at a time.

#### Value

An sf object with 2-dimensional points.

#### Examples

dom = acs5\_2013[c(1,5,8,12),]
pts = rdomain(10000, dom)

spacetime\_bisquare Space-Time Bisquare Basis

## Description

Space-time bisquare basis on point data.

#### Usage

spacetime\_bisquare(dom, knots, w\_s, w\_t)

## Arguments

dom	Space-time points $(u_1, v_1), \ldots, (u_n, v_n)$ to evaluate. See "Details".
knots	Spatio-temporal knots $(c_1, g_1), \ldots, (c_r, g_r)$ for the basis. See "Details".
W_S	Spatial radius for the basis.
w_t	Temporal radius for the basis.

## Details

Notes about arguments:

- Both dom and knots may be provided as either sf or sfc objects, or as matrices of points.
- If an sf or sfc object is provided for dom, n three-dimensional POINT entries are expected in st\_geometry(dom). Otherwise, dom will be interpreted as an  $n \times 3$  numeric matrix.
- If an sf or sfc object is provided for knots, r three-dimensional POINT entries are expected in st\_geometry(knots). Otherwise, knots will be interpreted as an  $r \times 3$  numeric matrix.
- If both dom and knots\_s are given as sf or sfc objects, they will be checked to ensure a common coordinate system.

For each  $(u_i, v_i)$ , compute the basis functions

$$\psi_j(\boldsymbol{u}, v) = \left[2 - \frac{\|\boldsymbol{u} - \boldsymbol{c}_j\|^2}{w_s^2} - \frac{|v - g_j|^2}{w_t^2}\right]^2 \cdot I(\|\boldsymbol{u} - \boldsymbol{c}_j\| \le w_s) \cdot I(|v - g_j| \le w_t)$$

for j = 1, ..., r.

Due to the treatment of  $u_i$  and  $c_j$  as points in a Euclidean space, this basis is more suitable for coordinates from a map projection than coordinates based on a globe representation.

#### Value

A sparse  $n \times r$  matrix whose *i*th row is

$$\boldsymbol{s}_i^{\top} = \Big(\psi_1(\boldsymbol{u}_i, v_i), \dots, \psi_r(\boldsymbol{u}_i, v_i)\Big).$$

#### spatial\_bisquare

## See Also

Other bisquare: areal\_spacetime\_bisquare(), areal\_spatial\_bisquare(), spatial\_bisquare()

## Examples

```
set.seed(1234)
# Create knot points
seq_x = seq(0, 1, length.out = 3)
seq_y = seq(0, 1, length.out = 3)
seq_t = seq(0, 1, length.out = 3)
knots = expand.grid(x = seq_x, y = seq_y, t = seq_t)
knots_sf = st_as_sf(knots, coords = c("x", "y", "t"), crs = NA, dim = "XYM", agr = "constant")
# Points to evaluate
x = runif(50)
y = runif(50)
t = sample(1:3, size = 50, replace = TRUE)
pts = data.frame(x = x, y = y, t = t)
dom = st_as_sf(pts, coords = c("x", "y", "t"), crs = NA, dim = "XYM", agr = "constant")
rad = 0.5
spacetime_bisquare(cbind(x,y,t), knots, w_s = rad, w_t = 1)
spacetime_bisquare(dom, knots_sf, w_s = rad, w_t = 1)
# Plot the (spatial) knots and the points at which we evaluated the basis
plot(knots[,1], knots[,2], pch = 4, cex = 1.5, col = "red")
text(x, y, labels = t, cex = 0.75)
# Draw a circle representing the basis' radius around one of the knot points
tseq = seq(0, 2*pi, length=100)
coords = cbind(rad * cos(tseq) + seq_x[2], rad * sin(tseq) + seq_y[2])
lines(coords, col = "red")
```

spatial\_bisquare Spatial Bisquare Basis

#### Description

Spatial bisquare basis on point data.

#### Usage

spatial\_bisquare(dom, knots, w)

#### Arguments

dom	Points $u_1, \ldots, u_n$ to evaluate. See "Details".
knots	Knots $c_1, \ldots, c_r$ for the basis. See "Details".
W	Radius for the basis.

#### Details

Notes about arguments:

- Both dom and knots may be provided as either sf or sfc objects, or as matrices of points.
- If an sf or sfc object is provided for dom, n two-dimensional POINT entries are expected in st\_geometry(dom). Otherwise, dom will be interpreted as an  $n \times 2$  numeric matrix.
- If an sf or sfc object is provided for knots, r two-dimensional POINT entries are expected in st\_geometry(knots). Otherwise, knots will be interpreted as an  $r \times 2$  numeric matrix.
- If both dom and knots are given as sf or sfc objects, they will be checked to ensure a common coordinate system.

0

For each  $u_i$ , compute the basis functions

$$\varphi_j(\boldsymbol{u}) = \left[1 - \frac{\|\boldsymbol{u} - \boldsymbol{c}_j\|^2}{w^2}\right]^2 \cdot I(\|\boldsymbol{u} - \boldsymbol{c}_j\| \le w)$$

for j = 1, ..., r.

Due to the treatment of  $u_i$  and  $c_j$  as points in a Euclidean space, this basis is more suitable for coordinates from a map projection than coordinates based on a globe representation.

#### Value

A sparse  $n \times r$  matrix whose *i*th row is  $\boldsymbol{s}_i^{\top} = (\varphi_1(\boldsymbol{u}_i), \dots, \varphi_r(\boldsymbol{u}_i)).$ 

## See Also

Other bisquare: areal\_spacetime\_bisquare(), areal\_spatial\_bisquare(), spacetime\_bisquare()

```
set.seed(1234)
# Create knot points
seq_x = seq(0, 1, length.out = 3)
seq_y = seq(0, 1, length.out = 3)
knots = expand.grid(x = seq_x, y = seq_y)
knots_sf = st_as_sf(knots, coords = c("x","y"), crs = NA, agr = "constant")
# Points to evaluate
x = runif(50)
y = runif(50)
pts = data.frame(x = x, y = y)
dom = st_as_sf(pts, coords = c("x","y"), crs = NA, agr = "constant")
```

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```
rad = 0.5
spatial_bisquare(cbind(x,y), knots, rad)
spatial_bisquare(dom, knots, rad)
# Plot the knots and the points at which we evaluated the basis
plot(knots[,1], knots[,2], pch = 4, cex = 1.5, col = "red")
points(x, y, cex = 0.5)
# Draw a circle representing the basis' radius around one of the knot points
tseq = seq(0, 2*pi, length=100)
coords = cbind(rad * cos(tseq) + seq_x[2], rad * sin(tseq) + seq_y[2])
lines(coords, col = "red")
```

```
stcos
```

stcos: Space-Time Change of Support

#### Description

An R Package for Space-Time Change of Support (STCOS) modeling.

## Details

Supports building and running STCOS and related models. A guide on package use is given in <arXiv:1904.12092>.

## References

Jonathan R. Bradley, Christopher K. Wikle, and Scott H. Holan (2015). Spatio-temporal change of support with application to American Community Survey multi-year period estimates. STAT 4 pp.255-270. https://doi.org/10.1002/sta4.94.

Andrew M. Raim, Scott H. Holan, Jonathan R. Bradley, and Christopher K. Wikle (2017). A model selection study for spatio-temporal change of support. In JSM Proceedings, Government Statistics Section. Alexandria, VA: American Statistical Association, pp.1524-1540.

Andrew M. Raim, Scott H. Holan, Jonathan R. Bradley, and Christopher K. Wikle (2020+). Spatio-Temporal Change of Support Modeling with R. https://arxiv.org/abs/1904.12092.

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